

Version
October 2013

Add-on Module

RF-CONCRETE Surfaces

Reinforced Concrete Design

Program Description

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1. Introduction

1.1 Add-on module RF-CONCRETE Surfaces

Although reinforced concrete is as frequently used for plate structures as for frameworks, standards and technical literature provide rather little information on the design of two-dimensional structural components. In particular, shell structures that are simultaneously subjected to moments and axial forces are rarely described in reference books. Since the finite element method allows for realistic modeling of two-direction objects, design assumptions and algorithms must be found to close this "regulatory gap" between member-oriented rules and computer-generated internal forces of plate structures.

DLUBAL SOFTWARE GMBH meets this challenge with the add-on module RF-CONCRETE Surfaces. Based on the compatibility equations by BAUMANN from 1972, a consistent design algorithm has been developed to dimension reinforcements with two and three directions of reinforcement. The module is more than just a tool for determining the statically required reinforcement: RF-CONCRETE Surfaces also includes regulations concerning the allowable minimum and maximum reinforcement ratios for different types of structural components (2D plates, 3D shells, walls, deep beams), as they can be found in the form of design specifications defined in the standards.

In the determination of reinforcing steel, **RF-CONCRETE Surfaces** checks if the concrete's plate thickness, which stiffens the reinforcement mesh, is sufficient to meet all requirements arising from bending and shear loading.

In addition to the ultimate limit state design, the serviceability limit state design is possible, too. These designs include the limitation of the concrete compressive and the reinforcing steel stresses, the minimum reinforcement for the crack control, as well as the crack control by limiting rebar diameter and rebar spacing. For this purpose, analytical and nonlinear design check methods are available for selection.

If you also have a license for **RF-CONCRETE Deflect**, you can calculate the deformations with the influence of creep, shrinkage, and tension stiffening according to the analytical method.

With a license of **RF-CONCRETE NL**, you can consider the influence of creep and shrinkage in the determination of deformations, crack widths, and stresses according to the nonlinear method.

The design is possible according to the following standards:

- EN 1992-1-1:2004/AC:2010
- DIN 1045-1:2008-08
- ACI 318-11
- SIA 262:2003
- GB 50010-2010

The figure on the right shows the National Annexes to EN 1992-1-1 that are currently implemented in RF-CONCRETE Surfaces.

All intermediate results for the design are comprehensively documented. In line with the DLUBAL philosophy, this provides a special transparency and traceability of design results.

We hope you will enjoy working with RF-CONCRETE Surfaces.

Your DLUBAL Team

CEN	EU
	BS United Kingdom
	CSN Czech Republic
	DIN Germany
	DK Denmark
	NBN Belgium
	NEN Netherlands
	NF France
	NP Portugal
	PN Poland
	SFS Finland
	SIST Slovenia
	SS Singapore
	SS Sweden
	STN Slovakia
	UNE Spain
	UNI Italy
	ÖNORM Austria

National Annexes to EN 1992-1-1

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1.3 Using the Manual

Topics like installation, graphical user interface, results evaluation, and printout are described in detail in the manual of the main program RFEM. The present manual focuses on typical features of the add-on module RF-CONCRETE Surfaces.

The descriptions in this manual follow the sequence and structure of the module's input and results windows. The text of the manual shows the described **buttons** in square brackets, for example [View mode]. At the same time, they are pictured on the left. In addition, **expressions** used in dialog boxes, tables, and menus are set in *italics* to clarify the explanations.

At the end of the manual, you find the index. However, if you do not find what you are looking for, please check our website www.dlubal.com, where you can go through our *FAQ* pages by selecting particular criteria.

1.4 Opening RF-CONCRETE Surfaces

RFEM provides the following options to start the add-on module RF-CONCRETE Surfaces.

Menu

To start the add-on module from of the RFEM menu, select

Add-on Modules → Design - Concrete → RF-CONCRETE Surfaces.

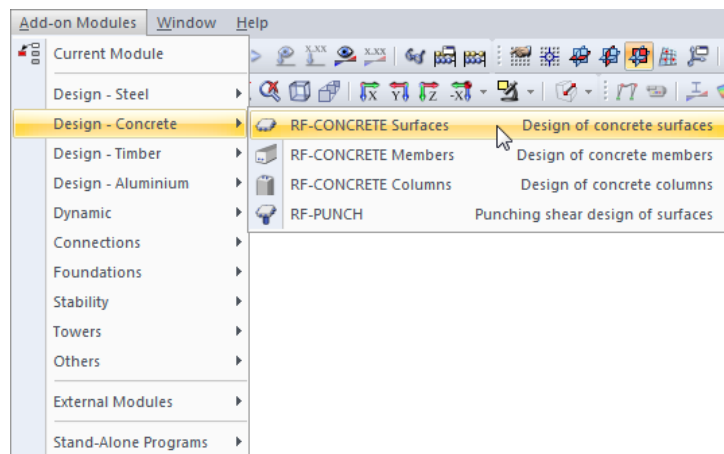


Figure 1.1: Menu: Add-on Modules → Design - Concrete → RF-CONCRETE Surfaces

Navigator

Alternatively, you can open the add-on module in the *Data* navigator by clicking the entry

Add-on Modules → RF-CONCRETE Surfaces.

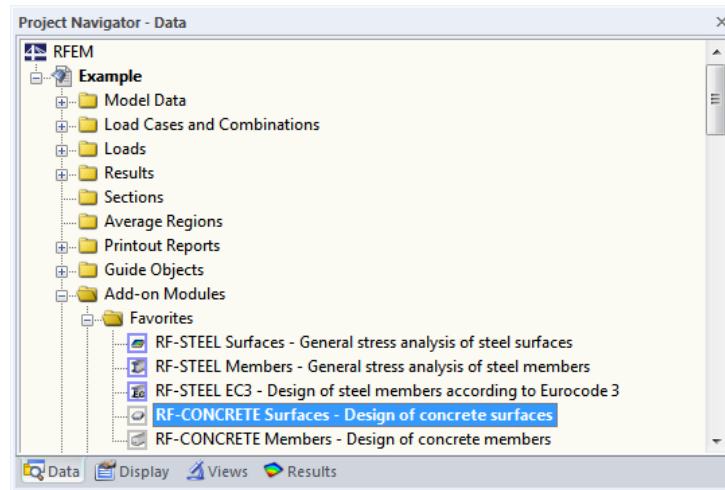


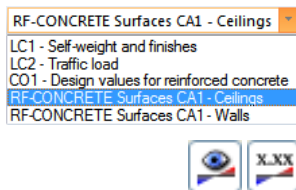
Figure 1.2: Data navigator: *Add-on Modules* → *RF-CONCRETE Surfaces*

Panel

If results from RF-CONCRETE Surfaces are already available in the RFEM model, you can return to the design module by using the panel:

Set the relevant RF-CONCRETE Surfaces design case in the load case list, which is located in the menu bar. Click [Show Results] to display the reinforcements graphically.

The panel appears, showing the button [RF-CONCRETE Surfaces] which you can use to open the module.



RF-CONCRETE Surfaces

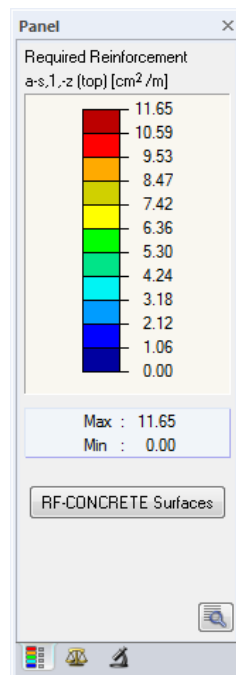


Figure 1.3: Panel button [RF-CONCRETE Surfaces]

2. Theoretical Background

2.1 Type of Model

The *Type of Model* that you define when creating a new model has a crucial influence on how the structural components will be stressed.

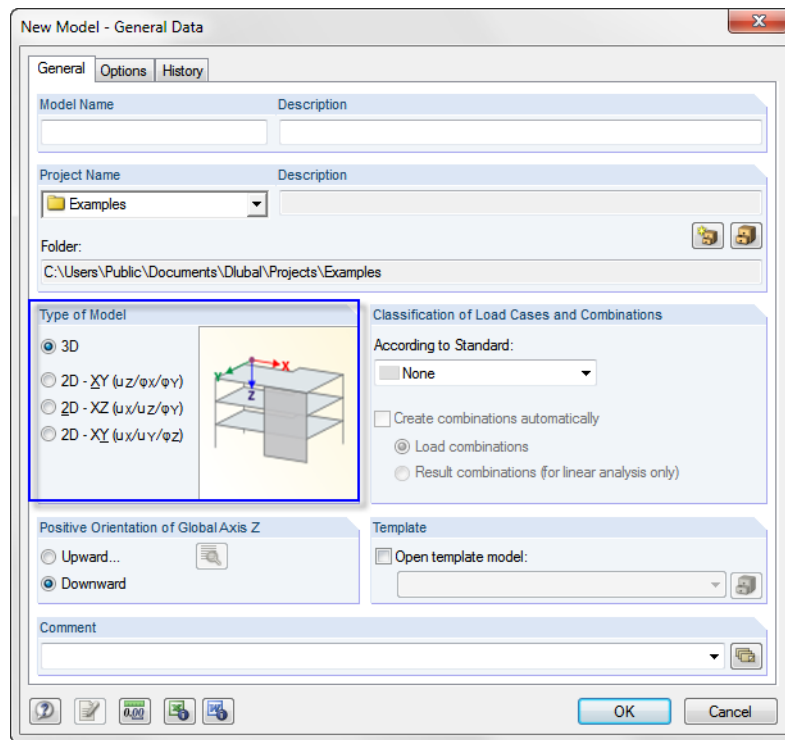


Figure 2.1: Dialog box *New Model - General Data*, section *Type of Model*

If you select the model type $2D - XY (u_z/\varphi_x/\varphi_y)$, the plate will be subjected to bending only. The internal forces to be designed will be exclusively represented by moments whose vectors lie in the plane of the component.

If you select $2D - XZ (u_x/u_z/\varphi_y)$ or $2D - XY (u_x/u_y/\varphi_z)$, the wall (diaphragm) will be subjected only to compression or tension. The internal forces used for the design will be represented exclusively by axial forces whose vectors lie in the plane of the structural component.

In a spatial $3D$ type of model, both internal forces (moments and axial forces) are combined. Therefore, the structural component can be subjected to tension/compression and bending simultaneously. Thus, the internal forces to be designed are represented by axial forces as well as by moments whose vectors lie in the component's plane.

2.2 Design of 1D and 2D Structural Components

To check the ultimate limit state of a one-dimensional or a two-dimensional structural component consisting of reinforced concrete, it is always necessary to find a state of equilibrium between the acting internal forces and resisting internal forces of the deformed component. In addition to this common feature in the ultimate limit state design of one-dimensional structural components (members) and two-dimensional components (surfaces), there is also a crucial difference:

1D structural component (member)

In a member, the acting internal force is always orientated in such a way that it can be compared to the resisting internal force that is determined from the design strengths of the materials. As an example, we can take a member subjected to the axial compressive force N .

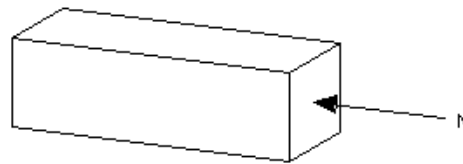


Figure 2.2: Design of a member

The dimensions of the structural component and the design value of the concrete strength can be used to determine the resisting compressive force. If it is smaller than the acting compressive force, the required area of the compressive reinforcement can be determined by means of the existing steel strain with an allowable concrete compressive strain.

2D structural component (surface)

For a surface, the direction of the acting internal force is only in exceptional cases (trajectory reinforcement) orientated in such a way that the acting internal force can be set in relation to the resisting internal force: In an orthogonally reinforced wall, for example, the directions of the two principal axial forces n_1 and n_2 are usually not identical with the reinforcement directions.

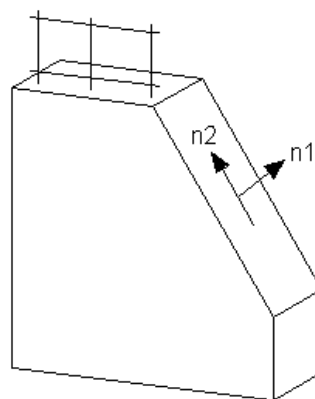


Figure 2.3: Design of a wall

Hence, for the dimensioning of the reinforcement of the reinforcement mesh, it is possible to use a procedure that is similar to the reinforcement of a member. The internal forces running in the reinforcement directions of the reinforcement mesh are required for the determination of the action-effects on concrete. These internal forces are termed **design internal forces**.

To better understand the design internal forces, we can look at an element of a loaded reinforcement mesh. For simplicity's sake, we assume the second principal axial force n_2 to be zero.

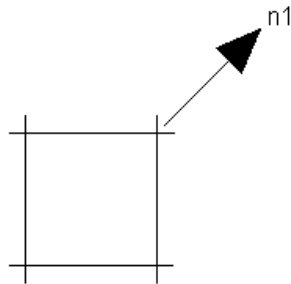


Figure 2.4: Reinforcement mesh element with loading

The reinforcement mesh deforms under the given loading as follows.

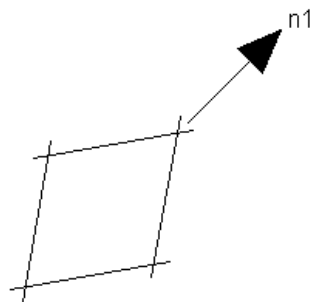


Figure 2.5: Deformation of the reinforcement mesh element

The size of the deformation is limited by introducing a concrete compression strut to the reinforcement mesh element.

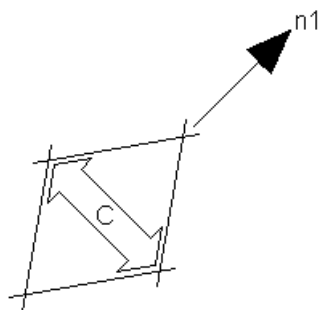


Figure 2.6: Reinforcement mesh element with concrete compression strut

The concrete strut induces tensile forces in the reinforcement.

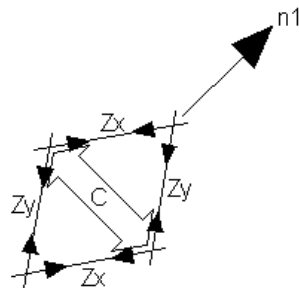


Figure 2.7: Tension forces in the reinforcement

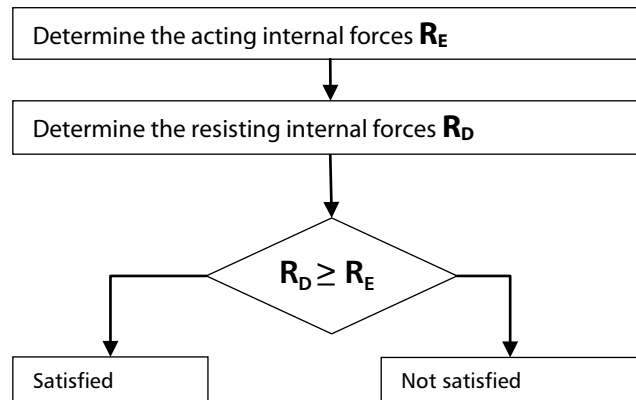
These tensile forces in the reinforcement and the compressive force in the concrete are the design internal forces.

Upon determination of the design internal forces, the design can be carried out like in a one-dimensional structural component.

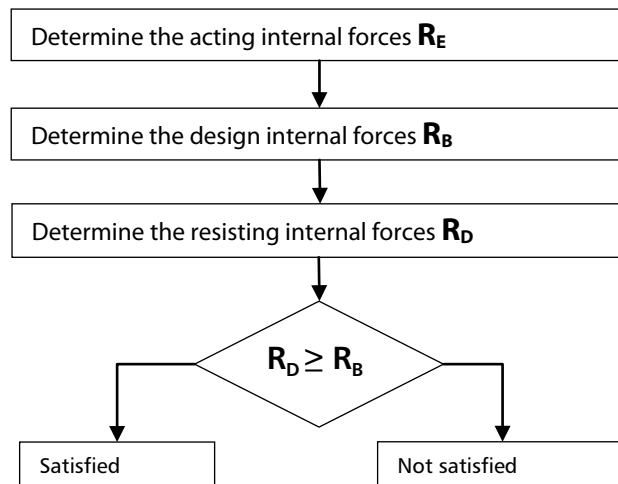
Thus, the main feature of the design of a two-dimensional structural component is the transformation of the acting internal forces (principal internal forces) into design internal forces. The direction of the design internal forces allows for the dimensioning of the reinforcement and checking of the load-bearing capacity of concrete.

The following flowcharts illustrate the main difference between the design of one-dimensional and two-dimensional structural components.

One-dimensional structural component



Two-dimensional structural component



2.3 Walls (Diaphragms)

2.3.1 Design Internal Forces

The determination of design internal forces for walls is carried out according to BAUMANN'S [1] method of transformation. In this method, the equations for the determination of design internal forces are derived for the general case of a reinforcement with three arbitrary directions. Then, these forces can be used for simpler cases like orthogonal reinforcement meshes with two reinforcement directions.

BAUMANN analyzed the equilibrium conditions with the following wall element.

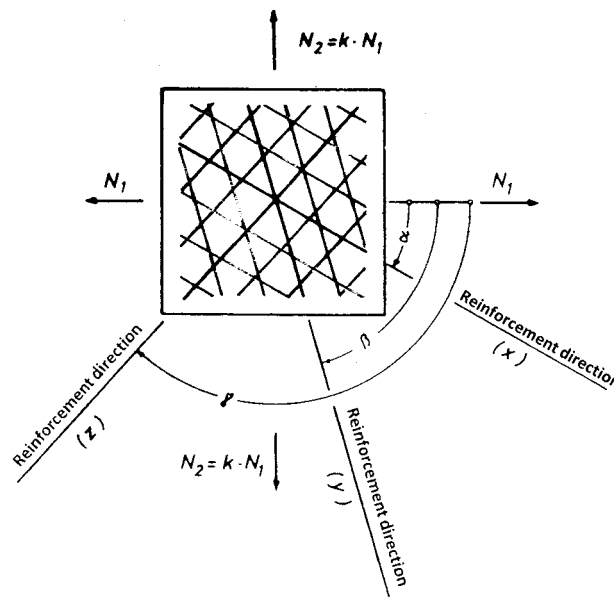


Figure 2.8: Equilibrium conditions according to BAUMANN

Figure 2.8 shows a rectangular segment of a wall. It is subjected to the principal axial forces N_1 and N_2 (tensile forces). By means of factor k , the principal axial force N_2 is expressed as a multiple of the principal axial force N_1 .

$$N_2 = k \cdot N_1$$

Equation 2.1

Three reinforcement directions are applied in the wall. The reinforcement directions are signified by x , y , and z . The clock-wise angle between the first principal axial force N_1 and the direction of the reinforcement direction x is signified by α . The angle between the first principal axial force N and the reinforcement direction y is called β . The angle to the remaining reinforcement set is called γ .

BAUMANN writes in his thesis: If the shear and tension in the concrete is neglected, the external loading (N_1 , $N_2 = k \cdot N_1$) of a wall element can usually be resisted by three internal forces oriented in any direction. In a reinforcement mesh with three reinforcement directions, these forces correspond to the three reinforcement directions (x), (y), and (z). Those directions form (with the greater main tensile force N_1) the angles α , β , γ , and are called Z_x , Z_y , Z_z (positive, because tensile forces).

To determine those forces Z_x , Z_y (and Z_z in case of a third reinforcement direction), we first define a section parallel to the third reinforcement direction.

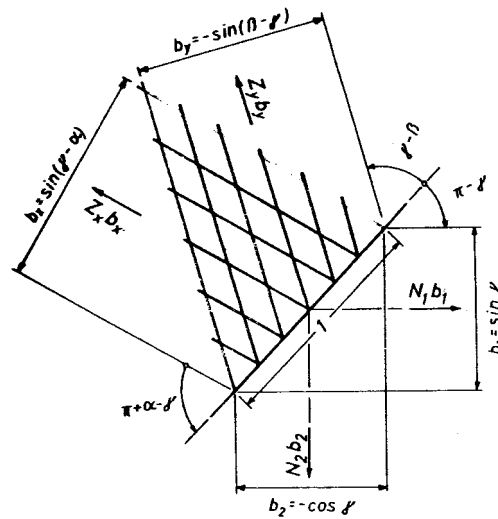


Figure 2.9: Section parallel to the third reinforcement direction z

The value of the section length is taken as 1. With this section length, we determine the projected section lengths running perpendicular to the respective force. In the case of the external forces, these are the projected section lengths b_1 (perpendicular to force N_1) and b_2 (perpendicular to force N_2). In the case of the tension forces in the reinforcement, these are the projected section forces b_x (perpendicular to tension force Z_x) and b_y (perpendicular to tension force Z_y).

The product of the respective force and the according projected section length yields the force that can be used to establish an equilibrium of forces.

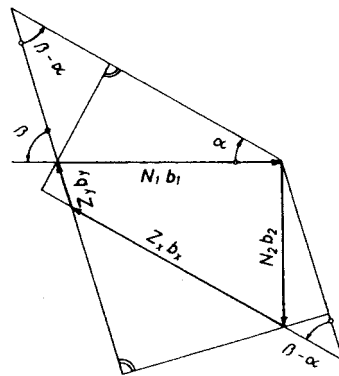


Figure 2.10: Equilibrium of force in a section parallel to the reinforcement in the z-direction

The equilibrium between the external forces (N_1, N_2) and the internal forces (Z_x, Z_y) can thus be expressed as follows.

$$Z_x \cdot b_x = \frac{1}{\sin(\beta - \alpha)} \cdot (N_1 \cdot b_1 \cdot \sin \beta - N_2 \cdot b_2 \cdot \cos \beta)$$

Equation 2.2

$$Z_y \cdot b_y = \frac{1}{\sin(\beta - \alpha)} \cdot (-N_1 \cdot b_1 \cdot \sin \alpha - N_2 \cdot b_2 \cdot \cos \alpha)$$

Equation 2.3

To determine the equilibrium between the external forces (N_1, N_2) and the internal force Z_z in the reinforcement direction z, we define a section parallel to the reinforcement direction x.

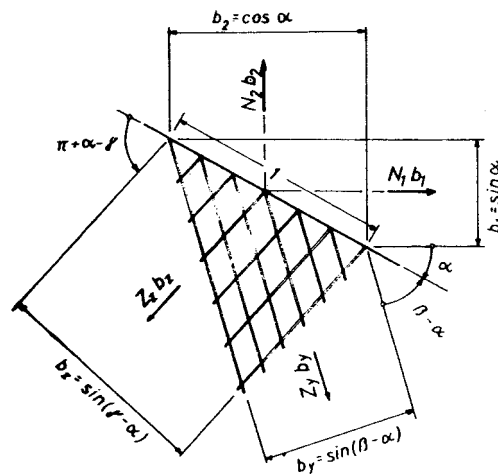


Figure 2.11: Section parallel to the reinforcement direction x

Graphically, we can determine the following equilibrium.

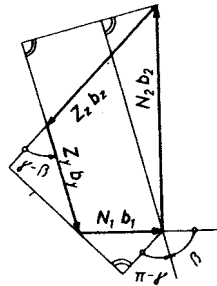


Figure 2.12: Equilibrium of reinforcement in a section parallel to the reinforcement in x-direction

From the equilibrium between the external forces (N_1, N_2) and the internal forces (Z_x, Z_y), we can express Z_x as follows.

$$Z_x \cdot b_z = \frac{1}{\sin(\beta - \gamma)} \cdot (N_1 \cdot b_1 \cdot \sin \beta + N_2 \cdot b_2 \cdot \cos \beta)$$

Equation 2.4

For Z_y , see equation 2.3.

If we replace the projected section lengths b_1, b_2, b_x, b_y, b_z by the values shown in the figure and use k as the quotient of the principal axial force N_2 divided by N_1 , we obtain the following equations.

$$\frac{Z_x}{N_1} = \frac{\sin \beta \cdot \sin \gamma + k \cdot \cos \beta \cdot \cos \gamma}{\sin(\beta - \alpha) \cdot \sin(\gamma - \alpha)}$$

Equation 2.5

$$\frac{Z_y}{N_1} = \frac{\sin \alpha \cdot \sin \gamma + k \cdot \cos \alpha \cdot \cos \gamma}{\sin(\beta - \alpha) \cdot \sin(\beta - \gamma)}$$

Equation 2.6

$$\frac{Z_z}{N_1} = \frac{-\sin \alpha \cdot \sin \beta - k \cdot \cos \alpha \cdot \cos \beta}{\sin(\beta - \gamma) \cdot \sin(\gamma - \alpha)}$$

Equation 2.7

These equations are at the core of the design algorithm for RF-CONCRETE Surfaces. Thus, we can determine from the acting internal forces N_1 and N_2 the design internal forces Z_x , Z_y , and Z_z for the respective reinforcement directions.

By adding Equation 2.5, Equation 2.6, and Equation 2.7, we obtain:

$$\frac{Z_x}{N_1} + \frac{Z_y}{N_1} + \frac{Z_z}{N_1} = 1 + k$$

Equation 2.8

By multiplying Equation 2.8 with N_1 and substituting k with N_2 / N_1 , we obtain the following equation that clarifies the equilibrium of the internal and external forces.

$$Z_x + Z_y + Z_z = N_1 + N_2$$

Equation 2.9

2.3.2 Two-Directional Reinforcement Meshes with $k > 0$

For a reinforcement with two reinforcement directions subjected to two positive principal axial forces N_1 and N_2 , we select the direction of the concrete compressive strut as follows.

$$\gamma = \frac{\alpha + \beta}{2}$$

Equation 2.10

Generally, there are two possibilities to arrange a concrete strut exactly at the center between the two crossing reinforcement directions.

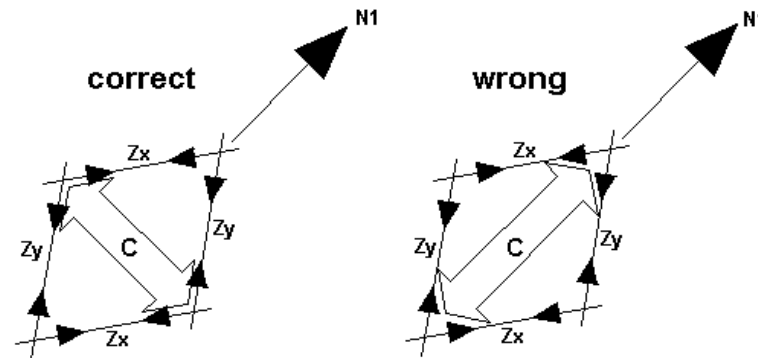


Figure 2.13: Correct and incorrect arrangement of the stiffening concrete compressive strut

In the figure on the left, the stiffening concrete strut divides the obtuse angle between the crossing reinforcement directions; in the figure on the right, it divides the acute angle. The strut on the left stiffens the reinforcement mesh in the desired way. In contrast to that, the concrete strut shown in the figure on the right has the result that the reinforcement mesh can be deformed arbitrarily by the force N_1 .

To ensure that the concrete strut divides the correct angle, the design forces Z_x , Z_y , and Z_z are determined by means of Equation 2.5, Equation 2.6, and Equation 2.7 for both geometrically possible directions of the concrete strut. A wrong direction of the concrete strut would result in a tensile force.

Therefore, the following directions of the concrete compression strut are analyzed.

$$\gamma_{1a} = \frac{\alpha + \beta}{2} \quad \text{and} \quad \gamma_{1b} = \frac{\alpha + \beta}{2} + 90^\circ$$

Equation 2.11

To distinguish the analyzed directions, the index "1a" is assigned to the simple arithmetic mean value and "1b" to the concrete strut that is rotated by 90° .

The following graph shows that for the equilibrium of forces in the two reinforcement directions we obtain a tension force, respectively, and a compression force in the selected direction of the compression strut.

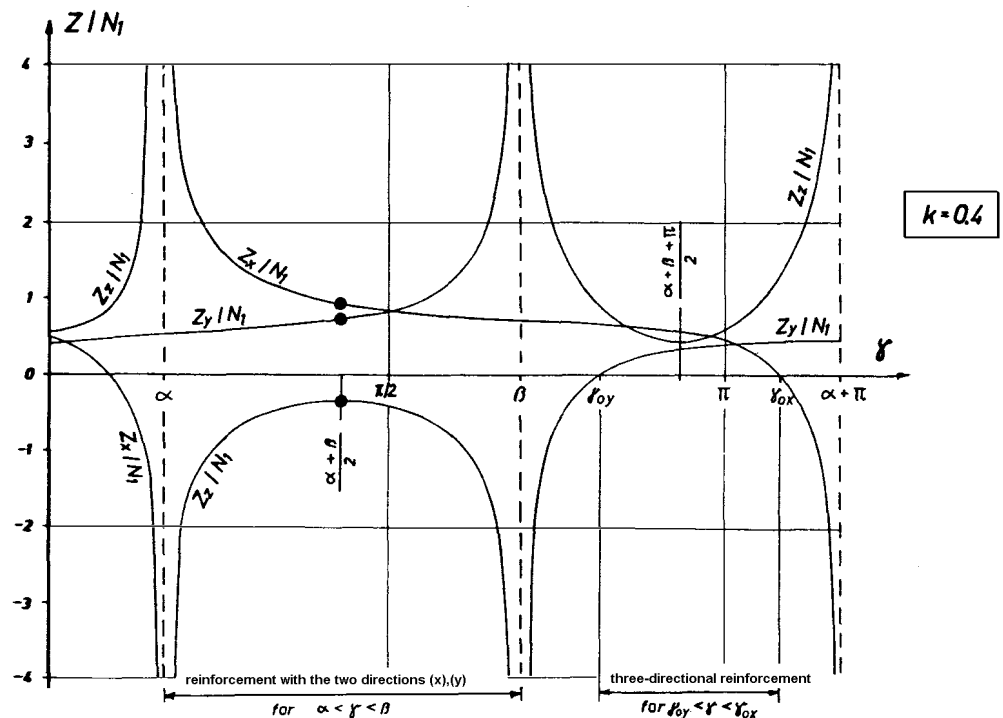


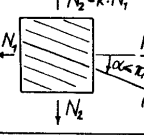
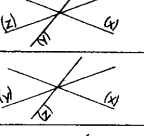
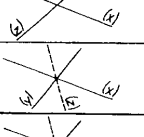
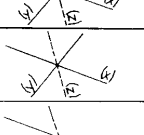
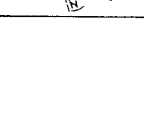
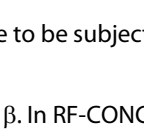
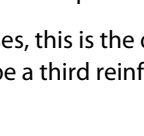
Figure 2.14: Two-directional reinforcement in pure tension

In his studies, BAUMANN [1] assumed certain ranges of values for the different angles. For example, the angle α (between the principal axial force N_1 and the reinforcement direction closest to it) is to be between 0 and $\pi/4$. The angle β must be greater than $\alpha + \pi/2$.

[1] gives Table IV with the possible states of equilibrium (see Figure 2.15). The rows 1 through 4 of this table show the possible states of equilibrium for walls that are subjected to tension alone. Row 4 shows the state of equilibrium with two directions of reinforcement subjected to tension and one compression strut. The rows 5 to 7 show walls for which the principal axial forces have different signs.

Table IV:

Appropriate selection of angle β and γ for a given direction $0 < \alpha < \pi/4$ of the reinforcement direction (x)

Row	Ratio $k = N_2/N_1$ of internal forces	Number of required directions of reinforcement	Direction β of reinforcement direction (y)	Direction γ of direction Z_z	Tension forces (to be resisted by the reinforcement) in the direction of	Tension force (to be resisted by the concrete) in the direction of	Reinforcement layout ¹⁾	Row
1	$0 < k < 1$	3	$\alpha < \beta < \beta_{oy}$	$\beta_{oy} < \gamma < \beta_{ox}$	(x), (y), (z)	-		1
2			$\beta_{oy} < \beta < \pi$	$\beta_{ox} < \gamma < \beta_{oy}$	(x), (y), (z)	-		2
3		2	—	$\gamma = \beta_{oy}$	(x), (z)	-		3
4			$\alpha + \pi/2 < \beta < \beta_{oy}$	$\gamma = \frac{\alpha + \beta}{2}$	(x), (y)	(z)		4
5	$-tg^2\alpha \leq k \leq 0$	2	$\alpha + \pi/2 < \beta < \pi - \alpha$	$\gamma = \frac{\alpha + \beta}{2}$	(x), (y)	(z)		5
6	$k < -tg^2\alpha$	2	$\alpha + \pi/2 < \beta < \pi - \alpha$ and $\beta > 2\beta_{oy} - \alpha$	$\gamma = \frac{\alpha + \beta}{2}$	(x), (y)	(z)		6
7		1	—	$\gamma = \beta_{oy}$	(x)	(z)		7

¹⁾ Reinforcement directions are indicated by continuous lines, concrete compression forces by dotted lines.

Figure 2.15: Possible states of equilibrium according to [1]

The second column of this table defines the value range of the loading.

The third column indicates the number of reinforcement directions that are to be subjected to a tension force.

The forth column (β) shows the value range of the reinforcement direction β . In RF-CONCRETE Surfaces, this range results from the directions of reinforcement specified in the input data.

The fifth column (γ) shows the direction of the internal force Z_z . In most cases, this is the direction of the concrete strut computed by the program. However, it can also be a third reinforcement direction to which a tension force is assigned.

The seventh row indicates whether or not γ is indeed a compression force.

The penultimate column shows the required internal forces together with their directions. Here, the reinforcement directions with a tension force are represented by simple lines. The possible compression struts are indicated by dashed lines.

2.3.3 Two-Directional Reinforcement Meshes with $k < 0$

If in a two-directional reinforcement mesh, the main axial forces N_1 and N_2 have different signs, for the equilibrium of forces this results in a tension force in the two reinforcement directions, respectively, and a compression forces in the direction of the compressive strut.

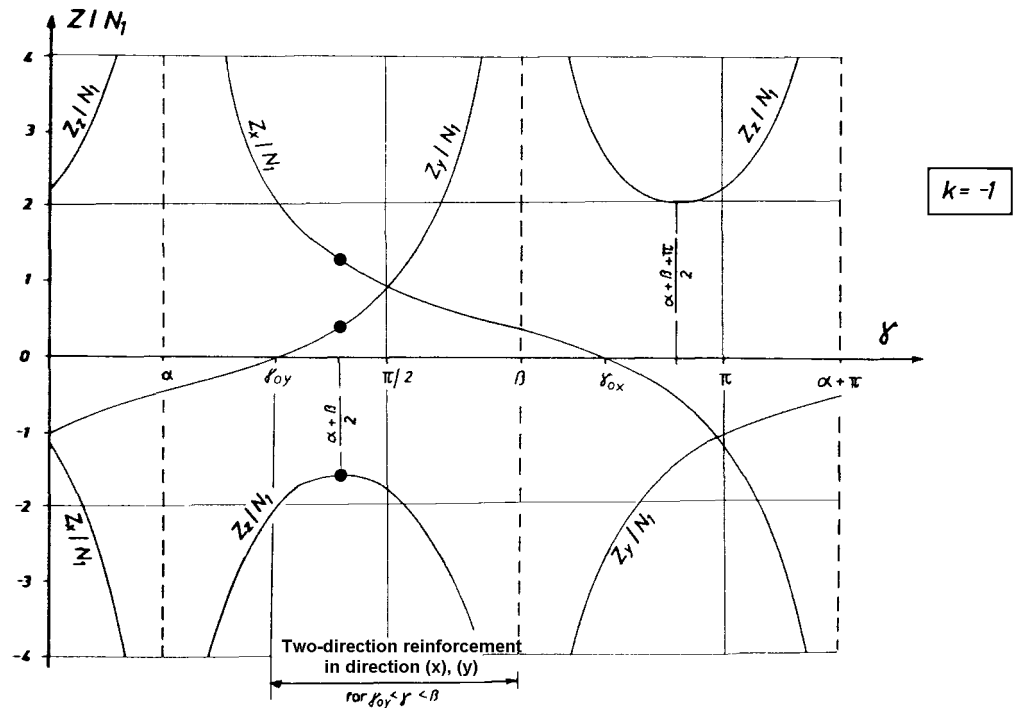


Figure 2.16: Two-directional reinforcement in tension and compression

Rows 5 and 6 of Table IV (Figure 2.15) give examples for this possible state of equilibrium.

For a wall subjected to tension as well as compression, however, it can happen that for the selected direction of the concrete strut (arithmetic mean between the two directions of reinforcement) a compression strut is obtained, as expected, in a direction γ and in a further direction β . This is the case if the arithmetic mean is to the left of the zero-crossing of the force distribution of Z_y in the diagram above. However, this kind of equilibrium is not possible. We determine the reinforcement of the conjugated direction, that is, the value γ_{0y} is used for the concrete strut direction γ .

$$\tan \gamma_{0y} = -k \cdot \cot \alpha$$

Equation 2.12

This means that no force occurs in the second reinforcement direction y under the angle β . Row 7 in Table IV (Figure 2.15) shows an example for this equilibrium of forces. In the add-on module RF-CONCRETE Surfaces, such a case of equilibrium is represented if a compression force in the direction of the reinforcement direction y is obtained for the routinely assumed direction of the concrete strut (arithmetic mean between the directions of the two reinforcement directions).

Thus, we have described all possible states of equilibrium for two-directional reinforcements.

2.3.4 Possible Load Situations

The load is obtained by applying the principal axial forces n_1 and n_2 , with the principal axial force n_1 under consideration of the sign always being greater than the principal axial force n_2 .

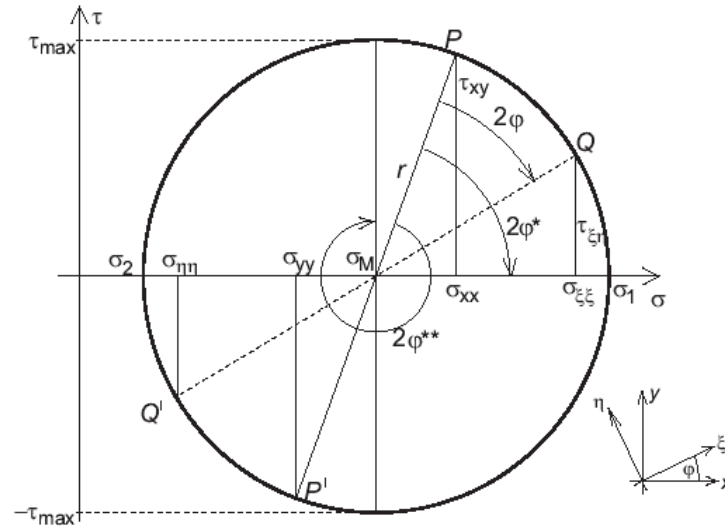


Figure 2.17: Mohr's circle

Different load situations are distinguished, depending on the sign of the principal axial forces.

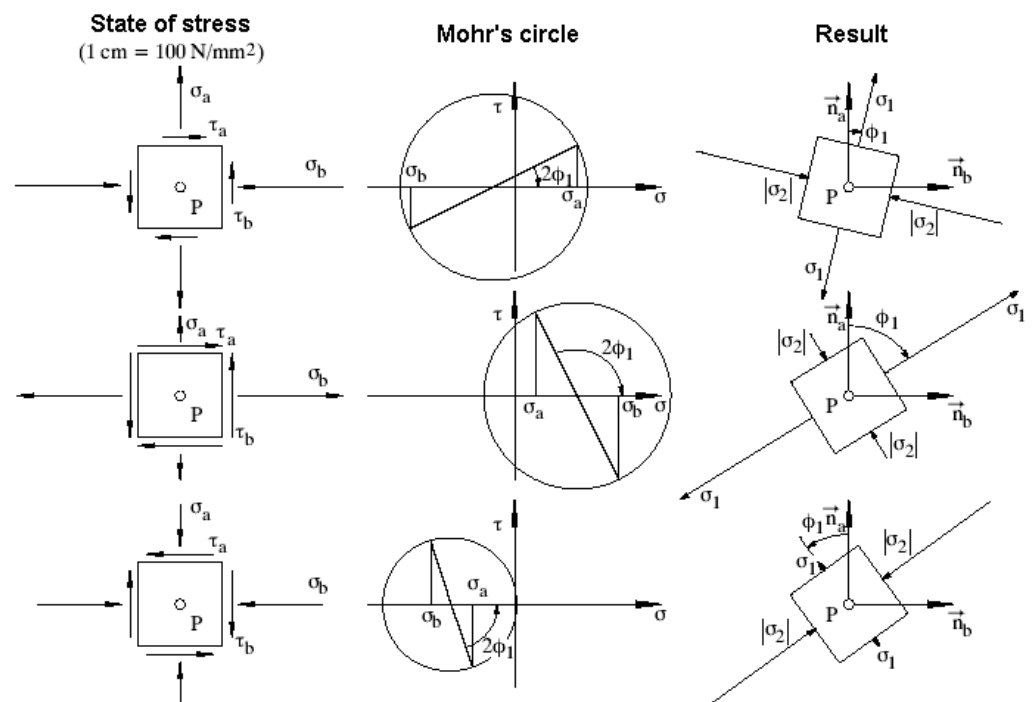


Figure 2.18: Load situations

In a matrix of principal axial force, we obtain the following designations of the individual design situations (n_1 is called here n_I , n_2 is called n_{II}):

$n_{II} \backslash n_I$	$n_I > 0$	$n_I = 0$	$n_I < 0$
$n_{II} > 0$	Elliptical tension (tension - tension)	Not possible	Not possible
$n_{II} = 0$	Parabolic tension	No loading	Not possible
$n_{II} < 0$	Hyperbolic tension (tension - compression)	Parabolic compression	Elliptical compression (compression - compression)

Figure 2.19: Matrix of principal axial force for load situations

The determination of design axial forces by means of Equation 2.5 through Equation 2.7 is described in the previous chapters for the load situations *Elliptical Tension* and *Hyperbolic State*. For the load situation *Parabolic Tension*, the design axial forces are obtained by using the same formulas. The value k is to be taken as zero in Equation 2.5 through Equation 2.7.

Now we will explain the design axial forces for the following design situations.

Elliptical compression in a mesh with three reinforcement directions

Equation 2.5 through Equation 2.7 are applied without changes, even if the two principal axial forces n_1 and n_2 are negative. If a negative design axial force results for each of the three reinforcement directions, none of the three provided reinforcement directions is activated. The concrete can transfer the principal axial forces by itself, that is, without the use of a reinforcement mesh in tension, stiffened by a concrete strut.

The assumption about the introduction of concrete compression forces in the direction of the provided reinforcement to resist the principal axial forces is purely hypothetical. It is based on the wish to obtain a distribution of the principal compression forces in the direction of the individual reinforcement directions in order to be able to determine the minimum compression reinforcement that is required, for example, by EN 1992-1-1, clause 9.2.1.1. To this end, a statically required concrete cross-section is necessary. It can only be determined by means of the previously determined concrete compression forces in the direction of the provided reinforcement.

In the determination of the minimum compressive reinforcement, other standards do without a statically required concrete cross-section resulting from the transformed principal axial force into a design axial force. However, for a unified transformation method across different standards, the principal compressive forces are transformed in the defined reinforcement directions for these standards, too. Studies have shown that the design with transformed compressive forces is on the safe side. The concrete pressures occurring in the direction of the individual reinforcement directions are verified.

However, if after the transformation at least one of the design axial forces is positive, the reinforcement mesh is activated for this load situation. Then, as described in chapter 2.3.2 and 2.3.3, an internal equilibrium of forces in the form of two reinforcement directions and one selected concrete compression strut is to be established.

Elliptical compression in a two-directional mesh

Equation 2.5 through Equation 2.7 are used without changes. If the direction of the two main axial forces is identical to the direction of both reinforcement directions, the design axial forces are equal to the principal axial forces.

If the principal axial forces deviate from the reinforcement directions, the equilibrium between a compression strut in the concrete and the design axial forces in the reinforcement directions is searched for again. For the two directions of the concrete strut, the two intermediate angles between the reinforcement directions are reanalyzed. The same applies as for the elliptical tension: The assumption of a concrete strut direction is assumed to be correct if a negative design force is indeed assigned to the concrete strut. If allowable solutions are obtained for both concrete strut directions, the smallest absolute value of all design axial forces decides which solution is chosen.

If the design axial force for a reinforcement direction is a compressive force, the program first checks whether the concrete can resist this design axial force. If this is not the case, the program determines a compression reinforcement ratio.

Parabolic compression in a two-directional mesh

In this load situation, the principal axial force n_1 is zero. Since the quotient $k = n_2 / n_1$ cannot be calculated anymore, we cannot use Equation 2.5 through Equation 2.7 as usually. The following modifications are necessary.

$$n_\alpha = \frac{n_1 \cdot \sin \beta \cdot \sin \gamma + n_2 \cdot \cos \beta \cdot \cos \gamma}{\sin(\beta - \alpha) \cdot \sin(\gamma - \alpha)}$$

$$n_\beta = \frac{n_1 \cdot \sin \alpha \cdot \sin \gamma + n_2 \cdot \cos \alpha \cdot \cos \gamma}{\sin(\beta - \alpha) \cdot \sin(\beta - \gamma)}$$

$$n_\gamma = \frac{-n_1 \cdot \sin \alpha \cdot \sin \beta + n_2 \cdot \cos \alpha \cdot \cos \beta}{\sin(\beta - \gamma) \cdot \sin(\gamma - \alpha)}$$

Equation 2.13

With the modified equations, the program search for the design axial forces in the two reinforcement directions and one design axial force for the concrete. If one reinforcement direction is identical to the acting principal axial force, then its design axial force is the principal axial force. Otherwise, solutions with one concrete strut between the two reinforcement directions are obtained.

Parabolic compression in a three-directional mesh

The formulas presented above are used according to Equation 2.13.

If the principal axial force runs in a reinforcement direction, solutions (like for the parabolic tension) for a concrete strut direction between the first and the second reinforcement direction or the first and third reinforcement direction are analyzed. Again, the smallest absolute value of all design axial forces values decides which solution is chosen.

2.3.5 Design of the Concrete Compression Strut

The concrete compression force in the selected direction of the concrete strut is one of the design forces. It is analyzed whether or not the concrete can resist the compression force. However, we do not apply the complete compression stress f_{cd} . Instead, the allowable concrete compression stress is reduced to 80%, thus following the sense of the recommendation by SCHLAICH/SCHÄFER ([13], page 373).

With the reduced concrete compression stress $f_{cd,08}$, the magnitude of the resisting axial force $n_{strut,d}$ is determined per meter. This is done by multiplying the concrete compression stress by the width of one meter and the wall thickness.

$$n_{strut,d} = f_{cd,08} \cdot b \cdot d$$

Equation 2.14

This resisting concrete compression force can now be compared to the acting concrete compression force n_{strut} . The analysis of the concrete compression strut is OK, if

$$n_{strut,d} \geq n_{strut}$$

Equation 2.15

The design of the concrete compression strut is carried out in the same way for all standards – of course, with the respective valid material properties.

2.3.6 Determination of Required Reinforcement

To determine the dimension of the reinforcement area to be used, the resisting design axial force n_{φ} in the respective reinforcement direction φ is divided by the reinforcing steel strength.

Depending on the standard and concrete strength class, the steel stress at yield is defined differently. For the design, the respective partial safety factor for the reinforcing steel has to be considered.

If the reinforcement is in compressive strain instead of tension, the steel stress for the allowable concrete compression at failure shall be determined. It is the same in all standards and equals 2 ‰. Thus, the steel stress can be determined by using the modulus of elasticity as follows:

$$\sigma = E_s \cdot 0.002$$

Equation 2.16

If the steel stress is greater than the steel stress at yielding, the steel stress at yielding is used. However, a compression reinforcement is determined only in the case if the resistant axial force $n_{strut,d}$ per meter of the concrete is smaller than the acting, compression-inducing design axial force. The compression reinforcement is then designed for the difference of the two axial forces.

2.3.7 Reinforcement Rules

All standards contain regulations for plate structures regarding size and direction of the reinforcement to be used. To this purpose, the standard classifies the plate structures in certain structural elements. For example, EN 1992-1-1 distinguishes the following elements of structures:

- Plate (slab)
- Wall (diaphragm)
- Deep beam

The following graphic illustrates the relation between the user-defined *Type of Model*, the model for the design, and the element of structure according to the standard, which is used to determine the size and direction of the minimum or maximum reinforcement.

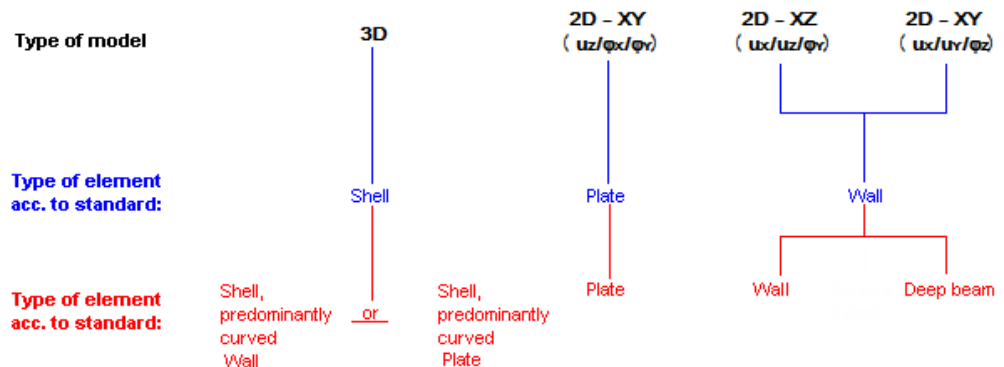


Figure 2.20: Relation between type of model, design model, and structural element

If 3D (see Figure 2.1, page 10) is selected as type of model, the structural component is always designed as shell – independent of whether both axial forces and moments occur in portions of the structural component or if there is only one of these internal forces or moments. A type of model defined as 2D - XY ($u_z/\varphi_x/\varphi_y$) is always designed as plate, the two types 2D - XZ ($u_x/u_z/\varphi_y$) and 2D - XY ($u_x/u_y/\varphi_z$) are designed as walls.

After selecting the structural element, the rules of the respective standard are automatically used in the determination of the required reinforcement. We will now briefly look at these rules acc. to **EN 1992-1-1**. The standard distinguishes between *solid plates*, *walls*, and *deep beams*.

Solid plates

For solid plates, EN 1992-1-1 specifies the following:

- Clause 9.2.1.1 (1): The area of longitudinal tension reinforcement should not be taken as less than $A_{s,min}$.

$$A_{s,min} = 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b_t \cdot d \geq 0.0013 \cdot b_t \cdot d$$

Equation 2.17

- Clause 9.2.1.1 (3): The cross-sectional area of tension or compression reinforcement should not exceed $A_{s,max}$ outside lap locations. The recommended value is $0.04 A_c$.

According to DIN EN 1992-1-1/NA:2010, the sum of the tension and compression reinforcement may not exceed $A_{s,max} = 0.08 A_c$. This is also true for the lap locations.

Walls

For walls, EN 1992-1-1 specifies the following:

- Clause 9.6.2 (1): The area of the vertical reinforcement should lie between $A_{s,vmin}$ and $A_{s,vmax}$. The recommended values are $A_{s,vmin} = 0.002 A_c$ and $A_{s,vmax} = 0.04 A_c$ outside lap locations.

DIN EN 1992-1-1/NA:2010 specifies

- General: $A_{s,vmin} = 0.15 |N_{Ed}| / f_{yd} \geq 0.0015 A_c$

- $A_{s,vmax} = 0.04 A_c$ (this value may be doubled at laps)

The percentage of reinforcement should be equal at both wall faces.

- Clause 9.6.3 (1): Horizontal reinforcement running parallel to the faces of the wall (and to the free edges) should be provided at the outer face. It should not be less than $A_{s,hmin}$. The recommended value is either 25 % of the vertical reinforcement or $0.001 A_c$.

DIN EN 1992-1-1/NA:2010 specifies

- General: $A_{s,hmin} = 0.20 A_{s,v}$

The diameter of the horizontal reinforcement should not be less than one quarter of the diameter of the perpendicular members.

Deep beam

According to EN 1992-1-1, clause 5.3.1 (3), a beam is considered as a deep beam if the span is less than three times the cross-section depth. If this is the case, the following applies:

- Clause 9.7 (1): Deep beams should normally be provided with an orthogonal reinforcement mesh near each face, with a minimum of $A_{s,dbmin}$. The recommended value is 0.1% but not less than $150 \text{ mm}^2/\text{m}$ in each face and each direction.

DIN EN 1992-1-1/NA:2010 specifies

- $A_{s,dbmin} = 0.075 \% \text{ of } A_c \geq 150 \text{ mm}^2/\text{m}$

User-defined, cross-standard rules of reinforcement detailing

In addition to the normative requirements (that means they cannot be modified) of reinforcement detailing, there is the possibility to specify user-defined rules. The minimum reinforcements can be specified in the *Reinforcement Ratios* tab of the 1.4 *Reinforcement* window.



Figure 2.21: Window 1.4 *Reinforcement*, tab *Reinforcement Ratios*

Calculation

If, for example, you specify a minimum secondary reinforcement of 20 % of the greatest placed longitudinal reinforcement, the [Calculation] determines the maximum longitudinal reinforcement first. In the results windows, this is shown as *Required Reinforcement*.

2.1 Required Reinforcement Total

Surface No.	A Grid Point	B Point-Coordinates [m] X	C Y	D Z	E Symbol	F Required Reinforcement	G Basic Reinforcement	H Additional Reinforcement Required	I Reinforcement Provided	J Unit	K Note
3	G253	6.000	6.000	0.000	a s, 1.-z (top)	7.76	2.57	5.19	-	cm ² /m	
4	G180	9.500	0.000	0.000	a s, 2.-z (top)	6.36	2.57	3.79	-	cm ² /m	
5	G171	5.000	4.000	0.000	a s, 1.-z (bottom)	8.54	2.57	5.97	-	cm ² /m	
5	G171	5.000	4.000	0.000	a s, 2.-z (bottom)	16.10	2.57	13.53	-	cm ² /m	
5	G171	5.000	4.000	0.000	a sw	19.82	-	-	-	cm ² /m ²	

Design details

☐ In FE nodes ☒ In grid points Required reinforcement for: ULS

Figure 2.22: Required longitudinal reinforcement and button [Design Details]

To check the minimum secondary reinforcement, click [Design Details].



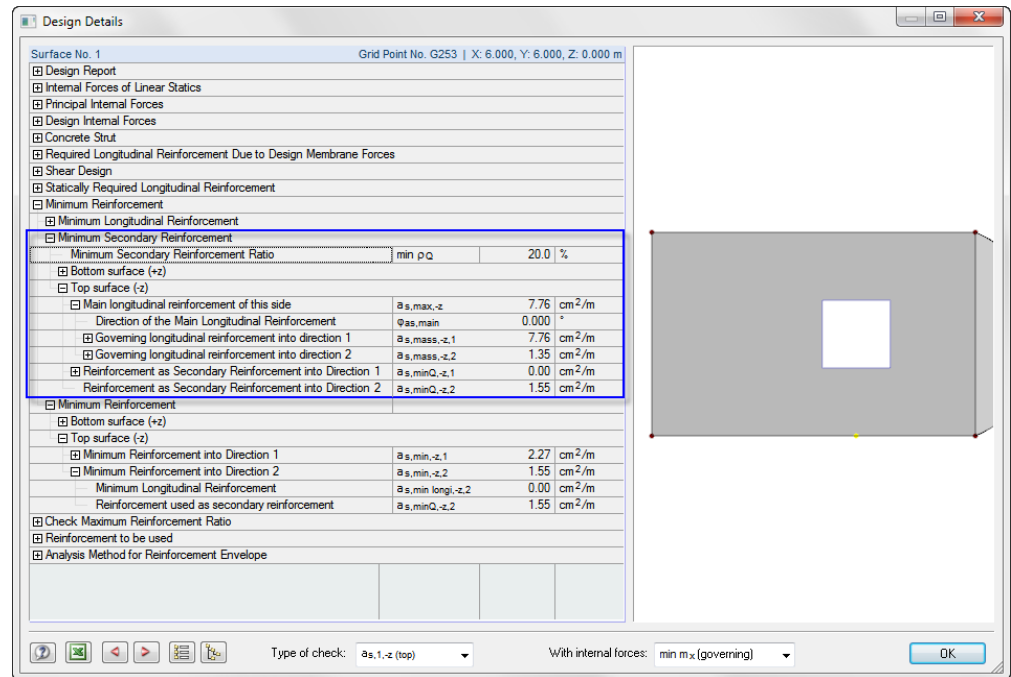


Figure 2.23: Dialog box *Design Details* for checking the minimum reinforcement

In the example above, the *Reinforcement as Secondary Reinforcement into Direction 2* is 20 % of the reinforcement that is provided in the reinforcement direction 1 (here the main direction): $7.76 \text{ cm}^2/\text{m} \cdot 0.2 = 1.55 \text{ cm}^2/\text{m}$. Since this value is greater than the *Governing longitudinal reinforcement into reinforcement direction 2* of $1.35 \text{ cm}^2/\text{m}$, the secondary reinforcement is governing.

2.4 Plates

2.4.1 Design Internal Forces

The most important formulas for the determination of design axial forces from the principal axial forces are presented in Equation 2.5 through Equation 2.7 in chapter 2.3. According to BAUMANN [1], these formulas can also be used for the moments, because they are just a couple of diametrically opposed forces with the same absolute value and at a certain distance from each other.

Plates differ from walls in that, amongst other things, the actions result in stresses with different signs on the opposing surfaces of the plate. Therefore, it would make sense to provide reinforcement meshes with different directions to both surfaces of the plates. The principal moments m_1 and m_2 are determined in the centroidal plane of the surface. Hence, they must be distributed on the surfaces of the plate in order to be able to determine the design moments for the reinforcement of the respective plate surface.

We want to look at a plate element with its loading. The local coordinate system of the surface is in the centroidal plane of the plate.

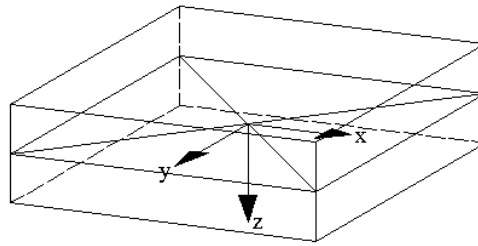


Figure 2.24: Plate element with local surface coordinate system in the centroidal plane of the plate



Top and Bottom surface

In RFEM, the bottom surface is always in the direction of the positive local surface axis z . Accordingly, the top surface is defined in the direction of the negative local z -axis. The surface axes can be switched on in the *Display* navigator by selecting

Model → **Surfaces** → **Surface Axis Systems x, y, z** .

Alternatively, you can use the context menu (see Figure 3.29, page 150).

The principal moments m_1 and m_2 are determined in RFEM for the centroidal plane of the plate.

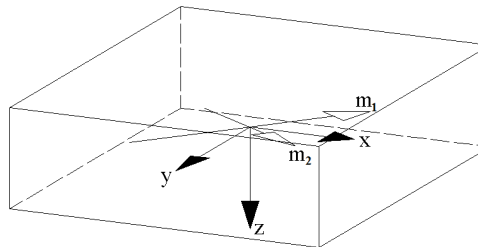


Figure 2.25: Principal moments m_1 and m_2 in the centroidal plane of the plate

The principal moments are indicated by simple arrows. They are oriented like the reinforcement that would be required for resisting them. To obtain design moments from these principal moments for the reinforcement mesh at the bottom surface of the plate, the principal moments are shifted to the bottom surface of the plate without being changed. For the design, they are signified by the Roman indexes m_I and m_{II} .

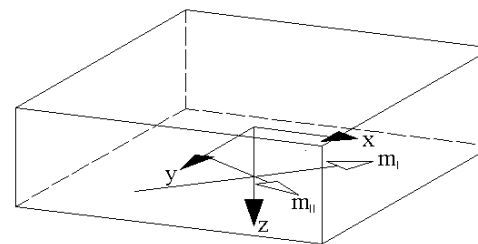


Figure 2.26: Principal moments shifted to bottom surface of the plate

To obtain the principal moments for determining the design moments for the reinforcement mesh at the top surface of the plate, the principal moments are shifted to the top surface of the plate. In addition, their direction is rotated by 180° .

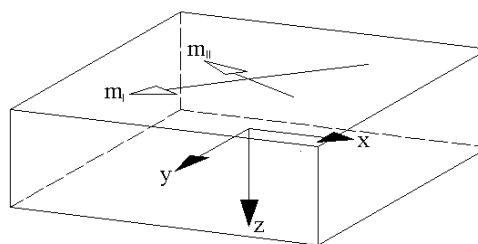


Figure 2.27: Principal moments shifted to top surface of plate

The principal moment is usually denoted m_i , which, considering the sign, is the greater one (see Figure 2.17, page 21). Hence, the denotations of the principal moments at the top surface of the plate must be reversed.

Thus, the principal moments for determining the design moments at both plate surfaces are as follows:

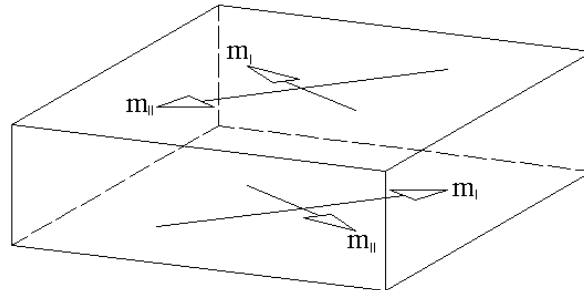


Figure 2.28: Final principal moments at bottom and top surface of the plate

If the principal moments for both plate surfaces are known, the design moments can be determined. To this end, the first step is to determine of the differential angle of the reinforcement directions to the direction of the principal moment at each plate surface.

The smallest differential angle specifies the positive direction. All other angles are determined in this positive direction, and then sorted by their size. In RF-CONCRETE Surfaces, they are denoted as $\alpha_{m,+z}$, $\beta_{m,+z}$, and $\gamma_{m,+z}$ (see the following example). The index +z indicates the bottom surface.

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Design Internal Forces			
Bottom surface (+z)			
Design Bending Moments			
Principal Moments			
Differential Angle Between $\alpha_{m,+z}$ and			
Reinforcement Direction 1	$\Delta\Phi_{+z,1,b}$	0.248	°
Reinforcement Direction 2	$\Delta\Phi_{+z,2,b}$	90.248	°
Reinforcement Direction 3	$\Delta\Phi_{+z,3,b}$	45.248	°
Differential Angle According to Baumann			
1st Differential Angle	$\alpha_{m,+z}$	0.248	°
2nd Differential Angle	$\beta_{m,+z}$	45.248	°
3rd Differential Angle	$\gamma_{m,+z}$	90.248	°

Figure 2.29: Differential angle according to [1] for bottom surface of plate (here, for three directions of reinforcement)

Then, Equation 2.5 through Equation 2.7 according to BAUMANN [1] are used in order to determine the design moments:

$$m_{\alpha} = m_1 \cdot \frac{\sin\beta \cdot \sin\gamma + k \cdot \cos\beta \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\gamma - \alpha)}$$

$$m_{\beta} = m_1 \cdot \frac{\sin\alpha \cdot \sin\gamma + k \cdot \cos\alpha \cdot \cos\gamma}{\sin(\beta - \alpha) \cdot \sin(\beta - \gamma)}$$

$$m_{\gamma} = m_1 \cdot \frac{-\sin\alpha \cdot \sin\beta + k \cdot \cos\alpha \cdot \cos\beta}{\sin(\beta - \gamma) \cdot \sin(\gamma - \alpha)}$$

Equation 2.18

RF-CONCRETE Surfaces obtains the following design moments $m_{\alpha,+z}$, $m_{\beta,+z}$ and $m_{\gamma,+z}$ for the bottom surface of the plate.

▣ Design Report
▣ Internal Forces of Linear Statics
▣ Principal Internal Forces
▣ Design Internal Forces
▣ Bottom surface (+z)
▣ Design Bending Moments
▣ Principal Moments
▣ Differential Angle Between $\alpha_{,+z}$ and $\beta_{,+z}$
▣ Differential Angle According to Baumann
1st Differential Angle
2nd Differential Angle
3rd Differential Angle

$\alpha_{m,+z}$	35.89	°
$\beta_{m,+z}$	-0.31	°
$\gamma_{m,+z}$	0.39	°

Figure 2.30: Design moments according to [1] for the bottom surface of the plate

In this example, the design moments are smaller than zero. Now, the program searches for a reinforcement mesh consisting of two reinforcement layers. The mesh is stiffened by a concrete strut.

The first assumed reinforcement mesh consists of the two reinforcement directions α_m and β_m . The direction γ of the stiffening concrete strut (of the stiffening moment that is producing compression at this surface of the plate) is assumed exactly between these two directions of reinforcement.

$$\gamma_{1a,m} = \frac{\alpha_m + \beta_m}{2}$$

Equation 2.19

With the adapted Equation 2.5 through Equation 2.7, the program redetermines the design moments in the selected reinforcement directions of the mesh and the moment that is stiffening it. In the example, the result for the bottom surface of the plate is the following.

▣ Internal Forces of Linear Statics
▣ Principal Internal Forces
▣ Design Internal Forces
▣ Bottom surface (+z)
▣ Design Bending Moments
▣ Principal Moments
▣ Differential Angle Between $\alpha_{,+z}$ and $\beta_{,+z}$
▣ Differential Angle According to Baumann
▣ Design Bending Moments by Baumann
1st Design Bending Moment
2nd Design Bending Moment
3rd Design Bending Moment
Does design Bending Moment have different sign
Determine strut direction?
First Assumption of the Strut Direction γ
New Differential Angle
▣ Design Bending Moments by Baumann
1st Design Bending Moment
2nd Design Bending Moment
3rd Design Bending Moment
Strut direction permissible?

$m_{\alpha,+z}$	35.89	kNm/m
$m_{\beta,+z}$	-0.31	kNm/m
$m_{\gamma,+z}$	0.39	kNm/m
sign $m_{,+z}$	Yes	
strut $m_{,+z}$	Yes	
$\gamma_{m,+z,1a}$	22.748	°
$m_{\alpha,+z,1a}$	36.82	kNm/m
$m_{\beta,+z,1a}$	1.02	kNm/m
$m_{\gamma,+z,1a}$	-1.87	kNm/m
strut $m_{,+z,1a}$	Yes	

Figure 2.31: First assumption for the direction γ of the concrete compression strut

The assumption of the reinforcement mesh results in a viable solution, because the direction of the concrete strut is permitted.

The analysis of further concrete strut directions must show if this is the energetic minimum with the least required reinforcement. These analyses are carried out in a similar way.

Once all sensible possibilities for a reinforcement mesh consisting of two reinforcement directions and a stiffening concrete strut have been analyzed, the sums of the absolute design moments are shown. For the example above, the overview looks as follows.

Internal Forces of Linear Statics			
Principal Internal Forces			
Design Internal Forces			
Bottom surface (+z)			
Design Bending Moments			
Principal Moments			
Differential Angle Between α , +z, +z and			
Differential Angle According to Baumann			
Design Bending Moments by Baumann			
First Assumption of the Strut Direction γ			
Second Assumption of the Strut Direction γ			
First Assumption of the Strut Direction β			
Second Assumption of the Strut Direction β			
Energy = Sum of abs(Design Bending Moments)			
Smallest Energy for All Valid Cases	$\Sigma_{min,+z}$	36.58	kNm/m
Energy for Differential Angle $\gamma_{m,+z,1a}$	$\Sigma_{\gamma m,+z,1a}$	39.71	kNm/m
Energy for Differential Angle $\beta_{m,+z,2a}$	$\Sigma_{\beta m,+z,2a}$	36.58	kNm/m

Figure 2.32: Sum of the absolute design moments

The *Smallest Energy for all Valid Cases* is given by $\Sigma_{min,+z}$ as minimum absolute sum of the determined design moments. In the example, the reinforcement mesh from the reinforcement directions for the differential angle $\beta_{m,+z,2a}$ yields the most favorable solution for the bottom surface of the plate.

The design details also show the direction of the governing concrete strut. This direction is related to the definition of the differential angles according to BAUMANN. Hence, the program also gives the direction Φ_{strut} related to the direction of the reinforcement. In the example, the following angle of the concrete strut is determined for the bottom surface of the plate.

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Design Internal Forces			
Bottom surface (+z)			
Design Bending Moments			
Principal Moments			
Differential Angle Between α , +z, +z and			
Differential Angle According to Baumann			
Design Bending Moments by Baumann			
First Assumption of the Strut Direction γ			
Second Assumption of the Strut Direction γ			
First Assumption of the Strut Direction β			
Second Assumption of the Strut Direction β			
Energy = Sum of abs(Design Bending Moments)			
Governing Strut			
First Assumption for Direction β	$\beta_{m,+z,2a}$	45.248	°
Strut Direction	$\Phi_{strut,m,+z}$	45.000	°

Figure 2.33: Governing concrete compression strut

For an optimized direction of the design moment stiffening the reinforcement mesh (see Figure 3.40, page 159), we obtain the design moments according to BAUMANN. As shown in the following figure, these design moments are applied to the defined reinforcement directions.

Surface No. 1		FE Mesh Point No. M15 X: 2.000, Y: 1.000, Z: 0.000 m	
☒ Design Report			
☒ Internal Forces of Linear Statics			
☒ Principal Internal Forces			
☒ Design Internal Forces			
☒ Bottom surface (+z)			
☒ Design Bending Moments			
☒ Principal Moments			
☒ Differential Angle Between α , +z, +z and			
☒ Differential Angle According to Baumann			
☒ Design Bending Moments by Baumann			
☒ First Assumption of the Strut Direction γ			
☒ Second Assumption of the Strut Direction γ			
☒ First Assumption of the Strut Direction β			
☒ Second Assumption of the Strut Direction β			
☒ Energy = Sum of abs(Design Bending Moments)			
☒ Governing Strut			
☒ Governing Design Bending Moments			
☐ into Direction 1	$m_{+z, \Phi 1}$	35.89	kNm/m
☐ into Direction 2	$m_{+z, \Phi 2}$	0.39	kNm/m
☐ into Direction 3	$m_{+z, \Phi 3}$	0.00	kNm/m
☐ into Strut Direction	$m_{end, +z, strut}$	-0.31	kNm/m
☐ Find optimal strut direction?	$Strut_{opti, m, +z}$	No	
☐ The strut found runs in a direction defined as the reinforcement direction.			
☐ The strut force determined will be the design force of this reinforcement direction.			
☐ The force for the design of the concrete strut will be set to zero.			
☒ Design Bending Moments by Baumann			
☐ into Direction 1	$m_{\alpha, +z}$	35.89	kNm/m
☐ into Direction 2	$m_{\beta, +z}$	-0.31	kNm/m
☐ into Direction 3	$m_{\gamma, +z}$	0.39	kNm/m
☒ Final Design Bending Moments			
☐ into Direction 1	$m_{end, +z, \Phi 1}$	35.89	kNm/m
☐ into Direction 2	$m_{end, +z, \Phi 2}$	0.39	kNm/m
☐ into Direction 3	$m_{end, \Phi, +z, 3}$	-0.31	kNm/m
☐ into Strut Direction	$m_{end, +z, strut}$	-0.31	kNm/m

Figure 2.34: Final design moments for bottom surface of plate

2.4.2 Design of Stiffening Moment

After determining the design moments, the program analyzes the compression struts. It checks if the moments for the stiffening of the reinforcement mesh can be resisted by the plate.

This check is shown in the *Concrete Strut* entry:

☒ Design Report			
☒ Internal Forces of Linear Statics			
☒ Principal Internal Forces			
☒ Design Internal Forces			
☒ Concrete Strut			
☒ Thickness of Surface		7.00	cm
☒ Bottom surface (+z)			
Design Membrane Forces in Strut Direction	$n_{S, strut, +z}$	0.000	kN/m
No check necessary: force inside the strut is zero.			
☒ Top surface (-z)			
Design Membrane Forces in Strut Direction	$n_{S, strut, -z}$	-1.921	kN/m
☒ Concrete Membrane Force Resistance	$n_{strut, d}$	-746.667	kN/m
Width of Surface	b	1.000	m
Thickness of Surface	h_E	7.00	cm
☒ Applied Concrete Compressive Strength	$f_{cd, 08}$	10.67	N/mm ²
Design Unconfined Concrete Compressive Strength	f_{cd}	13.33	N/mm ²
Coefficient of Maximum Utilization	ξ_{fod}	0.800	
Failure of concrete strut?	$\ln_{strut, d} < \ln_{S, strut, -z}$	No	

Figure 2.35: Analysis of the stiffening moment

The program performs a normal bending design for the determined moments at the bottom and top surface of the plate. However, this design does not aim at finding a reinforcement: The aim is rather to verify that the compression zone of concrete can yield a resulting compressive force. Multiplied by the lever arm of the internal forces, it results in a greater moment on the side of the resistance than the acting moment.

The analysis is not verified if the moment on the side of the resistance is smaller than the governing design moment m_{strut} even in the case of a maximum allowable bending compressive strain of the concrete and a maximum allowable retraction of an assumed reinforcement.

The current standards regulate the satisfaction of the allowable strain via the limit of the ratio between neutral axis depth x and effective depth d . For this, the stress-strain relationships for concrete and reinforcing steel as well as the limit strains of these standards are used (see the following explanations for EN 1992-1-1).

Stress-strain relationships for cross-section design

The parabola-rectangle diagram according to Figure 3.3 of EN 1992-1-1 is used as the calculation value of the stress-strain relation.

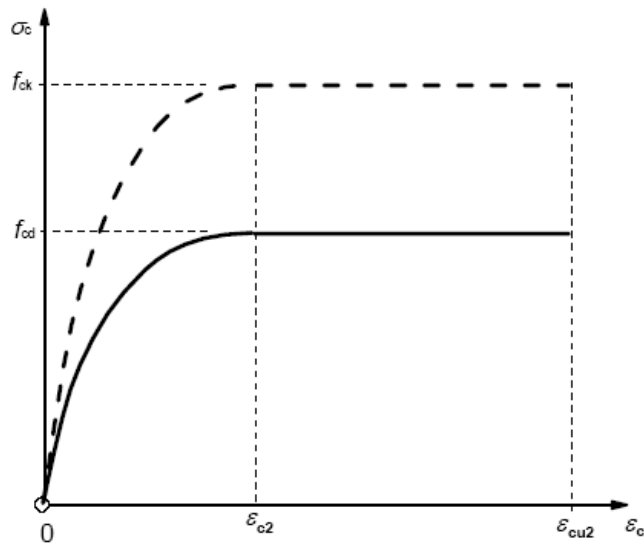


Figure 2.36: Stress-strain diagram for concrete under compression

The stress-strain diagram of the reinforcing steel is shown in Figure 3.8 of EN 1992-1-1.

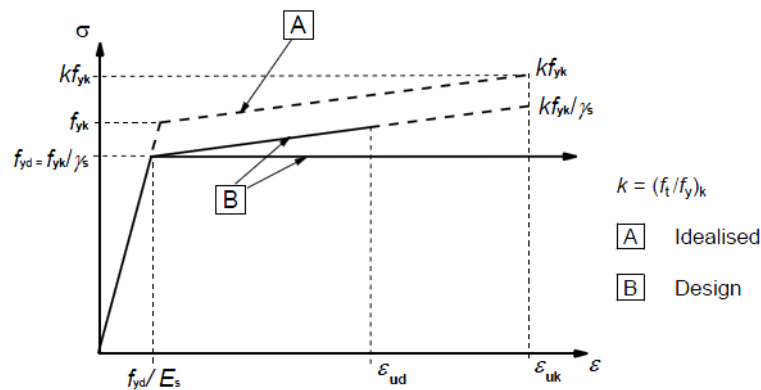
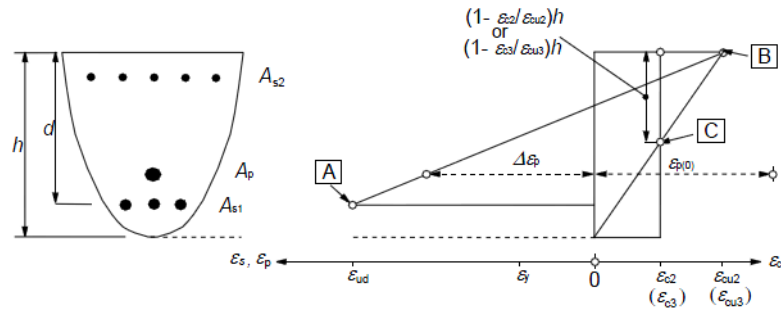


Figure 2.37: Stress-strain relation for reinforcing steel

The allowable limit deformations are shown in Figure 6.1 of EN 1992-1-1.



A - reinforcing steel tension strain limit

B - concrete compression strain limit

C - concrete pure compression strain limit

Figure 2.38: Possible strain distributions in the ultimate limit state

The ultimate limit state is determined by means of the limit strains. Either the concrete or the reinforcing steel fails, depending on where the limit strain occurs.

- Failure of concrete, for example, C30/37:
Limit strain in case of axial compression: $\varepsilon_{c2} = -2.0 \text{ ‰}$
Ultimate strain: $\varepsilon_{cu2} = -3.5 \text{ ‰}$
- Failure of reinforcing steel, for example B 500 S (A):
Steel strain under maximum load: $\varepsilon_{uk} = 25 \text{ ‰}$
- Simultaneous failure of concrete and reinforcing steel:
The limit compressive strains of concrete and steel occur simultaneously.

2.4.3 Determination of Statically Required Reinforcement

The stress-strain relationships described in chapter 2.4.2 together with the possible range of strain distributions (limit strains) are the basis for the determination of the required longitudinal reinforcement for the previously determined design moments. This process is also documented in the design details.

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Design Internal Forces			
Bottom surface (+z)			
Design Bending Moments			
Design Axial Forces			
Design Internal Forces			
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	0.150	m
Due to Design in Reinforcement Direction 1	$z_{+z,\Phi 1}$	0.161	m
Due to Design in Reinforcement Direction 2	$z_{+z,\Phi 2}$	0.159	m
Due to Design in Reinforcement Direction 3	$z_{+z,\Phi 3}$	0.150	m
Membrane Force			
Design Membrane Forces			
Top surface (-z)			
Concrete Strut			
Required Longitudinal Reinforcement Due to Design Membrane Forces			
Bottom surface (+z)			
into Reinforcement Direction 1	$a_{s,dim,+z,1}$	5.29	cm ² /m
Design Membrane Force	$n s_{end,+z,\Phi 1}$	240.025	kN/m
Design Stress	$\sigma_{s,+z,1}$	454.14	N/mm ²
Governing Range (see manual)	Range	III	
Design Internal Forces			
Strains			
Strain on Top (-z) of Cross-Section	$\epsilon_{o,-z (top),dim,+z,1}$	-3.187	‰
Strain of the Top (-z) Reinforcement	$\epsilon_{s,-z (top),dim,+z,1}$	1.346	‰
Strain of the Bottom (+z) Reinforcement	$\epsilon_{s,+z (bottom),dim,+z}$	22.500	‰
Strain on Bottom (+z) of Cross-Section	$\epsilon_{o,+z (bottom),dim,+z}$	27.033	‰
Ratio Depth of Neutral Axis/Effective Depth	$\eta_{dim,+z,1}$	0.124	
Depth of Neutral Axis	$\kappa_{dim,+z,1}$	0.021	m
Effective Depth	$d_{dim,+z,1}$	0.170	m
Stresses			
Stress on Top (-z) of Cross-Section	$\sigma_{o,-z (top),dim,+z,1}$	-13.33	N/mm ²
Stress of Top (-z) Longitudinal Reinforcement	$\sigma_{s,-z (top),dim,+z,1}$	269.23	N/mm ²
Stress of Bottom (+z) Longitudinal Reinforcement	$\sigma_{s,+z (bottom),dim,+z}$	454.14	N/mm ²
Stress on Bottom (+z) of Cross-Section	$\sigma_{o,+z (bottom),dim,+z}$	0.00	N/mm ²
into Reinforcement Direction 2	$a_{s,dim,+z,2}$	0.06	cm ² /m
into Reinforcement Direction 3	$a_{s,dim,+z,3}$	0.00	cm ² /m
Top surface (-z)			

Figure 2.39: Design details: required longitudinal reinforcement

The first subentries for the required longitudinal reinforcement are top surface and bottom surface of the plate. The *Bottom surface (+z)* and *Top surface (-z)* entries contain further details for each direction of reinforcement.

Figure 2.39 shows that the reinforcement directions 2 and 3 require only very little or no reinforcement at the bottom surface of the plate.

The *Reinforcement Direction 1* is to be designed for the design bending moment $m_{end,+z,\Phi 1} = 35.89$ kNm/m. The strains provide information about the determination of the longitudinal reinforcement.

We will check the example shown in Figure 2.39 by means of a design table for a dimensionless design procedure. The following input parameters are given:

- Cross-section [cm]: rectangle $b/h/d = 100/20/17$
- Materials: concrete C20/25
B 500 S (A)
- Design internal forces: $M_{Eds} = n s_{end,+z,\Phi 1} \cdot z_{+z,\Phi 1} = 240.005 \cdot 0.161 = 38.64$ kNm/m
 $N_{Ed} = 0.00$ kNm/m

$$f_{cd} = \frac{\alpha \cdot f_{ck}}{\gamma_c} = \frac{0.85 \cdot 2.0}{1.5} = 1.13 \text{ kN/cm}^2$$

$$\mu_{Eds} = \frac{M_{Eds}}{b \cdot d^2 \cdot f_{cd}} = \frac{3864}{100 \cdot 17^2 \cdot 1.13} = 0.1183$$

For $\mu_{Eds} = 0.1183$, it is possible to interpolate the following values from the design tables (for example [7] Annex A4):

$$\omega_1 = 0.1170 + \frac{(0.1285 - 0.1170) \cdot (0.1183 - 0.11)}{0.12 - 0.11} = 0.1265$$

$$\sigma_{sd} = 45.24 + \frac{(45.40 - 45.24) \cdot (0.1183 - 0.11)}{0.12 - 0.11} = 45.37 \text{ kN/cm}^2$$

With these values, it is possible to determine the required longitudinal reinforcement:

$$A_{s1} = \frac{\omega_1 \cdot b \cdot d \cdot f_{cd} + N_{Ed}}{\sigma_{sd}} = \frac{0.1265 \cdot 100 \cdot 17 \cdot 1.13 + 0}{45.37} = 5.36 \text{ cm}^2 / \text{m}$$

2.4.4 Shear Design

The shear design differs among the individual standards significantly. In the following, it is described for EN 1992-1-1.

The check of shear force resistance is only to be performed in the ultimate limit state (ULS). The actions and resistances are considered with their design values. The general check requirement is the following:

$$V_{Ed} \leq V_{Rd}$$

Equation 2.20

where

V_{Ed} design value of applied shear force
(principal shear force determined by RF-CONCRETE Surfaces)

V_{Rd} design value of shear force resistance

Depending on the failure mechanism, the design value of the shear force resistance is determined by one of the following three values:

$V_{Rd,c}$ design shear resistance of a structural component without shear reinforcement

$V_{Rd,s}$ design shear resistance of a structural component with shear reinforcement;
limitation of the resistance by failure of shear reinforcement (failure of tie)

$V_{Rd,max}$ design value of the maximum shear force which can be sustained by the member,
limited by crushing of the compression struts

If the applied shear force V_{Ed} remains below the value of $V_{Rd,c}$, then no calculated shear reinforcement is necessary and the check is verified.

If the applied shear force V_{Ed} is higher than the value of $V_{Rd,c}$, a shear reinforcement must be designed. The shear reinforcement must resist the entire shear force. In addition, the bearing capacity of the concrete compression strut must be analyzed.

$$V_{Ed} \leq V_{Rd,s}$$

$$V_{Ed} \leq V_{Rd,max}$$

Equation 2.21

2.4.4.1 Design Shear Resistance Without Shear Reinforcement

$$V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d \quad (6.2a)$$

Equation 2.22

where

$C_{Rd,c} = 0.18 / \gamma_c$ (recommended value; acc. to DIN EN 1992-1-1/NA:2010: $C_{Rd,c} = 0.15 / \gamma_c$)

$k = 1 + \sqrt{(200 / d)} \leq 2.0$ Scaling factor for considering the plate thickness

d Mean effective depth in [mm]

$\rho_1 = A_{sl} / (b_w \cdot d) \leq 0.02$ Longitudinal reinforcement ratio

A_{sl} Area of tensile reinforcement which extends at least by d beyond the considered cross-section and is effectively anchored there

f_{ck} Characteristic value of concrete compressive strength in [N/mm²]

b_w Cross-section width

d Effective depth of bending reinforcement in [mm]

$\sigma_{cp} = N_{Ed} / A_c < 0.2 \cdot f_{cd}$ Design value of concrete longitudinal stress in [N/mm²]

N_{Ed} Applied axial force in direction of principal shear force

You may apply the following minimum value of the shear force resistance $V_{Rd,c,min}$:

$$V_{Rd,c} = (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d \quad (6.2b)$$

Equation 2.23

where

$k_1 = 0.15$ (recommended value; acc. to DIN EN 1992-1-1/NA:2010: $k_1 = 0.12$)

$v_{min} = 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2}$ (recommended value) (6.3N)

according to DIN EN 1992-1-1/NA:2010:

$v_{min} = (0.0525 / \gamma_c) \cdot k^{3/2} \cdot f_{ck}^{1/2}$ for $d \leq 600\text{mm}$ (6.3aDE)

$v_{min} = (0.0375 / \gamma_c) \cdot k^{3/2} \cdot f_{ck}^{1/2}$ for $d > 800\text{mm}$ (6.3bDE)

for $600\text{ mm} < d \leq 800\text{ mm}$ interpolation possible

These equations are mostly intended for the one-dimensional design case (beam). There is only one provided longitudinal reinforcement from which the ratio of the longitudinal reinforcement is determined. For two-dimensional structural components with up to three reinforcement directions, it is not so easy to say how great the longitudinal reinforcement to be applied is.

The *Longitudinal Reinforcement* tab of the 1.4 *Reinforcement* window offers three possibilities to specify the provided longitudinal reinforcement for the shear force check.

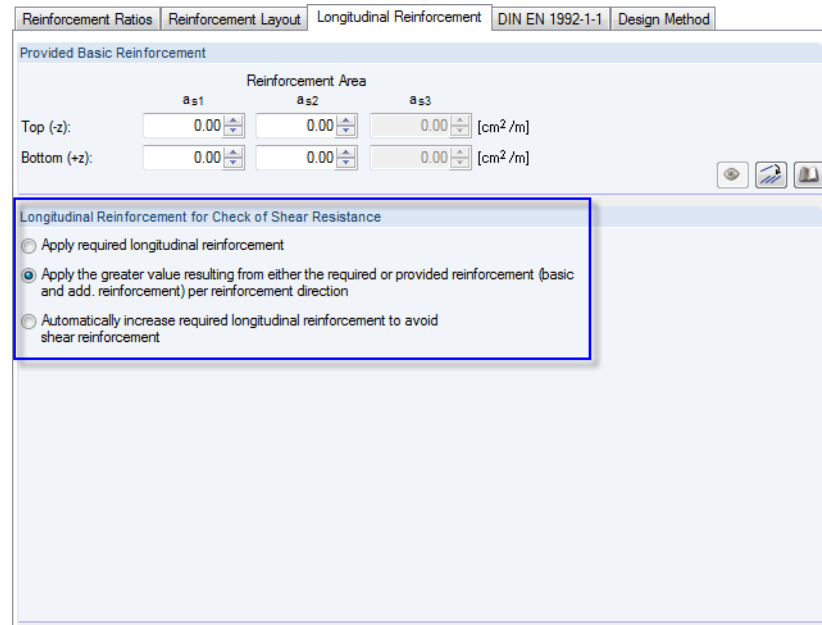


Figure 2.40: Window 1.4 *Reinforcement*, tab *Longitudinal Reinforcement*

Apply required longitudinal reinforcement

First, the program analyzes which reinforcement direction at the two surfaces of the plate after the design, including an applied tension force according to clause 6.2.3 (7), are subjected to tension. According to EN 1992-1-1, the provided ratio of longitudinal reinforcement can be determined only from the area of the provided tensile reinforcement.

In order to transform the reinforcement from the different reinforcement directions with tensile forces in direction β of the maximum shear force, the direction of the maximum shear force is determined as follows.

$$\beta = \arctan \frac{v_y}{v_x}$$

Equation 2.24

With this, the program determines the differential angle $\delta\phi_i$ between the respective reinforcement direction ϕ_i and the direction of the maximum shear force.

$$\delta\phi_i = \beta - \phi_i$$

Equation 2.25

With the differential angle $\delta\phi_i$, it is possible to determine the component $a_{sl,i}$ of a certain tensioned longitudinal reinforcement $a_{s,i}$.

$$a_{sl,i} = a_{s,i} \cdot \cos^2(\delta\phi_i)$$

Equation 2.26

In Equation 2.22, the tensile reinforcement a_{sl} to be applied for the determination of $V_{Rd,c}$ is the sum of the components from the individual reinforcement directions to which tension is assigned.

$$a_{sl} = \sum a_{s,i} \cdot \cos^2(\delta\phi_i)$$

Equation 2.27

Apply the greater value resulting from either required or provided reinforcement (basic and add. reinforcement) per reinforcement direction

The second option shown in Figure 2.40 on page 39 is used to determine the required tension reinforcement a_{sl} as described above. First, the program checks if a tension force is assigned to the required longitudinal reinforcement. The provided longitudinal reinforcement a_{sl} is then determined according to Equation 2.26 and Equation 2.27.

Then, the design shear resistance $V_{Rd,c}$ without shear reinforcement is determined. It might turn out that the check of shear force is possible without shear reinforcement. If the shear reinforcement $V_{Rd,ct}$ is negative and not sufficient, it is analyzed whether for a reinforcement direction the statically required longitudinal reinforcement $a_{s,dim}$ or the user-defined basic reinforcement $a_{s,def}$ is the greater reinforcement $a_{s,max}$.

With this greater reinforcement $a_{s,max}$, the provided longitudinal reinforcement a_{sl} is redetermined according to Equation 2.26 and Equation 2.27. Then, the shear resistance $V_{Rd,c}$ without shear reinforcement is redetermined.

If it turns out that the shear resistance $V_{Rd,c}$ without shear reinforcement and the respectively greater reinforcement (either the statically required or user-defined longitudinal reinforcement) is sufficient, the shear force check is satisfied. If despite this longitudinal reinforcement, the cross-section still cannot be designed because it is fully cracked, an according message appears.

If despite the application of the greater longitudinal reinforcement (statically required or user-defined longitudinal reinforcement) cannot be avoided, the shear resistance $V_{Rd,c}$ is redetermined with the statically required longitudinal reinforcement. It would not make much sense to apply the user-defined longitudinal reinforcement, and thus output it later than required, if by applying it a shear reinforcement cannot be avoided after all.

The shear force design comprises the check of the shear strength $V_{Rd,max}$ of the concrete strut and the design shear resistance $V_{Rd,s}$ of the shear reinforcement, as well as the determination of the required shear reinforcement.

Automatically increase longitudinal reinforcement to avoid shear reinforcement

In the third option (see Figure 2.40), the Equation 2.22 for $V_{Rd,c}$ is solved for the longitudinal reinforcement ratio ρ_l . $V_{Rd,c}$ is taken as the applied shear force V_{Ed} .

$$\rho_l = \frac{\left(\frac{V_{Ed} \cdot \gamma_c}{d \cdot b_w \cdot 0.15 \cdot \kappa \cdot \eta_1} + \frac{0.12 \cdot \gamma_c \cdot \sigma_{cd}}{0.15 \cdot \kappa \cdot \eta_1} \right)^3}{100 \cdot f_{ck}}$$

Equation 2.28

Thus, if the longitudinal reinforcement ratio is high enough, it becomes possible to do without shear reinforcement.

First, RF-CONCRETE Surfaces checks again the design shear resistance $V_{Rd,c}$ with the statically required longitudinal reinforcement. If this first design shear resistance is not enough, the longitudinal reinforcement a_{sl} is increased in the direction of the principal shear force. The longitudinal reinforcement a_{sl} cannot be increased arbitrarily.

The flowchart on the following page shows the cases in which shear reinforcement can be avoided and in which a shear reinforcement must be used with the statically required longitudinal reinforcement from the design.

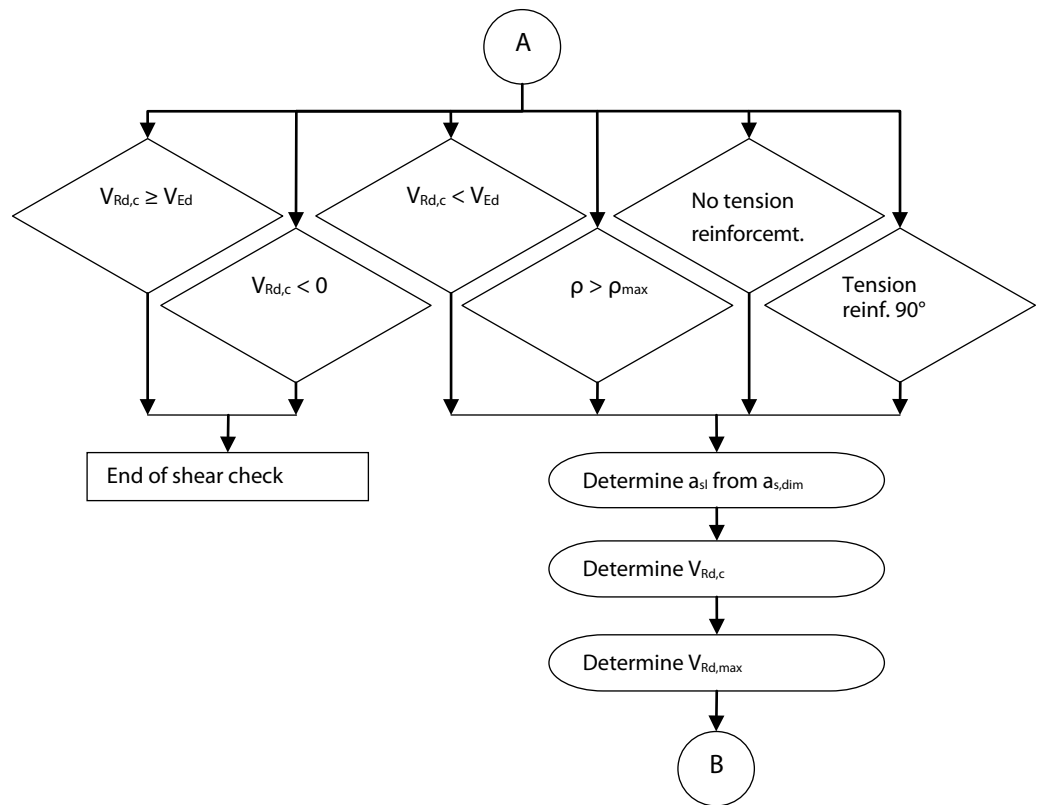


Figure 2.41: Flowchart for increase of longitudinal reinforcement to avoid shear reinforcement

The two paths on the left ($V_{Rd,c} \geq 0$, $V_{Rd,c} < 0$) indicate that the shear reinforcement is successfully avoided. The second path represents the possibility that even if the longitudinal reinforcement is increased, the design shear resistance $V_{Rd,c}$ remains negative, and therefore no check of shear force is possible for the fully cracked cross-section.

The other four paths ($V_{Rd,c} < V_{Ed}$, $\rho > \rho_{max}$, no tension reinforcement, tension reinforcement 90°) show the reasons why it is not possible to increase the longitudinal reinforcement. For example, despite the maximum longitudinal reinforcement ratio, shear reinforcement is unavoidable or the allowed longitudinal reinforcement ratio of the individual directions of reinforcement is exceeded. When the longitudinal reinforcement a_{sl} that is increased in the principal direction of the principal shear force is distributed to the individual directions of reinforcement, the program checks for each of these reinforcement directions if the user-defined longitudinal reinforcement ratio is not exceeded. If this is not the case, the longitudinal reinforcement ratio ρ_l is determined by using the option *Apply required longitudinal reinforcement*.

To better understand the two right paths, we must look at the longitudinal reinforcement increased in the direction of the principal shear force and distributed to the individual directions of reinforcement. If the determined longitudinal reinforcement ratio ρ_l is smaller than 0.02, the required longitudinal reinforcement ratio a_{sl} per meter is determined as follows.

$$a_{sl} = \rho_l \cdot d$$

Equation 2.29

The required longitudinal reinforcement is now applied to those reinforcement directions to tension is assigned. To this end, the program redetermines the angle deviation $\delta\varphi_i$ between the direction of the maximum shear force and the reinforcement direction with tension.

$$\delta\varphi_i = \beta - \varphi_i$$

Equation 2.30

The angle deviations $\delta\varphi_i$ are raised to the third power of the cosine and summed up as $\Sigma(\cos^3)$.

The component $a_{sl,i}$ of the required longitudinal reinforcement a_{sl} is then as follows.

$$a_{sl,i} = a_{sl} \cdot \frac{\cos(\delta\varphi_i)}{\sum \cos^3(\delta\varphi_i)}$$

Equation 2.31

This required reinforcement component $a_{sl,i}$ is now compared with the longitudinal reinforcement that is determined in the design. The greater reinforcement is governing.

In Equation 2.31, you can see that the denominator is problematic. This is the case if there is no reinforcement direction with tension (the sum of the third power of the angle deviations is calculated only with the tensioned directions); or because although there are reinforcement directions with tension, these run below 90° to the principal shear force direction and, thus, their cosine also yields the value zero. These possibilities are displayed in the two right paths of the flowchart.

In all cases, in which no solution is possible, the longitudinal reinforcement is not increased, and the option *Apply required longitudinal reinforcement* is used. The design shear resistance $V_{Rd,s}$ with shear reinforcement is to be determined with the shear reinforcement.

2.4.4.2 Design Shear Resistance with Shear Reinforcement

The following can be applied for structural components with shear reinforcement perpendicular to the component's axis ($\alpha = 90^\circ$):

$$V_{Rd,s} = (A_{sw} / s) \cdot z \cdot f_{ywd} \cdot \cot \theta \quad (6.8)$$

Equation 2.32

where

A_{sw}	Cross-sectional area of shear reinforcement
s	Spacing of links
z	Lever arm of internal forces
f_{ywd}	Design yield strength of the shear reinforcement
θ	Inclination of concrete strut

The inclination of the concrete strut θ may be selected within certain limits depending on the loading. In this way, the equation can take into account the fact that a part of the shear force is resisted by crack friction. Thus, the virtual truss is less stressed. These limits are specified in EN 1992-1-1, Equation (6.7N).

$$1.00 \leq \cot \theta \leq 2.5 \quad (6.7N)$$

Equation 2.33

Thus, the inclination of the concrete strut θ can vary between the following values:

	Minimum inclination	Maximum inclination
θ	21.8°	45.0°
$\cot \theta$	2.5	1.0

Table 2.1: Limits for concrete strut inclination according to EN 1992-1-1

DIN EN 1992-1-1/NA:2010 specifies the following:

$$1.00 \leq \cot \theta \leq (1.2 + 1.4 \cdot \sigma_{cd} / f_{cd}) / (1 - V_{Rd,cc} / V_{Ed}) \leq 3.0 \quad (6.7aDE)$$

Equation 2.34

where

$$V_{Rd,cc} = c \cdot 0.48 \cdot f_{ck}^{1/3} \cdot (1 - 1.2 \cdot \sigma_{cd} / f_{cd}) \cdot b_w \cdot z \quad (6.7bDE)$$

$$c = 0.5$$

$$\sigma_{cd} = N_{Ed} / A_c \quad N_{Ed} \text{ Design value of the longitudinal force in the cross-section due to external actions } (N_{Ed} > 0 \text{ as longitudinal compressive force})$$

The inclination of the concrete strut θ can vary between the following values:

	Minimum inclination	Maximum inclination
θ	18.4°	45.0°
$\cot\theta$	3.0	1.0

Table 2.2: Limits for inclination of concrete strut

A flatter concrete compression strut results in reduced tension forces within the shear reinforcement and thus in a reduced area of required reinforcement. In RF-CONCRETE Surfaces, the inclination of the concrete strut is defined in the *EN 1992-1-1* tab of the 1.4 *Reinforcement* window.

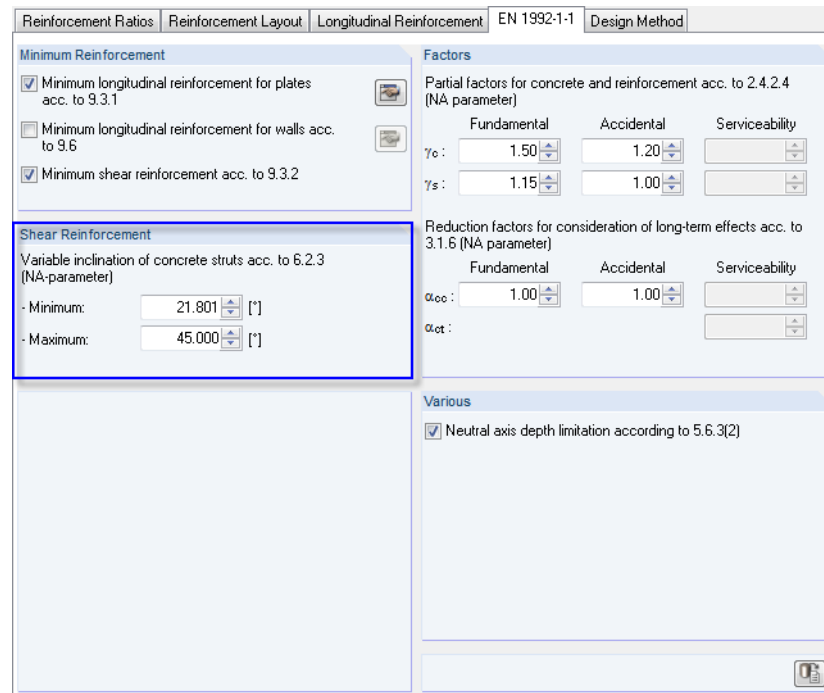


Figure 2.42: Window 1.4 *Reinforcement*, tab *EN 1992-1-1* with limits of the variable inclination of the strut

The measure of the minimum angle of the strut's inclination θ also depends on the applied internal forces V_{Ed} that can be taken into account only during the calculation. If the minimum angle of the strut's inclination is too small, the program shows an according message.

During the calculation, the given minimum value of the strut's inclination is used to determine the shear resistance $V_{Rd,max}$ of the concrete strut (see Equation 2.37). If it is smaller than the applied shear force V_{Ed} , a steeper strut inclination must be chosen. The strut inclination θ is increased until the following is given:

$$V_{Ed} \leq V_{Rd,max}$$

Equation 2.35

This angle of the strut inclination results in the smallest shear reinforcement.

2.4.4.3 Design of Concrete Strut

For structural components with **shear reinforcement perpendicular to the component's axis** ($\alpha = 90^\circ$), the shear resistance V_{Rd} is the smaller value from:

$$V_{Rd,s} = (A_{sw} / s) \cdot z \cdot f_{ywd} \cdot \cot \theta \quad (6.8)$$

Equation 2.36

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta) \quad (6.9)$$

Equation 2.37

where

A_{sw}	Cross-sectional area of shear reinforcement
s	Spacing of links
f_{ywd}	Design yield strength of the shear reinforcement
v_1	Reduction factor for concrete strength in case of shear cracks
α_{cw}	Coefficient taking account of the state of the stress in the compression chord

For structural components **with an inclined shear reinforcement**, the shear force resistance is the smaller value of:

$$V_{Rd,s} = (A_{sw} / s) \cdot z \cdot f_{ywd} \cdot (\cot \theta + \cot \alpha) \cdot \sin \alpha \quad (6.13)$$

Equation 2.38

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} \cdot (\cot \theta + \cot \alpha) / (1 + \cot^2 \theta) \quad (6.14)$$

Equation 2.39

2.4.4.4 Example for Shear Design

We want to look at the shear design of a plate according to EN 1992-1-1 by means of the design details (see the example for the statically required reinforcement, page 36).

In the details of the results, the shear forces determined in RFEM are shown at the beginning.

Design Report			
Internal Forces of Linear Statics			
Bending Moment			
x-Axis	m_x	-29.54	kNm/m
y-Axis	m_y	4.14	kNm/m
Differential Moment	m_{xy}	-1.03	kNm/m
Axial force with axial force vector in direction of			
Shear Force with Shear Force Vector in Direction of			
x-Axis	v_x	-6.932	kN/m
y-Axis	v_y	-72.968	kN/m
Principal Shear Force	v_{max}	73.297	kN/m
Principal Internal Forces			
Principal Moments			
Principal Axial Forces			
Principal Shear Force			
Principal Shear Force	v_{max}	73.297	kN/m
Direction	β_m	84.573	°

Figure 2.43: Internal forces of linear statics - shear forces

The required longitudinal reinforcement is determined from these internal forces.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Concrete Strut
Required Longitudinal Reinforcement Due to Design Membrane Forces
Bottom surface (+z)
into Reinforcement Direction 1
Design Membrane Force
Design membrane forces equal zero. No longitudinal reinforcement is required.
into Reinforcement Direction 2
into Reinforcement Direction 3
Top surface (-z)
into Reinforcement Direction 1
into Reinforcement Direction 2
into Reinforcement Direction 3

Figure 2.44: Required longitudinal reinforcement

The analysis of the shear resistance is shown in the details below. It starts with the determination of the allowed tensile reinforcement in the direction of the principal shear force.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Concrete Strut
Required Longitudinal Reinforcement Due to Design Membrane Forces
Shear Design
Applied tensile reinforcement determined from the required longitudinal reinforcement.
The shear reinforcement cannot be avoided in spite of the basic reinforcement.
Applied Longitudinal Reinforcement
Bottom surface (+z)
from reinforcement direction 1
from reinforcement direction 2
Required Longitudinal Reinforcement
State of Stress
Differential Angle in Direction of the Principal Shear
Second Power of Cosine of Differential Angle to 1
from reinforcement direction 3
Required Longitudinal Reinforcement
State of Stress
No Transformation (No Tension)
Top surface (-z)
from reinforcement direction 1
Required Longitudinal Reinforcement
State of Stress
Differential Angle in Direction of the Principal Shear
Second Power of Cosine of Differential Angle to 1
from reinforcement direction 2
from reinforcement direction 3

Figure 2.45: Shear Design - Applied tensile reinforcement

The second direction of reinforcement at the bottom surface of the plate and the first reinforcement direction at the top surface of the plate are the only directions of reinforcement to which tension is assigned and which approximately run parallel to the direction of the principal shear force.

These yield an *Applied Longitudinal Reinforcement* a_{sl} of 0.61 cm²/m.

The design shear force $V_{Rd,c}$ of the plate without shear reinforcement is determined with the following parameters:

$$C_{Rd,c} = 0.18 / \gamma_c = 0.18 / 1.15 = 0.12$$

$$k = 1 + \sqrt{(200 / d)} = 1 + \sqrt{(200 / 160)} = 2.11 \leq 2.00 \rightarrow k = 2.00 \quad d \text{ in [mm]}$$

$$d = 0.160 \text{ m}$$

$$\rho_l = a_{sl} / (b_w \cdot d) = 0.613 / (100 \cdot 16) = 0.000383 \leq 0.02$$

$$b_w = 1.00 \text{ m}$$

$$f_{ck} = 20.0 \text{ N/mm}^2 \quad \text{for concrete C20/25}$$

$$k_1 = 0.15$$

$$\sigma_{cp} = 0.00 \text{ N/mm}^2$$

$$V_{Rd,c} = [0.12 \cdot 2.00 \cdot (100 \cdot 0.000383 \cdot 20)^{1/3} + 0.15 \cdot 0.00] \cdot 1000 \cdot 160 = 35.135 \text{ kN/m}$$

The same result can be found in the design details:

▣ Design Report			
▣ Internal Forces of Linear Statics			
▣ Principal Internal Forces			
▣ Design Internal Forces			
▣ Concrete Strut			
▣ Required Longitudinal Reinforcement Due to Design Membrane Forces			
▣ Shear Design			
Applied tensile reinforcement determined from the required longitudinal reinforcement.			
The shear reinforcement cannot be avoided in spite of the basic reinforcement.			
▣ Applied Longitudinal Reinforcement	a_{sl}	0.61	cm ² /m
▣ Shear Resistance Without Shear Reinforcement			
▣ Shear Resistance According to Formula (6.2.a)			
Safety Factor	$C_{Rd,c}$	0.120	
▣ Factor of Size Effects	k	2.000	
Effective Depth	d	0.160	m
▣ Longitudinal Reinforcement Ratio	ρ_l	0.000	
Applied Longitudinal Reinforcement	a_{sl}	0.61	cm ² /m
Width of Member	b_w	1.000	m
Effective Depth	d	0.160	m
Characteristic Concrete Compressive Strength	f_{ck}	20.00	N/mm ²
Factor of Longitudinal Stress of Concrete	k_1	0.150	
▣ Longitudinal Reinforcement Ratio	σ_{cp}	0.00	N/mm ²
Axial Force in Direction of Principal Shear Force	n_{β}	0.000	kN/m
Width of Structural Member	b_w	1.000	m
Depth of Structural Member	h	20.00	cm
Width of Structural Member	b_w	1.000	m
Effective Depth	d	0.160	m
Shear Resistance According to Formula (6.2.a)	$V_{Rd,c,6.2a}$	35.142	kN/m

Figure 2.46: Shear design - shear resistance without shear reinforcement

The shear resistance $V_{Rd,c}$ of the plate without shear reinforcement is compared to the applied shear force V_{Ed} .

$$V_{Rd,c} = 35.142 \text{ kN/m} \geq V_{Ed} = 29.56 \text{ kN/m}$$

It has therefore been determined that the shear resistance of the plate without shear reinforcement is sufficient and no further checks are necessary.

2.4.5 Reinforcement Rules

For plates, the same reinforcement rules apply as presented in chapter 2.3.7, page 27.

In RF-CONCRETE Surfaces, user-defined specifications can be set in the 1.4 *Reinforcement* window. The following tabs are relevant:

- Tab *Reinforcement Layout* (see Figure 3.26, page 149)
- Tab *EN 1992-1-1* (see Figure 3.37, page 157)

If there are different specifications for the minimum shear reinforcement in the two tabs, the more unfavorable specification is applied.

The user-defined reinforcement specifications can be found in the design details.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Concrete Strut
Required Longitudinal Reinforcement Due to Design Membrane Forces
Shear Design
Statically Required Longitudinal Reinforcement
Minimum Reinforcement
Minimum Longitudinal Reinforcement
Minimum Reinforcement Ratio
Bottom surface (+z)
Top surface (-z)
Minimum Secondary Reinforcement
Minimum Secondary Reinforcement Ratio
Bottom surface (+z)
Top surface (-z)
Minimum Reinforcement
Bottom surface (+z)
Top surface (-z)
Check Maximum Reinforcement Ratio
Existing Longitudinal Reinforcement Ratio
Maximum longitudinal reinforcement ratio
Maximum longitudinal reinforcement ratio exceeded?

min ρ_Q	20.0	%
ρ_I	0.207	
max ρ_I	4.000	
(max ρ_I) < (ρ_I)	No	

Figure 2.47: Minimum reinforcement and maximum reinforcement ratio

Principal Internal Forces
Design Internal Forces
Concrete Strut
Required Longitudinal Reinforcement Due to Design Membrane Forces
Shear Design
Statically Required Longitudinal Reinforcement
Minimum Reinforcement
Check Maximum Reinforcement Ratio
Reinforcement to be used
Bottom surface (+z)
into Reinforcement Direction 1
Statically Required Reinforcement
Minimum Reinforcement
Minimum Longitudinal Reinforcement
Reinforcement used secondary reinforcement
into Reinforcement Direction 2
Statically Required Reinforcement
Minimum Reinforcement
Minimum Longitudinal Reinforcement
Reinforcement used as secondary reinforcement
into Reinforcement Direction 3
Statically Required Reinforcement
Minimum Reinforcement
Minimum Longitudinal Reinforcement
Reinforcement used as secondary reinforcement
Top surface (-z)
into Reinforcement Direction 1
into Reinforcement Direction 2
into Reinforcement Direction 3

$a_{s,+z,1}$	5.29	cm ² /m
$a_{s,stat,+z,1}$	5.29	cm ² /m
$a_{s,min,+z,1}$	2.08	cm ² /m
$a_{s,min longl,+z,1}$	2.08	cm ² /m
$a_{s,minQ,+z,1}$	0.00	cm ² /m
$a_{s,+z,2}$	1.06	cm ² /m
$a_{s,stat,+z,2}$	0.06	cm ² /m
$a_{s,min,+z,2}$	1.06	cm ² /m
$a_{s,min longl,+z,2}$	0.00	cm ² /m
$a_{s,minQ,+z,2}$	1.06	cm ² /m
$a_{s,+z,3}$	1.06	cm ² /m
$a_{s,stat,+z,3}$	0.00	cm ² /m
$a_{s,min,+z,3}$	1.06	cm ² /m
$a_{s,min longl,+z,3}$	0.00	cm ² /m
$a_{s,minQ,+z,3}$	1.06	cm ² /m
$a_{s,-z,1}$	0.00	cm ² /m
$a_{s,-z,2}$	0.00	cm ² /m
$a_{s,-z,3}$	0.00	cm ² /m

Figure 2.48: Reinforcement to be used

The reinforcement to be used is shown for the *Bottom surface (+z)* and *Top surface (-z)* in separate entries. The individual reinforcements in each direction indicate whether the reinforcement to be used is the statically required reinforcement or the minimum longitudinal reinforcement.

2.5 Shells

2.5.1 Design Concept



In terms of their internal forces, shells are a combination of walls (chapter 2.3) and plates (chapter 2.4), because they contain axial forces as well as moments.

All 3D model types (see Figure 2.1, page 10) are designed as shells. RF-CONCRETE Surfaces does this as follows: First, as shown in chapter 2.3 and 2.4, the design axial forces and design bending moments are determined separately. Again, they are based on the principal axial forces and principal bending moments of the linear RFEM plate analysis.

Such a design axial force and design moment is determined for each direction of reinforcement on each surface side. One or both internal forces can become zero – if the search for the optimal direction of the concrete strut in the determination of the design internal forces results in the fact that the reinforcement in this direction is not activated.

When the design internal forces are determined for the respective direction of reinforcement, the focus is on that direction of reinforcement for which design moments are available. For these moments, the program carries out a common one-dimensional design of a beam with the width of one meter. The goal of this design, however, is not to find a required reinforcement but to determine the lever arm of the internal forces.

When in this preliminary design the program has determined all lever arms of those design directions for which a design moment occurs, the program determines the smallest lever arm for each plate side. With this eccentricity, the moments of the linear plate analysis can now be transformed into membrane forces. To this end, the moments of the linear plate analysis is simply divided by the smallest lever arm z_{min} .

Now, if we add half the axial force from the linear plate analysis running perpendicular to the moment vector of the moment, which is divided by the lever arm of the internal forces, we obtain the final membrane force. This process can be expressed as follows:

$$n_{xs} = \frac{m_x}{z_{min}} + \frac{n_x}{2}$$

$$n_{ys} = \frac{m_y}{z_{min}} + \frac{n_y}{2}$$

$$n_{xys} = \frac{m_{xy}}{z_{min}} + \frac{n_{xy}}{2}$$

Equation 2.40

The moments at the top and bottom surface of the plate are considered with different signs.

The moments m_x , m_y , and m_{xy} and the axial forces n_x , n_y , and n_{xy} of the linear plate analysis are substituted by means of the lever arm z_{min} from the preliminary design by the membrane forces n_{xs} , n_{ys} , and n_{xys} . When this is done, the *principal membrane forces* n_{IIs} and n_{IIs} can be determined from these membrane forces for the bottom and top surface of the plate.

From the principal membrane forces n_{IIs} and n_{IIs} , the design membrane forces (see chapter 2.3, page 14) n_{α} , n_{β} , and n_{γ} are determined according to Equation 2.5 through Equation 2.7. The design membrane forces n_{α} , n_{β} , and n_{γ} are then assigned to the reinforcement directions φ_1 , φ_2 , and φ_3 . We obtain the design membrane forces n_1 , n_2 , and n_3 in the reinforcement directions.

From the design membrane forces, we can determine the required amount of steel. To do this, the design membrane forces are divided by the steel stresses σ_s that are determined during the determination of the minimum lever arm $z_{\min}$ in the respective reinforcement direction.

$$a_{s1} = \frac{n_1}{\sigma_s}$$

$$a_{s2} = \frac{n_2}{\sigma_s}$$

$$a_{s3} = \frac{n_3}{\sigma_s}$$

Equation 2.41

If the design member force is a compression force, the resisting axial force n_c is determined with the depth of the neutral axis x , which resulted from the determination of the lever arm.

$$n_c = f_{cd} \cdot b \cdot x$$

Equation 2.42

If the resisting axial force n_c of concrete is not sufficient, a compression reinforcement is determined for the differential force between the acting axial force and the resisting axial force. The design stress for this compression reinforcement results from the deformation of the compression reinforcement in the determination of the lever arm z .

If the lever arm was determined under the assumption of the strain range III, no compression reinforcement is determined, because it was not assumed. The strain ranges I through V are described in the following chapter in the part concerning the determination of the lever arm.

2.5.2 Lever arm of the Internal Forces

To this end, a rectangular cross-section is always designed with a width of one meter. The design is carried out directly with the rectangular stress distribution (see EN 1992-1-1, Figure 3.5). An iterative procedure would take too much time because of the high number of the necessary designs.

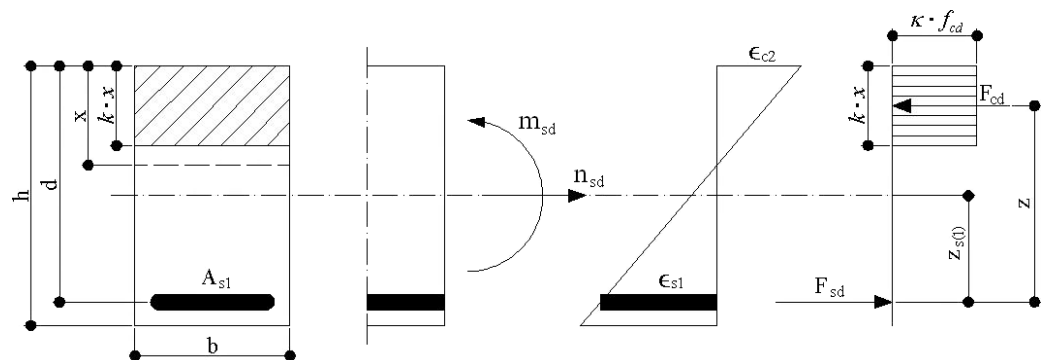


Figure 2.49: Calculation parameters of the design

For the figure above, the lever arm z is determined as follows:

$$z = d - \frac{k \cdot x}{2}$$

Equation 2.43

Figure 2.49 shows the state of strain than can result in the case of the simultaneous action of the moment and axial force. Five states of strain are possible (see Figure 2.50).

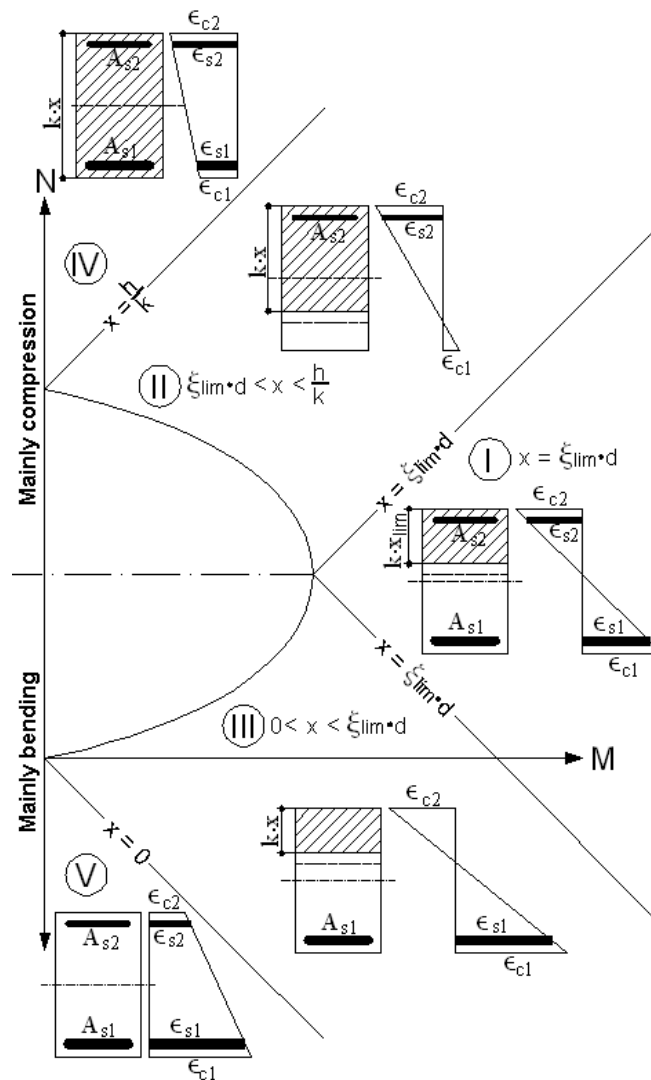


Figure 2.50: Areas of strain distribution

Range I

This shows a cross-section subjected to great bending. The depth of the neutral axis has reached its maximum value ($x = \xi_{\text{lim}} \cdot d$). Another increase of the section modulus is now only possible by using a compression reinforcement.

Range II

Mainly compression occurs in this range. The depth of the neutral axis is between the limits $\xi_{\text{lim}} \cdot d$ and h/k .

Range III

The applied moment is so small that the concrete compression zone (neutral axis) without the compression reinforcement can result in a sufficient section modulus. Depending on the applied moment, the limits of the neutral axis are between 0 and $\xi_{\text{lim}} \cdot d$.

Range IV

This range shows a fully compressed cross-section. The depth of the neutral axis is greater than h/k . This area also includes cross-sections that are subjected to compression only.

Range V

This state of strain is present if the tension force cracks a cross-section completely. This range also includes cross-sections that are subject to tension forces only.

The lever arm is determined for each strain range. With this, the moments of the linear plate analysis can be divided in membrane forces.

Lever arm for range I

For this range, the depth of the neutral axis is known: The concrete is fully utilized before compression reinforcement is applied.

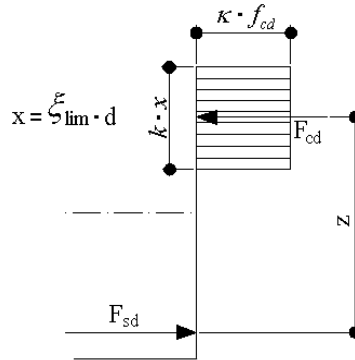


Figure 2.51: Lever arm z in case of maximum depth of neutral axis

For the maximum depth of the neutral axis x , the resisting concrete compressive force F_{cd} is obtained according to the following equation:

$$F_{cd} = \kappa \cdot f_{cd} \cdot k \cdot x_{lim} \cdot b$$

Equation 2.44

The limit section modulus $m_{sd,lim}$, which can be resisted by the cross-section without compression reinforcement, is determined as follows:

$$m_{sd,lim} = F_{cd} \cdot \left(d - \frac{k \cdot x_{lim}}{2} \right)$$

Equation 2.45

With the limit section modulus $m_{sd,lim}$, it is possible to determine the differential moment Δm_{sd} . This differential moment has to come from the compression reinforcement in order to reach an equilibrium with the applied moment $m_{sd(1)}$.

$$\Delta m_{sd} = m_{sd(1)} - m_{sd,lim}$$

Equation 2.46

The applied moment $m_{sd(1)}$ relates to the centroid of the tension reinforcement. It results from the applied moment m_{sd} , the acting axial force n_{sd} , and the distance $z_{s(1)}$ between the centroidal axis of the cross-section and the centroidal axis of the tension reinforcement.

$$m_{sd(1)} = m_{sd} - n_{sd} \cdot z_{s(1)}$$

Equation 2.47

With the differential moment Δm_{sd} , it is now possible to determine the required compression force $F_{sd(2)}$ in a compression reinforcement.

$$F_{sd(2)} = \frac{\Delta m_{sd}}{d - d_2}$$

Equation 2.48

where d is the effective depth of the tension reinforcement and d_2 the centroidal distance of the compression reinforcement from the edge of the concrete compression zone.

If we divide the applied moment $m_{sd(1)}$ that is related to the centroid of the tension reinforcement by the compression force F_{cd} and the force in the compression reinforcement $F_{sd(2)}$, we obtain the lever arm z .

$$z = \frac{m_{sd}}{F_{cd} + F_{sd(2)}}$$

Equation 2.49

Lever arm for range II

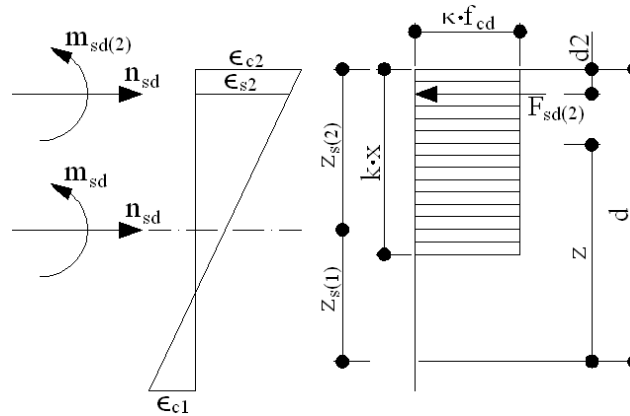


Figure 2.52: Determination of the lever arm for range II

In order to be able to determine the depth of the concrete neutral axis x , we first determine the design moment $m_{sd(2)}$ about the centroid of the compression reinforcement.

$$m_{sd(2)} = m_{sd} + n_{sd} \cdot Z_s(2)$$

Equation 2.50

Now the sum of the moments about the centroid of the compression reinforcement is calculated. These moments must result as equal to zero. On the side of the resistance, the moment is calculated from the resulting force F_{cd} of the concrete compression zones times its distance. In range II, there is no reinforcement in tension.

$$\sum m = F_{cd} \cdot \left(\frac{k \cdot x}{2} - d_2 \right) + m_{sd(2)} = 0$$

Equation 2.51

The resulting concrete compression force F_{cd} also contains the depth x of the concrete neutral axis.

$$F_{cd} = \kappa \cdot f_{cd} \cdot k \cdot x \cdot b$$

Equation 2.52

Thus, the equation for the determination of x is obtained as:

$$\kappa \cdot f_{cd} \cdot k \cdot x \cdot b \cdot \left(\frac{k \cdot x}{2} - d_2 \right) + m_{sd(2)} = \frac{\kappa \cdot f_{cd} \cdot b \cdot k^2 \cdot x^2}{2} - \kappa \cdot f_{cd} \cdot k \cdot x \cdot b \cdot d_2 + m_{sd(2)} = 0$$

$$x^2 - \frac{2 \cdot d_2 \cdot x}{k} + \frac{2 \cdot m_{sd(2)}}{\kappa \cdot f_{cd} \cdot b \cdot k^2} = 0 \quad \Rightarrow \quad x = \frac{d_2}{k} + \sqrt{\left(\frac{d_2}{k} \right)^2 - \frac{2 \cdot m_{sd(2)}}{\kappa \cdot f_{cd} \cdot b \cdot k^2}}$$

Equation 2.53

With the depth x of the neutral axis, it is possible to determine the lever arm z by subtracting from the effective height d half the depth of the neutral axis x , which is reduced by the factor x :

$$z = d - \frac{k \cdot x}{2}$$

Equation 2.54

Lever arm for range III

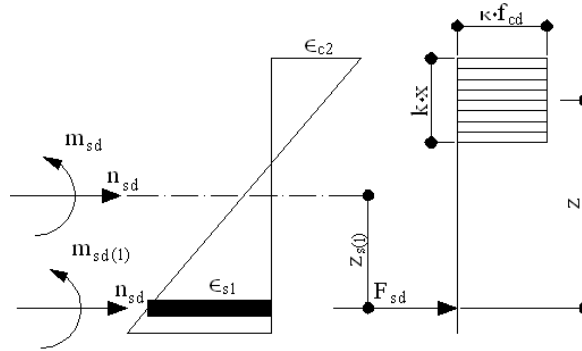


Figure 2.53: Determination of the lever arm for range III

To determine the depth x of the neutral axis, we first determine the design moment $m_{sd(1)}$ about the centroid of the tension reinforcement.

$$m_{sd(1)} = m_{sd} - n_{sd} \cdot z_{s(1)}$$

Equation 2.55

Now the sum of the moments about the centroid of the tension reinforcement is calculated. These moments must result as equal to zero. On the resistance side, the moment is calculated only from the resulting force F_{cd} of the concrete neutral axis times its distance. Then, the equilibrium of the moments about the position of the tension reinforcement is calculated.

$$\sum m = F_{cd} \cdot \left(d - \frac{k \cdot x}{2} \right) - m_{sd(1)} = 0$$

Equation 2.56

The depth x of the concrete neutral axis is also contained in the resulting concrete compression force F_{cd} (see Equation 2.52).

$$\kappa \cdot f_{cd} \cdot k \cdot b \cdot d \cdot x - \left(\frac{\kappa \cdot f_{cd} \cdot k^2 \cdot b}{2} \right) \cdot x^2 - m_{sd(1)} = x^2 - \frac{2 \cdot d}{k} \cdot x + \frac{2 \cdot m_{sd(1)}}{\kappa \cdot f_{cd} \cdot k^2 \cdot b} = 0$$

Equation 2.57

This quadratic equation can be solved as follows.

$$x = \frac{d}{k} + \sqrt{\frac{d^2}{k^2} - \frac{2 \cdot m_{sd(1)}}{\kappa \cdot f_{cd} \cdot k^2 \cdot b}}$$

Equation 2.58

With the depth x of the concrete neutral axis, the lever arm z can be determined by subtracting from the effective height d half the depth of the neutral axis x , which is multiplied by factor k :

$$z = d - \frac{k \cdot x}{2}$$

Equation 2.59

If the steel strain ϵ_s is greater than the maximum allowable steel strain ϵ_{ud} , x is calculated iteratively from the equilibrium conditions. The conversion factor κ and k for the concrete neutral axis is directly derived from the parabola-rectangle diagram of the concrete.

Lever arm for range IV

In a fully compressed cross-section, the lever arm is taken as the distance between both reinforcements.

$$z = d - d_2$$

Equation 2.60

For this area, a maximum utilization of the reinforcement is specified, that is, $\epsilon_s = \epsilon_{cu}$.

If the compression is approximately concentric ($e_d/h \leq 0.1$), the mean compressive strain should be limited to ϵ_{c2} according to EN 1992-1-1, clause 6.1 (5).

Lever arm of range V

In a fully cracked cross-section, the lever arm is also assumed as the distance between the two reinforcements (see Equation 2.60).

2.5.3 Determination of Design Membrane Forces

The design membrane forces for the abutment of a bridge are determined. For a closer analysis, we select the grid point No. 1 in surface No. 37.

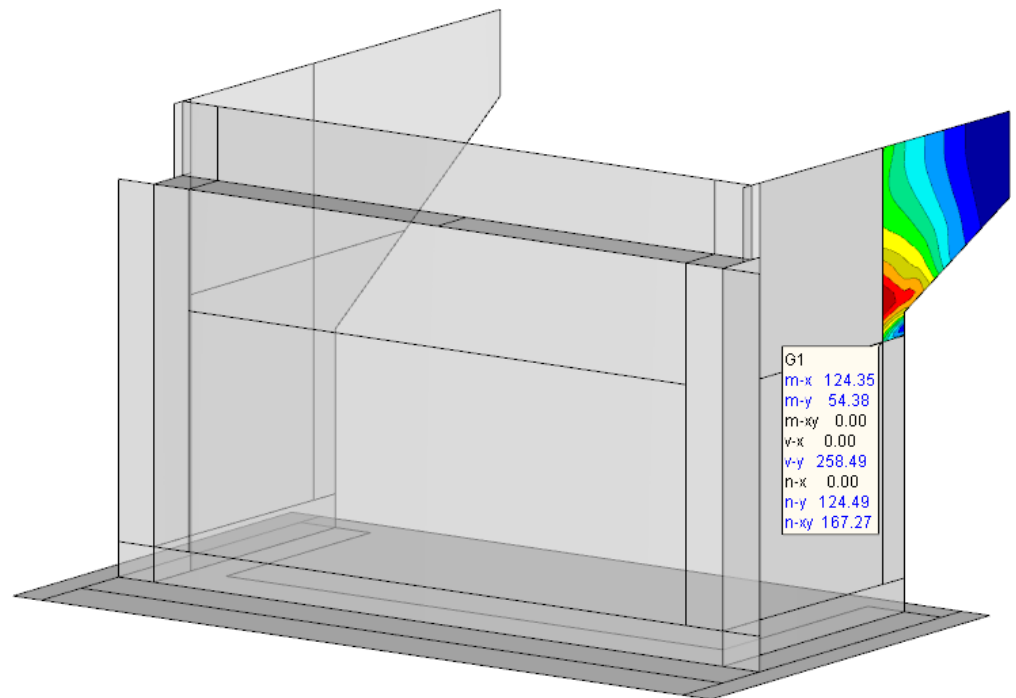
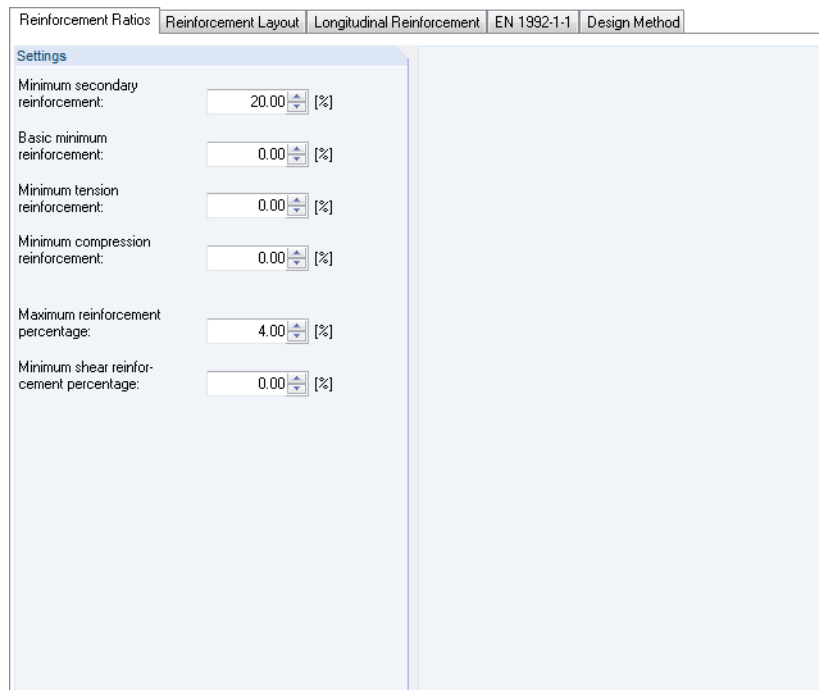


Figure 2.54: Bridge abutment - internal force in grid point R1

The analyzed surface No. 37 has a thickness of **129 cm**.

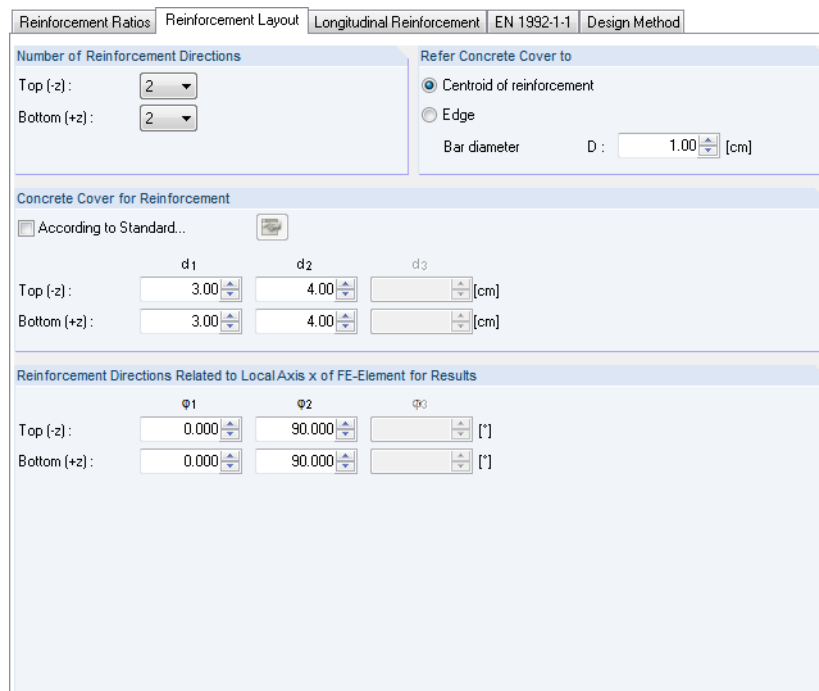
To design according to EN 1992-1-1, we select the concrete **C30/37** and the reinforcing steel **BSt 500 S (B)** in RF-CONCRETE Surfaces.

Further specifications in the 1.4 *Reinforcement* window:



Parameter	Value	Unit
Minimum secondary reinforcement:	20.00	[%]
Basic minimum reinforcement:	0.00	[%]
Minimum tension reinforcement:	0.00	[%]
Minimum compression reinforcement:	0.00	[%]
Maximum reinforcement percentage:	4.00	[%]
Minimum shear reinforcement percentage:	0.00	[%]

Figure 2.55: Window 1.4 *Reinforcement*, tab *Reinforcement Ratios*



Number of Reinforcement Directions		Refer Concrete Cover to	
Top (-z):	2	☒ Centroid of reinforcement	
Bottom (+z):	2	☐ Edge	
		Bar diameter	D : 1.00 [cm]

Concrete Cover for Reinforcement			
☐ According to Standard...			
	d1	d2	d3
Top (-z):	3.00	4.00	[cm]
Bottom (+z):	3.00	4.00	[cm]

Reinforcement Directions Related to Local Axis x of FE-Element for Results			
	φ1	φ2	φ3
Top (-z):	0.000	90.000	[°]
Bottom (+z):	0.000	90.000	[°]

Figure 2.56: Window 1.4 *Reinforcement*, tab *Reinforcement Layout*

Reinforcement Ratios | Reinforcement Layout | Longitudinal Reinforcement | EN 1992-1-1 | Design Method

Provided Basic Reinforcement

Reinforcement Area

	a_{s1}	a_{s2}	a_{s3}	
Top (-z):	0.00	0.00	0.00	[cm ² /m]
Bottom (+z):	0.00	0.00	0.00	[cm ² /m]

Longitudinal Reinforcement for Check of Shear Resistance

☐ Apply required longitudinal reinforcement
☒ Apply the greater value resulting from either the required or provided reinforcement (basic and add. reinforcement) per reinforcement direction
☐ Automatically increase required longitudinal reinforcement to avoid shear reinforcement

Figure 2.57: Window 1.4 Reinforcement, tab Longitudinal Reinforcement

Reinforcement Ratios | Reinforcement Layout | Longitudinal Reinforcement | EN 1992-1-1 | Design Method

Minimum Reinforcement

☒ Minimum longitudinal reinforcement for plates acc. to 9.3.1
☒ Minimum longitudinal reinforcement for walls acc. to 9.6
☒ Minimum shear reinforcement acc. to 9.3.2

Shear Reinforcement

Variable inclination of concrete struts acc. to 6.2.3 (NA-parameter)

- Minimum: 21.801 [°]
 - Maximum: 45.000 [°]

Factors

Partial factors for concrete and reinforcement acc. to 2.4.2.4 (NA parameter)

	Fundamental	Accidental	Serviceability
γ_c :	1.50	1.20	
γ_s :	1.15	1.00	

Reduction factors for consideration of long-term effects acc. to 3.1.6 (NA parameter)

	Fundamental	Accidental	Serviceability
α_{cc} :	1.00	1.00	
α_{ct} :			

Various

☒ Neutral axis depth limitation according to 5.6.3(2)

Figure 2.58: Window 1.4 Reinforcement, tab EN 1992-1-1

Reinforcement Ratios | Reinforcement Layout | Longitudinal Reinforcement | EN 1992-1-1 | Design Method

Design Internal Forces

☐ No optimization of design internal forces (Recommended for parts loaded mostly by pressure)
☒ Optimization of design internal forces (Recommended for parts loaded mostly by bending or tension)

☐ Apply reinforcement close to strut up to maximum difference: 8.000 [°]

☒ Equilibrium of deformations
☐ Equilibrium of forces
☐ Automatic

☐ Increase of tension reinforcement due to shear according to 6.2.3(7)

Figure 2.59: Window 1.4 Reinforcement, tab Design Method

2.5.3.1 Design Moments

The design details of the grid points show the internal forces interpolated from the FE nodes. In the General Data, the 3D type of model was specified (see Figure 2.1, page 10). Hence, there are the moments m_x , m_y , and m_{xy} as well as the axial forces n_x , n_y , and n_{xy} in the surface.

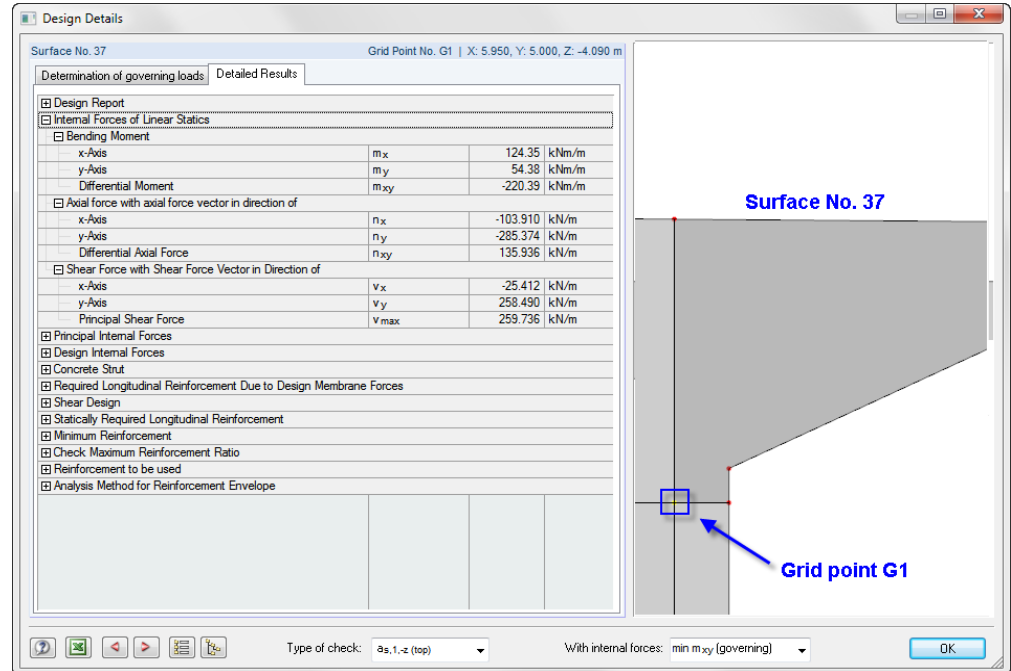


Figure 2.60: Internal forces of linear analysis

The principal internal forces are determined from the RFEM internal forces of the linear analysis. The principal internal forces are determined according to the equations described in the chapters 2.3 and 2.4.

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Principal Moments			
Bottom surface (+z)			
First Principal Moment	$m_{I,+z}$	312.51	kNm/m
Second Principal Moment	$m_{II,+z}$	-133.78	kNm/m
Direction	$\alpha_{m,+z}$	-40.490	°
Top surface (-z)			
First Principal Moment	$m_{I,-z}$	133.78	kNm/m
Second Principal Moment	$m_{II,-z}$	-312.51	kNm/m
Direction	$\alpha_{m,-z}$	49.510	°
Principal Axial Forces			
Bottom surface (+z)			
First Principal Axial Force	$n_{I,+z}$	-31.207	kN/m
Second Principal Axial Force	$n_{II,+z}$	-358.076	kN/m
Direction	$\alpha_{n,+z}$	28.139	°
Top surface (-z)			
First Principal Axial Force	$n_{I,-z}$	-31.207	kN/m
Second Principal Axial Force	$n_{II,-z}$	-358.076	kN/m
Direction	$\alpha_{n,-z}$	28.139	°
Principal Shear Force			
Principal Shear Force	v_{max}	259.736	kN/m
Direction	β_m	95.615	°

Figure 2.61: Principal internal forces

For shells, the principal axial forces are shown for both plate surfaces, because they are required for the shell design. Unlike the moments, the principal axial forces at the bottom and top surface of the plate are the same.

Now the design moments are determined from the principal moments $m_{I,+z}$ and $m_{II,+z}$ at the bottom surface. For this purpose, the program first determines the differential angles $\alpha_{m,+z}$ and $\beta_{m,+z}$ between the direction $\alpha_{m,+z}$ of the first principal axial force $m_{I,+z}$ at the bottom surface and both the directions of reinforcement $\varphi_1 = 0^\circ$ and $\varphi_2 = 90^\circ$.

Design Report		
Internal Forces of Linear Statics		
Principal Internal Forces		
Design Internal Forces		
Bottom surface (+z)		
Design Bending Moments		
Principal Moments		
Differential Angle Between $\alpha_{,+z,+z}$ and		
Reinforcement Direction 1	$\Delta\Phi_{+z,1,b}$	40.490 °
Reinforcement Direction 2	$\Delta\Phi_{+z,2,b}$	130.490 °
Differential Angle According to Baumann		
1st Differential Angle	$\alpha_{m,+z}$	40.490 °
2nd Differential Angle	$\beta_{m,+z}$	130.490 °

Figure 2.62: Differential angle

Now we search for the direction of a moment that stiffens the two-directional reinforcement mesh. As already shown for walls and plates, the moment directions can only be one of the two angles between the reinforcement directions. The analysis for the bottom surface yields these directions for the assumed concrete struts:

Design Report		
Internal Forces of Linear Statics		
Principal Internal Forces		
Design Internal Forces		
Bottom surface (+z)		
Design Bending Moments		
Principal Moments		
Differential Angle Between $\alpha_{,+z,+z}$ and		
Differential Angle According to Baumann		
First Assumption of the Strut Direction γ		
New Differential Angle	$\gamma_{m,+z,1}$	85.490 °
Design Bending Moments by Baumann		
1st Design Bending Moment	$m_{\alpha,+z,1}$	344.73 kNm/m
2nd Design Bending Moment	$m_{\beta,+z,1}$	274.76 kNm/m
3rd Design Bending Moment	$m_{\gamma,+z,1}$	-440.77 kNm/m
Strut direction permissible?	$m_{strut,+z,1}$	Yes
Second Assumption of the Strut Direction γ		
New Differential Angle	$\gamma_{m,+z,2}$	175.490 °
Design Bending Moments by Baumann		
1st Design Bending Moment	$m_{\alpha,+z,2}$	-96.04 kNm/m
2nd Design Bending Moment	$m_{\beta,+z,2}$	-166.01 kNm/m
3rd Design Bending Moment	$m_{\gamma,+z,2}$	440.77 kNm/m
Strut direction permissible?	$m_{strut,+z,2}$	No

Figure 2.63: Directions γ of the concrete strut

Only the assumption of the direction $\gamma_{m,+z,1}$ of 85.489° proves to be permissible. No optimization of this angle is carried out anymore. Hence, the final design moments $m_{end,+z,\varphi 1}$ and $m_{end,+z,\varphi 2}$ result in both directions of reinforcement:

Design Report		
Internal Forces of Linear Statics		
Principal Internal Forces		
Design Internal Forces		
Bottom surface (+z)		
Design Bending Moments		
Principal Moments		
Differential Angle Between $\alpha_{,+z,+z}$ and		
Differential Angle According to Baumann		
First Assumption of the Strut Direction γ		
Second Assumption of the Strut Direction γ		
Energy = Sum of abs(Design Bending Moments)		
Governing Strut		
Governing Design Bending Moments		
Final Design Bending Moments		
into Direction 1	$m_{end,+z,\Phi 1}$	344.73 kNm/m
into Direction 2	$m_{end,+z,\Phi 2}$	274.76 kNm/m
into Strut Direction	$m_{end,+z,strut}$	-440.77 kNm/m

Figure 2.64: Final design moments

2.5.3.2 Design Axial Forces

The design axial forces $n_{end,+z,\varphi 1}$ and $n_{end,+z,\varphi 2}$ are determined according to the same principle.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Bottom surface (+z)
Design Bending Moments
Design Axial Forces
Principal Axial Forces
Differential Angle Between α , +z, +z and
Differential Angle According to Baumann
First Assumption of the Strut Direction γ
Second Assumption of the Strut Direction γ
Energy = Sum of abs(Design Axial Forces)
Governing Strut
Governing Design Axial Forces
into Direction 1
into Direction 2
into Strut Direction
Find optimal strut direction?
Final Design Axial Forces
into Direction 1
into Direction 2
into Strut Direction

$n_{+z,\varphi 1}$	32.026	kN/m
$n_{+z,\varphi 2}$	-149.438	kN/m
$n_{end,+z,strut}$	-271.872	kN/m
Strut opti,n,+z	No	
$n_{end,+z,\varphi 1}$	32.026	kN/m
$n_{end,+z,\varphi 2}$	-149.438	kN/m
$n_{end,+z,strut}$	-271.872	kN/m

Figure 2.65: Design axial forces

2.5.3.3 Lever of Internal Forces

With the design internal forces for the directions of reinforcement $\varphi_1 = 0^\circ$ and $\varphi_2 = 90^\circ$, it is now possible to determine the lever arm of the internal forces.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Bottom surface (+z)
Design Bending Moments
Design Axial Forces
Design Internal Forces
into Reinforcement Direction 1
Design Bending Moment
Design Axial Force
into Reinforcement Direction 2
Design Bending Moment
Design Axial Force

$m_{end,+z,\varphi 1}$	344.73	kNm/m
$n_{end,+z,\varphi 1}$	32.026	kN/m
$m_{end,+z,\varphi 2}$	274.76	kNm/m
$n_{end,+z,\varphi 2}$	-149.438	kN/m

Figure 2.66: Design internal forces

As described in chapter 2.5.2 on page 49, a preliminary design is carried out with both determined internal forces for both directions of reinforcement. This preliminary design serves to determine the lever arm of the internal forces. The lever arm is determined from the state of strain due to the design internal forces (see the following figure).

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Design Internal Forces			
Bottom surface (+z)			
Design Bending Moments			
Design Axial Forces			
Design Internal Forces			
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Due to Design in Reinforcement Direction 1	$z_{+z, \Phi 1}$	1.250	m
Due to Design in Reinforcement Direction 2	$z_{+z, \Phi 2}$	1.239	m
Moment about the center of tension reinforcement	$m_{Sd(1),+z,2}$	365.17	kNm/m
Moment about the center of compression reinforcement	$m_{Sd(2),+z,2}$	184.35	kNm/m
Maximum Depth of Compression Zone	$x_{lim,+z,2}$	0.563	m
Limiting Axial Force	$n_{Sd,lim,+z,2}$	-9107.140	kN/m
Limiting Moment	$m_{Sd,lim,+z,2}$	9253.03	kNm/m
Limiting axial force larger than the design axial force?	$n_{Sd,lim,+z,2} > n_{Sd,+z,2}$	No	
Radicand (Value Below the Root)	$Radicand_{+z,2}$	22033.60	cm ²
Radicand less than zero?	$Radicand_{+z,2} < 0$	No	
Calculated Depth of the Compression Zone	$x_{calc,+z,2}$	0.031	m
Calculated depth of compression zone negative?	$x_{calc,+z,2} < 0$	No	
Existing Ratio Depth of the Compression Zone / Effective Depth	$\psi_{+z,2}$	0.025	
Existing ratio greater than maximum ratio of x / d ?	$\psi_{+z,2} > \psi_{lim}$	No	
Governing Range (see manual)	Range	III	
Lever Arm of the Internal Forces	$z_{+z,2}$	1.239	m

Figure 2.67: Lever arm of internal forces

The value 1.239 m is obtained for the smaller and, therefore, governing lever arm $z_{min,+z}$.

2.5.3.4 Membrane Forces

With the governing lever arm from the preliminary design, it is now possible to transform the internal forces of the linear plate analysis into membrane forces. To this end, the equations presented on page 48 are used.

$$n_{sx,+z} = \frac{m_x}{z_{min,+z}} + \frac{n_x}{2} = \frac{124.35}{1.239} + \frac{-103.911}{2} = 48.408 \text{ kN/m}$$

$$n_{sy,+z} = \frac{m_y}{z_{min,+z}} + \frac{n_y}{2} = \frac{54.36}{1.239} + \frac{-285.386}{2} = -98.819 \text{ kN/m}$$

$$n_{xy,+z} = \frac{m_{xy}}{z_{min,+z}} + \frac{n_{xy}}{2} = \frac{-220.39}{1.239} + \frac{135.935}{2} = -109.910 \text{ kN/m}$$

These membrane forces can also be found in the design details.

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Design Internal Forces			
Bottom surface (+z)			
Design Bending Moments			
Design Axial Forces			
Design Internal Forces			
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Membrane Force			
into Direction of the x-Axis	$n_{sx,+z}$	48.414	kN/m
Bending Moment	m_x	124.35	kNm/m
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Axial Force	n_x	-103.910	kN/m
into Direction of the y-Axis	$n_{sy,+z}$	-98.796	kN/m
Moment	m_y	54.38	kNm/m
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Axial Force	n_y	-285.374	kN/m
Differential Membrane Force	$n_{xy,+z}$	-109.923	kN/m
Moment	m_{xy}	-220.39	kNm/m
Minimum Lever Arm of the Internal Forces	$z_{min,+z}$	1.239	m
Axial Force	n_{xy}	135.936	kN/m

Figure 2.68: Membrane forces

2.5.3.5 Design Membrane Forces

From the membrane forces $n_{sx,+z}$, $n_{sy,+z}$ and $n_{xy,+z}$ (which replace the moments m_{sx} , m_{sy} , m_{xy} , and axial forces n_x , n_y , n_{xy} of the linear slab analysis), the principal membrane forces $ns_{I,+z}$ and $ns_{II,+z}$ are determined.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Bottom surface (+z)
Design Bending Moments
Design Axial Forces
Design Internal Forces
Minimum Lever Arm of the Internal Forces
Membrane Force
Design Membrane Forces
Principal Membrane Forces
First Principal Membrane Force
Second Principal Membrane Force
Direction
Quotient $k = ns_{II,+z}/ns_{I,+z}$

Figure 2.69: Design membrane forces

The design membrane forces are determined from the principal membrane forces according to Equation 2.5 through Equation 2.7 (see page 16). They can be found in the design details.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Bottom surface (+z)
Design Bending Moments
Design Axial Forces
Design Internal Forces
Minimum Lever Arm of the Internal Forces
Membrane Force
Design Membrane Forces
Principal Membrane Forces
Differential Angle Between $\alpha_{,+z,+z}$ and
Differential Angle According to Baumann
First Assumption of the Strut Direction γ
New Differential Angle
Design Membrane Forces by Baumann
1st Design Membrane Force
2nd Design Membrane Force
3rd Design Membrane Force
Strut direction permissible?
Second Assumption of the Strut Direction γ
Energy = Sum of abs(Design Membrane Forces)
Governing Strut
Governing Design Membrane Forces
Final Design Membrane Forces
into Direction 1
into Direction 2
into Strut Direction

Figure 2.70: Final design membrane forces

With the final design membrane forces $ns_{end,+z,q1}$ and $ns_{end,+z,q2}$, the program determines the required reinforcement areas of a two-directional reinforcement mesh for the surface side.

The reinforcement mesh is stiffened by a concrete strut.

The magnitude of the stiffening strut force $ns_{end,+z,strut}$ is shown under the final design membrane forces. The force magnitude is -219.859 kN/m.

Similarly, the design membrane forces and the stiffening force of the concrete strut are determined for the top surface of the plate.

2.5.4 Analysis of Concrete Struts

To analyze the concrete strut of a shell, it is divided in three surface layers that are subjected to the design membrane forces.

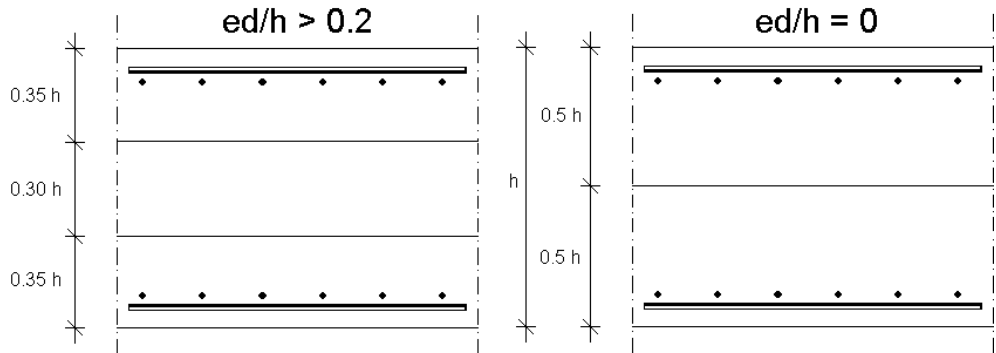


Figure 2.71: Surface layers thicknesses for shells mainly subjected to moments (left) and compression force (right)

For shells, where the applied moment is relatively great in relation to the applied axial force ($e_d/h > 0.2$), the thickness h_E of the two outer layers is reduced to $0.35 \cdot d$. For shells subjected to approximately concentric compression, the equivalent layer thickness h_E is increased to half the plate thickness h . If the relative eccentricity of the axial force e_d/h is between 0 and 0.2, then the thickness of the layers is interpolated.

For e_d , the greater value of the quotients of m_x/n_x and m_y/n_y is taken.

For the analysis of the concrete strut, the strut's compression force to be resisted, $n_{strut,z}$ is compared to the resisting axial force of the equivalent layer $n_{strut,d}$.

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Design Internal Forces			
Concrete Strut			
Thickness of Surface	h_E	45.15	cm
Related Load Eccentricity	e_d/h	0.928	
Predominant Strain	Under Stress	Compression	
Factor of Surface Thickness	f_{hE}	0.350	
Bottom surface (+z)			
Design Membrane Forces in Strut Direction	$n_{strut,z}$	-219.846	kN/m
Concrete Membrane Force Resistance	$n_{strut,d}$	-7224.000	kN/m
Width of Surface	b	1.000	m
Thickness of Surface	h_E	45.15	cm
Applied Concrete Compressive Strength	$f_{cd,08}$	16.00	N/mm ²
Design Unconfined Concrete Compressive Strength	f_{cd}	20.00	N/mm ²
Coefficient of Maximum Utilization	ξ_{fcd}	0.800	
Failure of concrete strut?	$n_{strut,d} < n_{strut,z}$	No	

Figure 2.72: Concrete strut and thickness of equivalent layer

The resisting axial force $n_{strut,d}$ depends on the thickness h_E of the equivalent layer and the applied concrete strength $f_{cd,08}$.

The first step to determine the thickness of the equivalent layer is to determine the provided load eccentricities in x- and y-direction from the internal forces of the linear plate analysis:

$$e_{dx} = \left| \frac{m_x}{n_x} \right| = \left| \frac{124.35}{-103.911} \right| = 1.197 \text{ m}$$

$$e_{dy} = \left| \frac{m_y}{n_y} \right| = \left| \frac{54.36}{-285.386} \right| = 0.190 \text{ m}$$

The greatest load eccentricity in x-direction is computed as governing. This can be used to determine the relative load eccentricity e_d/h .

$$e_d/h = \frac{1.197}{1.29} = 0.928 > 0.2$$

Since the relative load eccentricity is greater than 0.2, this is regarded as a shell that is predominantly subjected to bending. The factor f_{hE} for the determination of the equivalent layer thickness is 0.35.

The thickness h_E of the equivalent layer is therefore determined as follows:

$$h_E = f_{hE} \cdot h = 0.35 \cdot 129 = 45.15 \text{ cm}$$

The design value of the concrete compressive strength is reduced according to the recommendations from SCHLAICH/SCHÄFER (in: Betonkalender 1993/II, page 378) to 80 %. This recommendation can also be found in EN 1992-1-1, clause 6.5.2, regulating the design of struts in strut-and-tie models.

$$f_{cd} = f_{ck} / \gamma_c = 30 / 1.5 = 20 \text{ N/mm}^2$$

$$f_{cd,08} = 0.8 \cdot 20 = 16 \text{ N/mm}^2$$

This value can also be found in the design details (see Figure 2.72).

With this value, it is possible to determine the resisting force of the concrete strut $n_{\text{strut},d}$.

$$n_{\text{strut},d} = b \cdot h_E \cdot f_{cd,08} = 100 \cdot 45.15 \cdot 16 = 7224.00 \text{ kN/m}$$

The analysis of the concrete strut for the top surface is done similarly.

2.5.5 Required Longitudinal Reinforcement

The longitudinal reinforcement to be used at the bottom surface is determined from the design membrane forces. In the design details, the output is subdivided for the two directions of reinforcement.

☐ Design Report			
☐ Internal Forces of Linear Statics			
☐ Principal Internal Forces			
☐ Design Internal Forces			
☐ Concrete Strut			
☐ Required Longitudinal Reinforcement Due to Design Membrane Forces			
☐ Bottom surface (+z)			
☐ into Reinforcement Direction 1	$a_{s,dim,+z,1}$	3.40	cm ² /m
Design Membrane Force	$n_{s, end,+z, \phi 1}$	158.337	kN/m
☐ Design Stress	$\sigma_{s,+z,1}$	465.93	N/mm ²
☐ into Reinforcement Direction 2	$a_{s,dim,+z,2}$	0.24	cm ² /m
Design Membrane Force	$n_{s, end,+z, \phi 2}$	11.127	kN/m
☐ Design Stress	$\sigma_{s,+z,2}$	465.93	N/mm ²

Figure 2.73: Required longitudinal reinforcement

$$a_{s,dim+z,1} = \frac{n_{s, end,+z, \phi 1}}{\sigma_{s,+z,1}} = \frac{158.344}{465.93} = 3.40 \text{ cm}^2/\text{m}$$

$$a_{s,dim+z,2} = \frac{n_{s, end,+z, \phi 2}}{\sigma_{s,+z,2}} = \frac{11.116}{465.93} = 0.24 \text{ cm}^2/\text{m}$$

The reinforcement for the top surface is determined similarly.

2.5.6 Shear Design

In the shear design, the applied tension reinforcement is determined first.

Shear Design			
Applied tensile reinforcement determined from the required longitudinal reinforcement.			
Application of the basic reinforcement is not necessary because the required longitudinal reinforcement is $s_{l,req} = 0.00 \text{ cm}^2/\text{m}$			
Applied Longitudinal Reinforcement	a_{sl}	1.54	cm^2/m
Bottom surface (+z)			
from reinforcement direction 1	$a_{sl,+z,1}$	0.03	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim+z,1}$	3.40	cm^2/m
State of Stress	Stress +z,1	Tension	
Differential Angle in Direction of the Principal Shear	$\Delta\phi_{+z,1}$	84.385	°
Second Power of Cosine of Differential Angle to 1	$\cos^2(\Delta\phi_{+z,1})$	0.010	
from reinforcement direction 2	$a_{sl,+z,2}$	0.24	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim+z,2}$	0.24	cm^2/m
State of Stress	Stress +z,2	Tension	
Differential Angle in Direction of the Principal Shear	$\Delta\phi_{+z,2}$	5.615	°
Second Power of Cosine of Differential Angle to 1	$\cos^2(\Delta\phi_{+z,2})$	0.990	
Top surface (-z)			
from reinforcement direction 1	$a_{sl,-z,1}$	0.02	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim-z,1}$	2.00	cm^2/m
State of Stress	Stress -z,1	Tension	
Differential Angle in Direction of the Principal Shear	$\Delta\phi_{-z,1}$	84.385	°
Second Power of Cosine of Differential Angle to 1	$\cos^2(\Delta\phi_{-z,1})$	0.010	
from reinforcement direction 2	$a_{sl,-z,2}$	1.26	cm^2/m
Required Longitudinal Reinforcement	$a_{s,dim-z,2}$	1.27	cm^2/m
State of Stress	Stress -z,2	Tension	
Differential Angle in Direction of the Principal Shear	$\Delta\phi_{-z,2}$	5.615	°
Second Power of Cosine of Differential Angle to 1	$\cos^2(\Delta\phi_{-z,2})$	0.990	

Figure 2.74: Applied tension reinforcement

From all reinforcement layers and directions, a total of $1.54 \text{ cm}^2/\text{m}$ tension reinforcement can be applied. With this, the resisting shear force $V_{Rd,c}$ without shear reinforcement is determined.

Shear Design			
Applied tensile reinforcement determined from the required longitudinal reinforcement.			
Application of the basic reinforcement is not necessary because the required longitudinal reinforcement is $s_{l,req} = 0.00 \text{ cm}^2/\text{m}$			
Applied Longitudinal Reinforcement	a_{sl}	1.54	cm^2/m
Shear Resistance Without Shear Reinforcement			
Design Shear Resistance Without Shear Reinforcement			
Safety Factor	$C_{Rd,c}$	0.120	
Factor of Size Effects	k	1.399	
Effective Depth	d	1.255	m
Longitudinal Reinforcement Ratio	ρ_l	0.000	
Applied Longitudinal Reinforcement	a_{sl}	1.54	cm^2/m
Width of Member	b_w	1.000	m
Effective Depth	d	1.255	m
Characteristic Concrete Compressive Strength	f_{ck}	30.00	N/mm^2
Factor of Longitudinal Stress of Concrete	k_1	0.150	
Longitudinal Concrete Stress	σ_{cp}	0.33	N/mm^2
Axial Force in Direction of Principal Shear Force	n_{β}	-425.773	kN/m
Width of Structural Member	b_w	1.000	m
Depth of Structural Member	h	129.00	cm
Width of Structural Member	b_w	1.000	m
Effective Depth	d	1.255	m
Design Shear Resistance Without Shear Reinforcement	$V_{Rd,c,6.2a}$	213.271	kN/m
Minimum shear resistance according to (6.2.b)			
Factor of Compressive Strength	v_{min}	0.317	
Factor of Size Effects	k	1.399	
Characteristic Concrete Compressive Strength	f_{ck}	30.00	N/mm^2
Factor of Longitudinal Stress of Concrete	k_1	0.150	
Longitudinal Concrete Stress	σ_{cp}	0.33	N/mm^2
Axial Force in Direction of Principal Shear Force	n_{β}	-425.773	kN/m
Width of Structural Member	b_w	1.000	m
Depth of Structural Member	h	129.00	cm
Width of Structural Member	b_w	1.000	m
Effective Depth	d	1.255	m
Minimum shear resistance according to (6.2.b)	$V_{Rd,c,6.2b}$	460.326	kN/m
Shear Resistance Without Shear Reinforcement	$V_{Rd,c}$	460.326	kN/m
Shear reinforcement required?	Check $V_{Rd,c}$	No	
Design Shear Resistance of a Member Without Shear	$V_{Rd,c}$	460.326	kN/m
Design Shear Force	V_{Ed}	259.736	kN/m
Shear capacity without shear reinforcement sufficient. No further checks.			

Figure 2.75: Design shear resistance without shear reinforcement

With the applied longitudinal reinforcement ratio, the longitudinal reinforcement ratio ρ_l is determined:

$$\rho_l = A_{sl} / (b_w \cdot d) = 1.54 / (100 \cdot 125.5) = 0.00012 \leq 0.02$$

In a 3D type of model (unlike in a plate), an additional axial force can occur. It must be considered by using the according concrete longitudinal stress.

$$\sigma_{cp} = N_{\beta} / (b_w \cdot h) = -425.783 / (100 \cdot 129) = -0.33 \text{ N/mm}^2$$

The factor for taking account of the plate depth is calculated as follows:

$$k = 1 + \sqrt{(200 / d)} = 1 + \sqrt{(200 / 1255)} = 1.399 \leq 2.0 \quad d \text{ in [mm]}$$

Moreover, the following factors are included in the design:

Factor of concrete longitudinal stress	$k_1 = 0.15$
Concrete compressive strength for C30/37	$f_{ck} = 30.0 \text{ N/mm}^2$
Safety factor	$C_{rd,c} = 0.18 / \gamma_c = 0.18 / 1.5 = 0.12$

Thus, the design shear resistance $V_{Rd,c}$ without shear reinforcement can be determined according to Equation (6.2a):

$$\begin{aligned} V_{Rd,c} &= [C_{rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d = \\ &= [0.12 \cdot 1.399 \cdot (100 \cdot 0.00012 \cdot 30.0)^{1/3} + 0.15 \cdot 0.33] \cdot 1000 \cdot 1255 = 212.00 \text{ kN/m} \end{aligned}$$

The minimum value of the design shear resistance $V_{Rd,c}$ without shear reinforcement is determined according to Equation (6.2b) from the minimum reinforcement ratio v_{min} :

$$v_{min} = 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2} = 0.035 \cdot 1.399^{3/2} \cdot 30.0^{1/2} = 0.317$$

$$V_{Rd,c} = (0.317 + 0.15 \cdot 0.33) \cdot 1000 \cdot 1255 = 459.96 \text{ kN/m}$$

The design shear resistance of the plate, $V_{Rd,c} = 459.96 \text{ kN/m}$, is greater than the applied shear force $V_{Ed} = 259.726 \text{ kN/m}$. Hence, no shear reinforcement is required in the example.

If the shear resistance of the plate is not sufficient, the program first checks if the maximum shear resistance of the concrete strut $V_{Rd,max}$ is sufficient. $V_{Rd,max}$ is determined with the minimum inclination of the strut θ . If the design shear resistance of the concrete strut is greater than the applied shear force V_{Ed} , it is possible to determine the statically required shear reinforcement a_{sw} . Then, the design for the shear reinforcement $V_{Rd,sv}$ is carried out.

2.5.7 Statically Required Longitudinal Reinforcement

The table of the design details summarizes the statically required longitudinal reinforcement.

▣ Design Report
▣ Internal Forces of Linear Statics
▣ Principal Internal Forces
▣ Design Internal Forces
▣ Concrete Strut
▣ Required Longitudinal Reinforcement Due to Design Membrane Forces
▣ Shear Design
▣ Statically Required Longitudinal Reinforcement
▣ Bottom surface (+z)
▣ into Reinforcement Direction 1
Due to Design
▣ into Reinforcement Direction 2
Due to Design
▣ Top surface (-z)
▣ into Reinforcement Direction 1
Due to Design
▣ into Reinforcement Direction 2
Due to Design

Figure 2.76: Statically required longitudinal reinforcement

For each direction of reinforcement, the table shows which design is governing for the statically required reinforcement.

In the example, all longitudinal reinforcements result from the bending design as shell. In other cases, a required longitudinal reinforcement to avoid shear reinforcement would also be possible.

2.5.8 Minimum Longitudinal Reinforcement

The statically required longitudinal reinforcement is now compared to the minimum reinforcement. Unfortunately, none of the standards available in RF-CONCRETE Surfaces provide any regulations on the minimum reinforcement for shells. Therefore, we have to analyze in which constellation of moment and axial force we regard the element as a wall (mainly subjected to compression) or a plate (mainly subjected to bending). The distinguishing criterion is the related load eccentricity e_d / h in the ultimate limit state (ULS):

$$\frac{e_d}{h} = \frac{\frac{m}{n}}{h}$$

Equation 2.61

where

- m Moment of linear plate analysis (ULS)
- n Axial force of linear plate analysis (ULS)
- h Plate depth

In a design point, there are moments and axial forces in x- as well as y-direction. Hence, the relative load eccentricity for each design point is the greatest quotient from moment over axial force of both directions.

In RF-CONCRETE Surfaces, the following is equally specified for all standards:

$$\frac{e_d}{h} > 3.5: \quad \text{Mainly subjected to bending} \Rightarrow \text{reinforcement rules for **plates**}$$

$$\frac{e_d}{h} \leq 3.5: \quad \text{Mainly subjected to compression} \Rightarrow \text{reinforcement rules for **walls**}$$

This rule can be found in EN 1992-1-1, clause 9.3: Solid slabs, and clause 9.6: Walls. The minimum reinforcements are described in chapter 2.3.7, page 25, and chapter 2.4.5, page 46 in the reinforcement rules for walls and plates.

In our example, the system is mainly subjected to flexure. The following minimum reinforcement is shown in the design details.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Concrete Strut
Required Longitudinal Reinforcement Due to Design Membrane Forces
Shear Design
Statically Required Longitudinal Reinforcement
Minimum Reinforcement
Minimum Longitudinal Reinforcement
Minimum Reinforcement Ratio
Minimum Tension Longitudinal Reinforcement Ratio
Minimum Compression Longitudinal Reinforcement Ratio
General Minimum Reinforcement Ratio
Referred to Cross-Section
Bottom surface (+z)
Main longitudinal reinforcement with tension of this side
Direction of the Main Longitudinal Reinforcement
Statically Required Reinforcement
Statically Required Reinforcement
Minimum Longitudinal Reinforcement into Direction 1
State of Stress
Minimum Tension Reinforcement
Minimum Compression Reinforcement
General Minimum Reinforcement
Minimum Reinforcement for Walls
Minimum Longitudinal Reinforcement for Parts with Ductile
1st Calculated Value for Minimum Longitudinal Reinforc
2nd Calculated Value for Minimum Longitudinal Reinfor
Minimum Longitudinal Reinforcement into Direction 2

Figure 2.77: Minimum reinforcement

2.5.9 Reinforcement to be Used

The reinforcement to be used is determined from the statically required reinforcement and the minimum reinforcement.

Design Report
Internal Forces of Linear Statics
Principal Internal Forces
Design Internal Forces
Concrete Strut
Required Longitudinal Reinforcement Due to Design Membrane Forces
Shear Design
Statically Required Longitudinal Reinforcement
Minimum Reinforcement
Check Maximum Reinforcement Ratio
Reinforcement to be used
Bottom surface (+z)
into Reinforcement Direction 1
Statically Required Reinforcement
Minimum Reinforcement
into Reinforcement Direction 2
Statically Required Reinforcement
Minimum Reinforcement
into Reinforcement Direction 3
Top surface (-z)
into Reinforcement Direction 1
Statically Required Reinforcement
Minimum Reinforcement
into Reinforcement Direction 2
Statically Required Reinforcement
Minimum Reinforcement
into Reinforcement Direction 3

Figure 2.78: Reinforcement to be used

It is also possible to display the reinforcement area for the grid point No. 1 graphically.

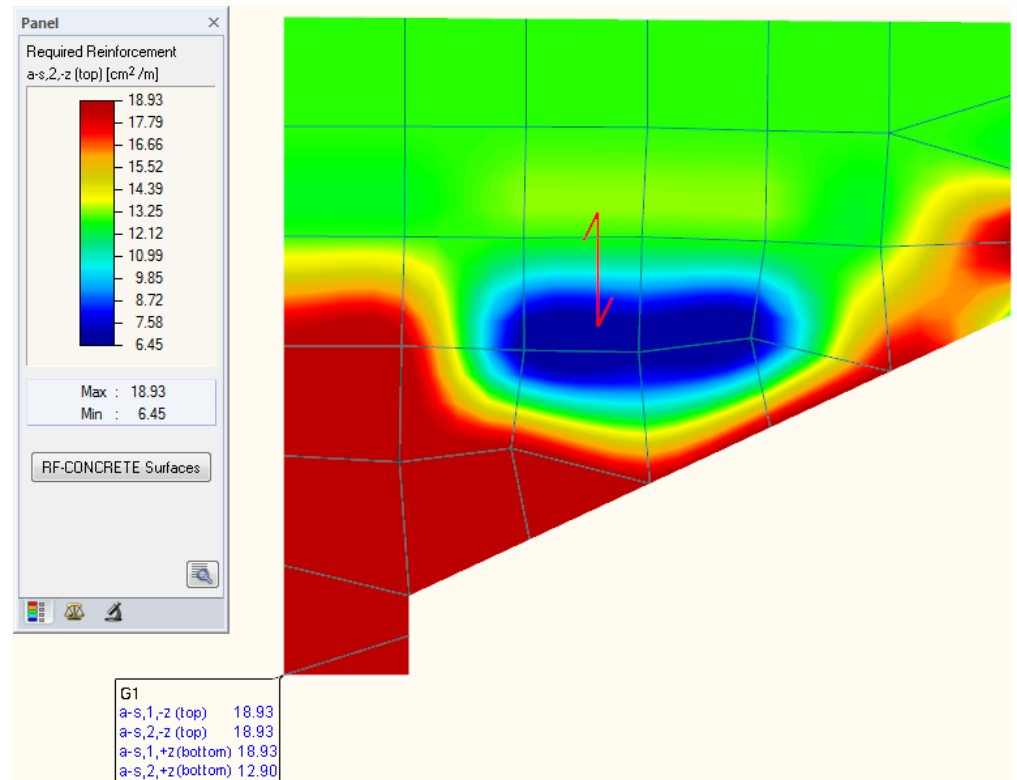


Figure 2.79: Graphic of the reinforcement for surface No. 37

2.6 Serviceability Limit State

The serviceability limit state designs consist of various individual designs that are specified in the following Eurocode chapters. The relevant clauses for EN 1992-1-1 are:

- Limitation of stresses: clause 7.2
- Crack control: clause 7.3
- Deflection control: clause 7.4

In the reinforced concrete standards, the designs listed above are always described for linear elements. In the ultimate limit state, as mentioned in the previous part of this manual, the design situation of a surface element is transformed into the design of several linear elements in the individual reinforcement directions. The transformation procedure is also used in the serviceability limit state.

2.6.1 Design Internal Forces

Unlike the transformation procedure for the ultimate limit state, it is not possible to carry out a purely geometrical division of the principal internal forces into forces in the individual directions of reinforcement. Such a division assumes a deformation ratio of 1.0 for the actual provided reinforcement. To have the same strain in both reinforcement directions for different design forces in these directions of reinforcement, however, according reinforcement areas would have to be provided. In the serviceability limit state, however, the design internal forces are searched for a provided reinforcement.

In the serviceability limit state, no required reinforcement is determined. Instead, the actual provided deformation ratio is determined with the provided reinforcement. In all cases in which the applied reinforcement deviates from the required reinforcement, the actual provided deformation ratio of the reinforcements does not equal the value 1.0.

Thus, the assumption of an identical deformation ratio is therefore not valid. A different deformation ratio must be found that confirms its resulting design internal forces. In solving this problem, the geometrical relationship between the deformation ratio and the direction of the concrete strut plays an important role.

BAUMANN [1] writes the following on this point: If we neglect the compression strain of the concrete because it usually small compared to the strain of the reinforcement, we obtain from Figure 38 the compatibility condition

$$\frac{\varepsilon_y}{\varepsilon_x} = \frac{\sin^2(\beta - \gamma)}{\sin^2(\gamma - \alpha)}$$

Equation 2.62

The figure on the following page represents the "Figure 38" mentioned by Baumann. It represents the compatibility condition of the strains in a two-directional reinforcement mesh.

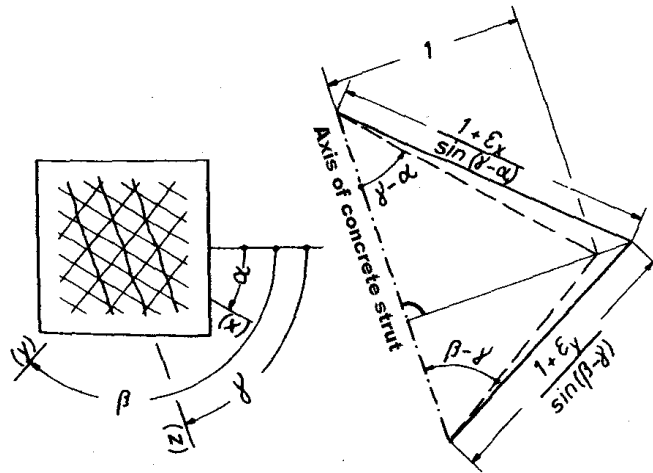


Figure 2.80: Compatibility of the strains

In Equation 2.62, ϵ_y and ϵ_x are the strains of two reinforcement directions. The angles α and β represent the intermediate angles between the principal force direction and the respective direction of reinforcement. The smaller intermediate angle is called α . The angle γ refers to the differential angle between the direction of the concrete strut and the direction of the first principal internal force.

The angles α and β cannot be changed by selecting the reinforcement direction. In contrast to this, the angle γ changes if due to the differently stiff directions of reinforcement a different direction of the concrete strut is necessary to stiffen the reinforcement mesh.

The design internal forces in the individual directions of reinforcement depend on the selected direction of the concrete strut. With these design internal forces, it is possible to determine the stresses in the reinforcements of the individual directions. Based on these stresses, the various standards give equations with which it is possible to determine the mean strains of the reinforcement relative to the concrete. In EN-1992-1-1, this is done according to Equation (7.9):

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \cdot \frac{f_{ct,eff}}{\rho_{p,eff}} \cdot (1 + \alpha_e \cdot \rho_{p,eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

Equation 2.63

Then, it is possible to determine the quotient from the differences of the strains between concrete and reinforcing steel of the second and first reinforcement direction.

$$Q_\epsilon = \frac{(\epsilon_{sm} - \epsilon_{cm})_{\varphi 2}}{(\epsilon_{sm} - \epsilon_{cm})_{\varphi 1}}$$

Equation 2.64

Equation 2.62 also gives a quotient of the strains, derived from the geometric relationships.

$$Q_{\epsilon,geo} = \frac{\epsilon_{\varphi 2}}{\epsilon_{\varphi 1}} = \frac{\sin^2(\beta - \gamma)}{\sin^2(\gamma - \alpha)}$$

Equation 2.65

For both quotients, the strain of the second direction of reinforcement is in the numerator. This is based on the assumption that the first reinforcement direction forms the smaller differential angle with the first principal internal force. If the second reinforcement direction with the first principal internal force forms the smaller differential angle, the strain of the first reinforcement direction would be in the numerator of the quotient.

Both quotients depend on the selected direction of the concrete strut. Now the program tries to select the direction of the concrete strut in such a way that both quotients become identical.

$$Q_{\varepsilon} = Q_{\varepsilon, \text{geo}}$$

Equation 2.62

If after one calculation run the geometrical deformation ratio $Q_{\varepsilon, \text{geo}}$ still does not correspond to the actual deformation ratio, the program specifies a new direction of the concrete strut and determines its resulting geometric deformation ratio. This process is repeated iteratively until a convergence is reached.

The determination of the design internal forces by means of the selection of the appropriate concrete strut direction is the most demanding part of the serviceability limit state design. If the selected provided reinforcement approximately corresponds to the statically required reinforcement for the analyzed service load magnitudes, the design internal forces only marginally differ from those internal forces that would result from an assumed strain ratio of 1.0. Therefore, RF-CONCRETE Surfaces offers the possibility to determine the design internal forces with an assumed deformation ratio of 1.0



The design internal forces for the serviceability limit state design are determined only if the cracking of the concrete leads to an activation of the reinforcement. To this end, the program analyzes the concrete tensile stresses caused by the first principal internal force.

2.6.2 Principal Internal Forces

If the first principal internal force is negative, we assume uncracked concrete in the area of the analyzed surface element. For walls, only the magnitude of the concrete stress is checked. For plates, at least at this surface side no serviceability limit state design is carried out.

If for a **Wall**, the first principal axial force is a tension force, the provided concrete tensile stress is determined according to the following equation.

$$\sigma_{c,l} = \frac{n_l}{A_c} = \frac{n_l}{b \cdot h}$$

Equation 2.62

If for a **Plate** the first principal moment is a positive moment, the provided concrete tensile stress is determined as follows.

$$\sigma_{c,l} = \frac{m_l}{W} = \frac{m_l \cdot 6}{b \cdot h^2}$$

Equation 2.62

If the linear-elastically determined stress $\sigma_{c,l}$ is greater than the mean axial tensile strength f_{ctm} , a cracked concrete is assumed. Only then does RF-CONCRETE Surfaces determine the design internal forces for the individual directions of reinforcement and performs the serviceability limit state designs mentioned at the beginning of chapter 2.6.

2.6.3 Provided Reinforcement

Before RF-CONCRETE Surfaces designs the serviceability limit state, it checks the provided reinforcement: First, the program uses the internal forces of the serviceability to perform a design like for the ultimate limit state. The design results in a structurally required reinforcement which is then compared to the user-defined provided reinforcement.

If the provided reinforcement is smaller than the statically required reinforcement, or if the design reveals any non-designable situations, the serviceability limit state design will not be performed. The problematic zones of the surface elements are indicated as non-designable.

2.6.4 Serviceability Limit State Designs

The following example illustrates how the various serviceability limit state designs are implemented in RF-CONCRETE Surfaces. In this example, we analyze a rectangular slab. The first applied principal moment m_I is greater than zero, the second applied moment m_{II} equals zero.

The design is carried out according to EN 1992-1-1 according to the analytical method.

2.6.4.1 Input Data for Example

Geometric specifications

Slab depth:	$d = 20 \text{ cm}$	
Rectangular reinforcement:	$\phi_1 = 30^\circ$	$\phi_2 = 120^\circ$
Centroid of concrete cover:	$d_1 = 3.0 \text{ cm}$	$d_2 = 4.2 \text{ cm}$

Material

Concrete:	C30/37
Reinforcing steel:	B 500 S (B)

2.6.4.2 Check of Principal Internal Forces

First, the program checks if the concrete cracks under the principal moment at the ULS. In the serviceability design details of the relevant grid point, you can see that this is indeed the case:



Internal Forces of Linear Statics			
Check if internal forces cause cracks			
Bottom surface (+z)			
First Principal Moment	$m_{I,+z}$	-7.16	kNm/m
The first principal moment is negative. No check of this side.			
Top surface (-z)			
Tensile Stress of Concrete	$\sigma_{c,I,-z}$	5.05	N/mm ²
First Principal Moment	$m_{I,-z}$	33.65	kNm/m
Section modulus	S	6666.67	cm ³
Width of the Element	b_w	1.000	m
Depth of Structural Member	h	0.200	m
Mean Axial Tensile Strength	f_{ctm}	2.90	N/mm ²
Concrete cracks on this side. Longitudinal reinforcement is activated.			
Concrete cracks on one side. Longitudinal Reinforcement is activated and check is completed.			

Figure 2.81: Checking the principal internal forces

The linear-elastically determined stress $\sigma_{c,I,-z}$ at the upper concrete edge is compared to the mean axial tensile strength f_{ctm} of 2.9 N/mm² for a concrete C30/37.

$$\sigma_{c,I,-z} = \frac{m_{I,-z}}{W} = \frac{m \cdot 6}{b \cdot h^2} = \frac{33.65 \cdot 6}{1.0 \cdot 0.2^2} = 5.05 \text{ N/mm}^2$$

The concrete edge strain $\sigma_{c,I,-z} = 5.05 \text{ N/mm}^2$ thus clearly exceeds the tensile strength f_{ctm} . Therefore, the reinforcement is activated for the serviceability limit state, too.

2.6.4.3 Required Reinforcement for ULS

The ultimate limit state design for the top surface of the plate is carried out with the following values:

Design Report			
Internal Forces of Linear Statics			
Principal Internal Forces			
Design Bending Moments			
Bottom surface (+z)			
Top surface (-z)			
Principal Moments			
First Principal Moment	$m_{I,-z}$	56.08	kNm/m
Second Principal Moment	$m_{II,-z}$	11.93	kNm/m
Direction	$\alpha_{b,-z}$	0.000	°
Quotient $k = m_{II,-z}/m_{I,-z}$	$k_{m,-z}$	0.213	
Differential Angle Between $\alpha_{,-z,-z}$ and			
Differential Angle According to Baumann			
First Assumption of the Strut Direction γ			
Second Assumption of the Strut Direction γ			
Energy = Sum of abs(Design Bending Moments)			
Governing Strut			
First Assumption for Direction γ	$\gamma_{m,-z,1}$	75.000	°
Strut Direction	$\phi_{strut,m,-z}$	75.000	°
Governing Design Bending Moments			
into Direction 1	$m_{-z,\phi 1}$	64.16	kNm/m
into Direction 2	$m_{-z,\phi 2}$	42.08	kNm/m
into Strut Direction	$m_{end,-z,strut}$	-38.23	kNm/m
Find optimal strut direction?	Strut opti,m,-z	No	
Final Design Bending Moments			
into Direction 1	$m_{end,-z,\phi 1}$	64.16	kNm/m
into Direction 2	$m_{end,-z,\phi 2}$	42.08	kNm/m
into Strut Direction	$m_{end,-z,strut}$	-38.23	kNm/m

Figure 2.82: Design internal forces ULS

Final design bending moments:

$$m_{end,-z,\phi 1} = 64.16 \text{ kNm/m}$$

$$m_{end,-z,\phi 2} = 42.08 \text{ kNm/m}$$

$$m_{end,-z,strut} = -38.23 \text{ kNm/m}$$

Direction of concrete strut:

$$\phi_{strut,m,-z} = 75.0^\circ$$

From the design internal forces, we obtain the following required reinforcement for the top surface:

2.1 Required Reinforcement Total											
Surface No.	A	B	C	D	E	F	G	H		I	J
	Grid Point	Point-Coordinates [m]			Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement		Provided	Unit
	X	Y	Z								
1	G4	0.000	0.500	0.000	$\bar{a}_{s,1,-z} \text{ (top)}$	8.97	11.31	0.00	0.00	0.00	cm ² /m
1	G4	0.000	0.500	0.000	$\bar{a}_{s,2,-z} \text{ (top)}$	6.15	11.31	0.00	0.00	0.00	cm ² /m
1	G6	1.000	0.500	0.000	$\bar{a}_{s,1,-z} \text{ (bottom)}$	0.14	0.00	0.14	0.14	0.14	cm ² /m
1	G6	1.000	0.500	0.000	$\bar{a}_{s,2,-z} \text{ (bottom)}$	0.70	0.00	0.70	0.70	0.70	cm ² /m

Figure 2.83: Required reinforcement

2.6.4.4 Specification of a Reinforcement

At the top surface of the plate, we select for both directions a reinforcement from the rebars with the diameter d_s of 12 mm at a distance l_s of 10.0 cm.

The following provided reinforcement results:

$$\text{prov } a_{s1,-z} = \frac{d_s^2}{4} \cdot \pi \cdot \frac{100 \text{ cm/m}}{l_s} = \frac{(1.2 \text{ cm})^2}{4} \cdot \pi \cdot \frac{100 \text{ cm/m}}{10.0 \text{ cm}} = 11.31 \text{ cm}^2/\text{m}$$



We enter these values in the *Longitudinal Reinforcement* tab of the *1.4 Reinforcement* window. Alternatively, they can be selected by means of the [Rebars] button (see Figure 2.105, page 86).

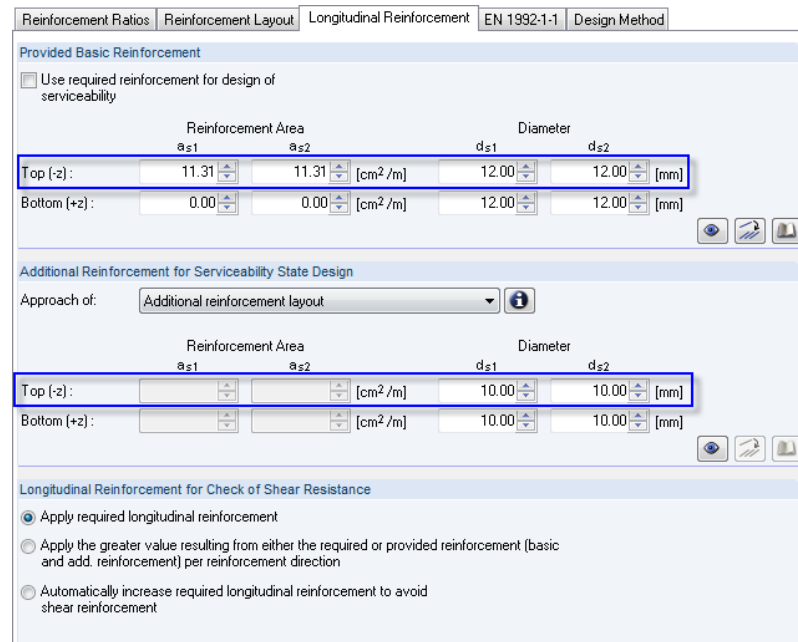


Figure 2.84: Window 1.4 Reinforcement, tab Longitudinal Reinforcement to define basic and additional reinforcement

With this reinforcement diameter, we obtain the following centroids of the concrete cover:

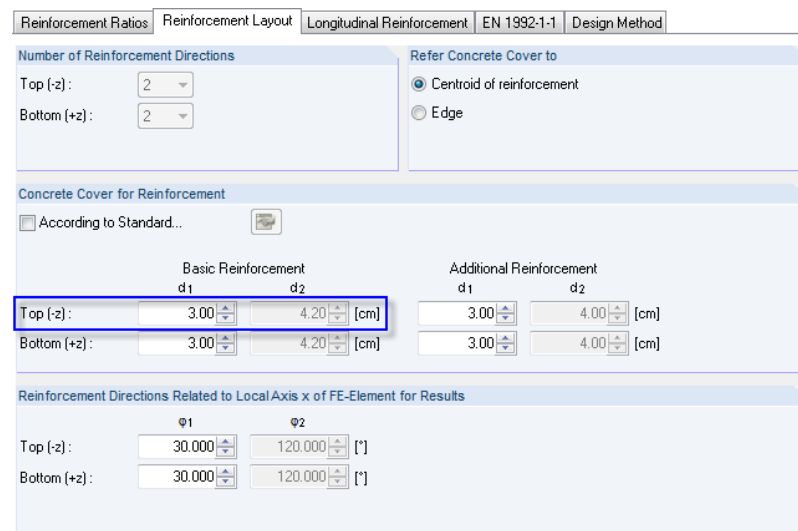


Figure 2.85: Window 1.4 Reinforcement, tab Reinforcement Layout

The effective depth for the individual directions of reinforcement is determined as follows:

$$d_{1,-z} = h - d_1 = 20 - 3 = 17 \text{ cm}$$

$$d_{2,-z} = h - d_2 = 20 - 4.2 = 15.8 \text{ cm}$$

2.6.4.5 Check of the Provided Reinforcement for SLS

First, the deformation ratio $\varepsilon_{\phi 2} / \varepsilon_{\phi 1} = 1.0$ is assumed for the serviceability limit state. Thus, the following values are determined:

Internal Forces of Linear Statics				
Check if internal forces cause cracks				
Check Existing Longitudinal Reinforcement				
Design Bending Moments				
Bottom surface (+z)				
Differential Angle According to Baumann				
1st Differential Angle	$\alpha_{m,+z}$	30.000	°	
2nd Differential Angle	$\beta_{m,+z}$	120.000	°	
Governing Strut				
First Assumption for Direction γ	$\gamma_{m,+z,1a}$	75.000	°	
Strut Direction	$\phi_{strut,m,+z}$	75.000	°	
Governing Design Bending Moments				
into Direction 1	$m_{+z,\phi 1}$	38.49	kNm/m	
into Direction 2	$m_{+z,\phi 2}$	25.25	kNm/m	
into Strut Direction	$m_{end,+z,strut}$	-22.94	kNm/m	
Design Bending Moments by Baumann				
into Direction 1	$m_{\alpha,+z}$	38.49	kNm/m	
into Direction 2	$m_{\beta,+z}$	25.25	kNm/m	
into Strut Direction	$m_{\gamma,+z}$	-22.94	kNm/m	
Find optimal strut direction?	Strut opti,m,+z	Yes		
Final Design Bending Moments				
into Direction 1	$m_{end,+z,\phi 1}$	38.49	kNm/m	
into Direction 2	$m_{end,+z,\phi 2}$	25.25	kNm/m	
into Strut Direction	$m_{end,+z,strut}$	-22.94	kNm/m	

Figure 2.86: Design moments in SLS for deformation ratio of 1.0

Design internal forces:

$$m_{end,-z,\phi 1} = 38.49 \text{ kNm/m}$$

$$m_{end,-z,\phi 2} = 25.25 \text{ kNm/m}$$

Stiffening compression moment:

$$m_{end,-z,strut} = -22.94 \text{ kNm/m}$$

Direction of stiffening compression moment

$$\phi_{strut,m,-z} = 75.0^\circ$$

For these design moments, the program determines at the top surface of the plate a required reinforcement of $a_{s,dim,-z,1} = 4.54 \text{ cm}^2/\text{m}$ in the first direction of reinforcement. The required reinforcement in the second direction is $a_{s,dim,-z,1} = 3.17 \text{ cm}^2/\text{m}$.

Internal Forces of Linear Statics				
Check if internal forces cause cracks				
Check Existing Longitudinal Reinforcement				
Design Bending Moments				
Statically Required Reinforcement				
Bottom surface (+z)				
Top surface (-z)				
into Reinforcement Direction 1	$a_{s,dim,-z,1}$	4.33	cm ² /m	
into Reinforcement Direction 2	$a_{s,dim,-z,2}$	3.04	cm ² /m	
Existing Longitudinal Reinforcement				
Bottom surface (+z)				
Top surface (-z)				
into Reinforcement Direction 1	$a_{s,exist,-z,1}$	11.31	cm ² /m	
into Reinforcement Direction 2	$a_{s,exist,-z,2}$	11.31	cm ² /m	

Figure 2.87: Statically required reinforcement for internal forces in SLS

The required reinforcement for the internal forces of the serviceability limit state is smaller than the user-defined provided reinforcement. Thus, it is possible to continue with the analysis.

2.6.4.6 Selection of the Concrete Strut

With the design internal forces $m_{\text{end},-z,\phi 1} = 38.9 \text{ kNm/m}$ and $m_{\text{end},-z,\phi 2} = 25.25 \text{ kNm/m}$, we obtain the strains $\varepsilon_{\phi 1} = 0.735 \text{ ‰}$ in the first direction of reinforcement and $\varepsilon_{\phi 2} = 0.527 \text{ ‰}$ in the second direction of reinforcement. Thus, the deformation ratio $Q_{s,-z}$ is 0.717.

Therefore, the assumed deformation ratio of 1.00 does not correspond to the actual deformation ratio. The inclination of the stiffening compression moment is therefore increased from 75.0° to 79.746° .

Geometrically, this inclination of the stiffening compression moment can appear only if the geometrical relation $Q_{s,\text{geo},-z}$ of the strain in the reinforcement direction ϕ_2 to the strain in the direction of the ϕ_1 is ca. 0.717. This is indeed the case in our example.

When determining the crack width w_k , it is shown that with the design moments for an inclination of the stiffening compression moment of 79.746° , strains result in the individual directions of reinforcement that lead to the deformation ratio $Q_{s,\text{geo},-z}$ of 0.717.

Internal Forces of Linear Statics			
Check if internal forces cause cracks			
Check Existing Longitudinal Reinforcement			
Design Internal Forces in Serviceability Limit State			
Bottom surface (+z)			
Top surface (-z)			
Concrete Strut Direction	$\gamma_{\text{strut,exist},2,-z}$	79.746	°
Concrete Strut Direction acc. to Baumann	$\gamma_{\text{strut,Bau,exist},2}$	79.746	°
Ratio of Reinforcement Deformation	$R_{s,-z}$	0.717	
Supposed Geometrical Ratio of Deformation	$R_{s,\text{geo},-z}$	0.717	
Modified design Moments			
into Direction 1	$m_{\text{exist},-z,\phi 1}$	36.74	kNm/m
into Direction 2	$m_{\text{exist},-z,\phi 2}$	27.33	kNm/m
into Strut Direction	$m_{\text{exist,strut},-z}$	-23.26	kNm/m
Check Steel Stress			
Check Final Crack Spacing			
Determination of difference in the mean strain			
Calculation Parameter for All Directions			
Bottom surface (+z)			
Top surface (-z)			
Difference in the mean strain into the direction 1	$(\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}})_{-z,\phi 1}$	0.735	‰
Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\phi 1}$	208.18	N/mm ²
Difference in the mean strain into the direction 2	$(\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}})_{-z,\phi 2}$	0.527	‰
Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\phi 2}$	167.09	N/mm ²
Resulting Difference in the Mean Strain	$(\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}})_{-z,\text{res}}$	1.291	‰

Figure 2.88: Direction of the concrete strut and deformation ratios

The selected inclination of the stiffening compression moment of 79.746° results in modified design moments in the individual directions of reinforcement. This corresponds to the method for the determination of the design internal forces in SLS taking into account the deformation ratio of the longitudinal reinforcement that was selected in the dialog box *Settings for Analytical Method of Serviceability Limit State Design* (see the following figure).



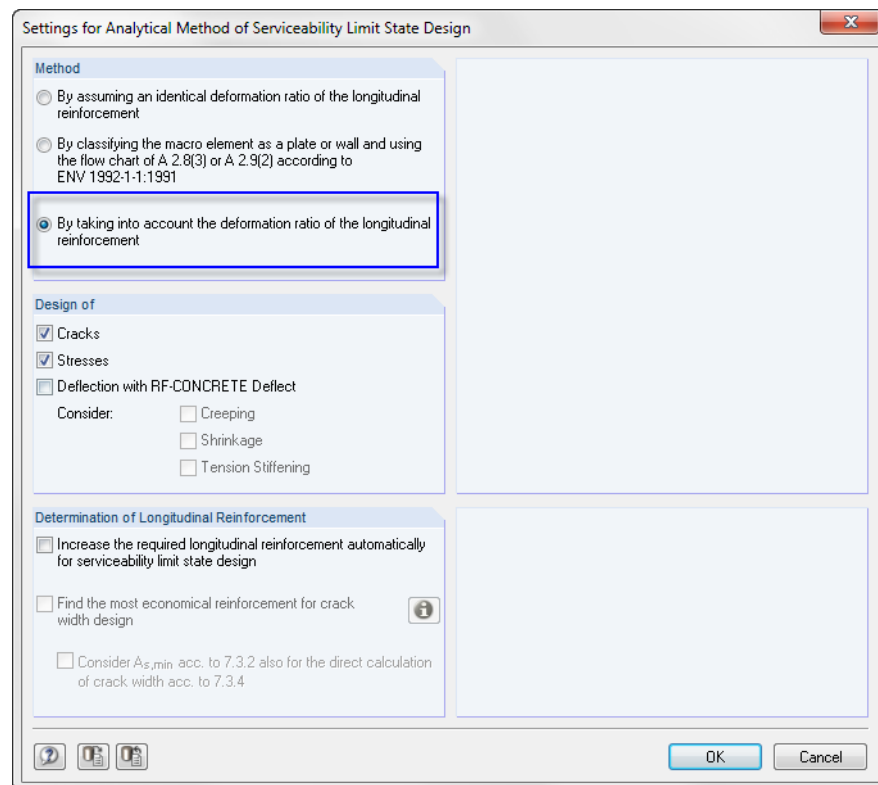


Figure 2.89: Dialog box Settings for Analytical Method of Serviceability Limit State Design

2.6.4.7 Limitation of Concrete Pressure Stress

In the 1.3 Surfaces window, the concrete pressure stress is limited to $\sigma_c = 0.45 \cdot f_{ck}$.

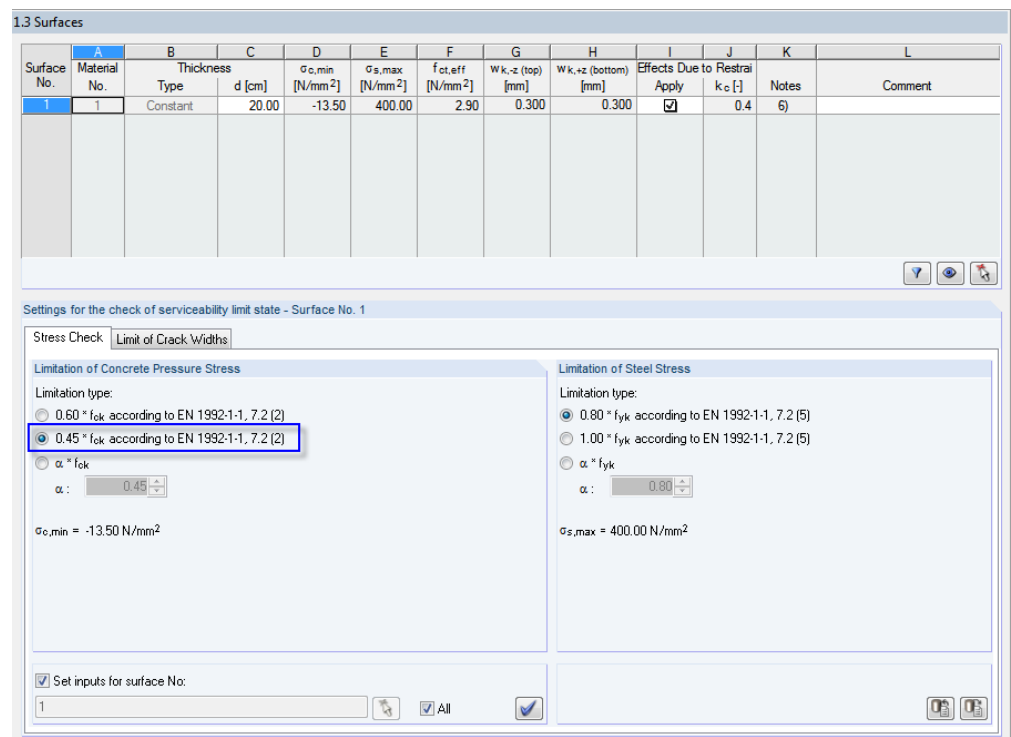


Figure 2.90: Limiting the concrete pressure stress in the Stress Check tab of the 1.3 Surfaces window

For concrete C30/37, the minimum concrete stress is determined as $\sigma_{c,min}$:

$$\sigma_{c,min} = 0.45 \cdot f_{ck} = 0.45 \cdot -30.0 = -13.5 \text{ N/mm}^2$$

The provided concrete pressure stress is determined under the assumption of a linear stress distribution because the great number of iterations to determine the suitable direction of the concrete strut would be too time-consuming. A linear distribution is sufficiently exact because there are usually negative strains of maximum 0.3 to 0.5 ‰ in the SLS.

The minimum stress $\sigma_{c,min}$ is to be compared to the provided stress of the concrete compression zone for both directions of reinforcement.

The provided concrete pressure stress σ_c is determined as follows:

$$\sigma_c = \frac{m_{Ed}}{I_{i,II}} \cdot x$$

Equation 2.66

where

m_{Ed}

Applied moment

$$I_{i,II} = \frac{1}{3} \cdot b \cdot x^3 + \alpha_e \cdot a_s \cdot (d - x)^2$$

Ideal moment of inertia in the state II

B

width of (for plates always 1 m)

α_E

Ratio of the elastic moduli

a_s

Provided tension reinforcement

d

Statically effective depth

$$x = \frac{\alpha_e \cdot a_s}{b} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot b \cdot d}{\alpha_e \cdot a_s}} \right)$$

Depth of the concrete neutral axis

For direction of reinforcement ϕ , we obtain the following depth of neutral axis $x_{-z,\phi 1}$:

$$x_{-z,\phi 1} = \frac{6.061 \cdot 11.31}{100} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot 100 \cdot 17}{6.061 \cdot 11.31}} \right) = 4.19 \text{ cm}$$

The same value and the according intermediate values can also be found in the details table.

Internal Forces of Linear Statics			
Check if internal forces cause cracks			
Check Existing Longitudinal Reinforcement			
Design Internal Forces in Serviceability Limit State			
Determination of concrete compressive stress in particular reinforcement directions			
Bottom surface (+z)			
Top surface (-z)			
Concrete compressive stress in reinforcement direction 1	$\sigma_{c,+z,\phi 1}$	-11.23	N/mm ²
Design Moment in the reinforcement direction 1	$m_{d,-z,\phi 1}$	36.74	kNm/m
Ideal Moment of Inertia	$I_{i,II,-z,\phi 1}$	13701.00	cm ⁴
Compression reinforcement from reinforcement of opposite surface is possible.			
Width of the Element	b_w	100.00	cm
Depth of the Concrete Compression Zone	$x_{-z,\phi 1}$	4.19	cm
Ratio of Modulus of Elasticity	α_e	6.061	
Tension Reinforcement into Direction 1	$a_{s,exist,-z,1}$	11.31	cm ² /m
Effective Depth	$d_{-z,\phi 1}$	0.170	m
Depth of the Concrete Compression Zone	$x_{-z,\phi 1}$	4.19	cm
Compression Reinforcement into Direction 1	$a_{s,exist,+z,1}$	0.14	cm ² /m
Concrete Cover of Compression Reinforcement	$c_{-z,\phi 1}$	0.030	m
Depth of the Concrete Compression Zone	$x_{-z,\phi 1}$	4.19	cm
Concrete compressive stress in reinforcement direction 2	$\sigma_{c,+z,\phi 2}$	-9.40	N/mm ²

Figure 2.91: Depth of the neutral axis for reinforcement direction 1

For the reinforcement direction ϕ , we obtain the depth of neutral axis $x_{-z,\phi 2}$:

$$x_{-z,\phi 2} = \frac{6.061 \cdot 11.31}{100} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot 100 \cdot 15.8}{6.061 \cdot 11.31}} \right) = 4.02 \text{ cm}$$

This value and the according intermediate values can also be read in the details.

Internal Forces of Linear Statics			
Check if internal forces cause cracks			
Check Existing Longitudinal Reinforcement			
Design Internal Forces in Serviceability Limit State			
Determination of concrete compressive stress in particular reinforcement directions			
Bottom surface (+z)			
Top surface (-z)			
Concrete compressive stress in reinforcement direction 1	$\sigma_{c,+z,\phi 1}$	-11.23	N/mm ²
Concrete compressive stress in reinforcement direction 2	$\sigma_{c,+z,\phi 2}$	-9.40	N/mm ²
Design Moment in the reinforcement direction 2	$m_{d,-z,\phi 2}$	27.33	kNm/m
Ideal Moment of Inertia	$I_{i,II,-z,\phi 2}$	11677.20	cm ⁴
Compression reinforcement from reinforcement of opposite surface is possible.			
Width of the Element	b_w	100.00	cm
Depth of the Concrete Compression Zone	$x_{-z,\phi 2}$	4.02	cm
Ratio of Modulus of Elasticity	α_e	6.061	
Modulus of Elasticity of Reinforcement	E_s	200000.00	N/mm ²
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm ²
Tension Reinforcement into Direction 2	$a_{s,exist,-z,2}$	11.31	cm ² /m
Compression Reinforcement into Direction 2	$a_{s,exist,+z,2}$	0.70	cm ² /m
Width of the Element	b_w	100.00	cm
Effective Depth	$d_{-z,\phi 2}$	0.158	m
Concrete Cover of Compression Reinforcement	$c_{+z,\phi 2}$	0.040	m
Ratio of Modulus of Elasticity	α_e	6.061	
Tension Reinforcement into Direction 2	$a_{s,exist,-z,2}$	11.31	cm ² /m
Effective Depth	$d_{-z,\phi 2}$	0.158	m
Depth of the Concrete Compression Zone	$x_{-z,\phi 2}$	4.02	cm
Compression Reinforcement into Direction 2	$a_{s,exist,+z,2}$	0.70	cm ² /m
Concrete Cover of Compression Reinforcement	$c_{-z,\phi 2}$	0.040	m
Depth of the Concrete Compression Zone	$x_{-z,\phi 2}$	4.02	cm

Figure 2.92: Depth of the neutral axis for reinforcement direction 2

For the two directions of reinforcement, the ideal moments of inertia $I_{i,II}$ in the state II (cracked section) are determined as follows:

$$I_{i,II,-z,\phi 1} = \frac{1}{3} \cdot 100.0 \cdot 4.19^3 + 6.061 \cdot 11.31 \cdot (17 - 4.19)^2 = 13701 \text{ cm}^4$$

$$I_{i,II,-z,\phi 2} = \frac{1}{3} \cdot 100 \cdot 4.02^3 + 6.061 \cdot 11.31 \cdot (15.8 - 4.02)^2 = 11678 \text{ cm}^4$$

For the two directions of reinforcement ϕ_1 and ϕ_2 , we obtain according to Equation 2.66 the following concrete pressure stresses σ_c in the concrete compression zone (i.e. the top surface):

$$\sigma_{c,o,\phi 1} = \frac{3676 \cdot 4.19}{13701} = -11.24 \text{ N/mm}^2$$

$$\sigma_{c,o,\phi 2} = \frac{2733 \cdot 4.02}{11678} = -9.41 \text{ N/mm}^2$$

These values are also shown in Figure 2.92.

Therefore, the existing compressive stresses $\sigma_{c,+z,\phi 1}$ and $\sigma_{c,+z,\phi 2}$ are smaller than the minimum concrete stress $\sigma_{c,min}$ (see Figure 2.90, page 77). The governing quotient of existing and allowable concrete compressive stress is in the direction of reinforcement ϕ_1 . The design criterion is satisfied.

Maximum Concrete Compressive Stress	max σ_c	-11.23	N/mm ²
Allowable Concrete Compressive Stress			
Check			
Maximum Concrete Compressive Stress	max σ_c	-11.23	N/mm ²
Allowable Concrete Compressive Stress	perm σ_c	-13.50	N/mm ²
Criterion of Check	Criterion	0.832	

Figure 2.93: Analysis of the concrete compression stress

2.6.4.8 Limitation of the Reinforcing Steel Stress

In the 1.3 *Surfaces* window, according to EN 1992-1-1, clause 7.2(5), tension stresses of the reinforcing steel are limited to $\sigma_{s,max} = 0.8 \cdot f_{yk}$ (see Figure 2.90, page 77). For BSt 500 S (B), the maximum steel stress $\sigma_{s,max}$ is determined as:

$$\sigma_{s,max} = 0.8 \cdot f_{yk} = 0.8 \cdot 500 = 400 \text{ N/mm}^2$$

The maximum stress $\sigma_{s,max}$ is to be compared to the provided tension stress for both directions of reinforcement.

The provided tension stress σ_s is determined as follows:

$$\sigma_s = \frac{\alpha_E \cdot m_{Ed} \cdot (d - x)}{I_{i,II}}$$

Equation 2.67

where

α_E

Relation of the elastic moduli (E_s / E_{cm})

m_{Ed}

Applied moment

d

Statically effective depth

$$x = \frac{\alpha_E \cdot a_s}{b} \cdot \left(-1.0 + \sqrt{1.0 + \frac{2.0 \cdot b \cdot d}{\alpha_E \cdot a_s}} \right)$$

Depth of the concrete neutral axis

b

Width of the element (for plates always 1 m)

a_s

Provided tension reinforcement

$$I_{i,II} = \frac{1}{3} \cdot b \cdot x^3 + \alpha_E \cdot a_s \cdot (d - x)^2$$

Ideal moment of inertia in the state II

With the values calculated in chapter 2.6.4.6, it is possible to determine the provided tension stresses $\sigma_{s,u,\phi 1}$ and $\sigma_{s,u,\phi 2}$ in the two reinforcement directions $\phi 1$ and $\phi 2$ as follows:

$$\sigma_{s,u,\phi 1} = \frac{6.061 \cdot 3674 \cdot (17 - 4.19)}{13701} = 208.18 \text{ N/mm}^2$$

$$\sigma_{s,u,\phi 2} = \frac{6.061 \cdot 2733 \cdot (15.8 - 4.02)}{11677} = 167.09 \text{ N/mm}^2$$

Internal Forces of Linear Statics				
Check if internal forces cause cracks				
Check Existing Longitudinal Reinforcement				
Design Internal Forces in Serviceability Limit State				
Check Steel Stress				
Bottom surface (+z)				
Top surface (-z)				
Concrete cracks. Longitudinal reinforcement is activated.				
Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\phi 1}$	208.18	N/mm ²	
Design Moment in the reinforcement direction 1	$m_{d,-z,\phi 1}$	36.74	kNm/m	
Ratio of Modulus of Elasticity	α_E	6.061		
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm ²	
Ideal Moment of Inertia	$I_{i,II,-z,\phi 1}$	13701.00	cm ⁴	
Depth of the Concrete Compression Zone	$x_{-z,\phi 1}$	4.19	cm	
Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\phi 2}$	167.09	N/mm ²	
Design Moment in the reinforcement direction 2	$m_{d,-z,\phi 2}$	27.33	kNm/m	
Ratio of Modulus of Elasticity	α_E	6.061		
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm ²	
Ideal Moment of Inertia	$I_{i,II,-z,\phi 2}$	11677.20	cm ⁴	
Depth of the Concrete Compression Zone	$x_{-z,\phi 2}$	4.02	cm	
Maximum Steel Stress	$\max \sigma_s$	208.18	N/mm ²	

Figure 2.94: Maximum steel stresses in reinforcement directions 1 and 2

Therefore, the existing tension stresses $\sigma_{s,z,\phi 1}$ and $\sigma_{s,z,\phi 2}$ are smaller than the maximum steel stress $\sigma_{s,max}$ (see Figure 2.90, page 77). The governing quotient of existing and allowable steel stress is in the reinforcement direction ϕ_1 . The design criterion is satisfied.

⊞ Design Internal Forces in Serviceability Limit State				
⊞ Check Steel Stress				
⊞ Allowable Steel Stress				
⊞ Check				
Maximum Steel Stress	max σ_s	208.18	N/mm ²	
Allowable Steel Stress	perm σ_s	400.00	N/mm ²	
Criterion of Check	Criterion	0.520		

Figure 2.95: Analysis of reinforcing steel stress

2.6.4.9 Minimum Reinforcement for Crack Control

The minimum reinforcement cross-section for the crack control is determined according to EN 1992-1-1, clause 7.3.2, equation (7.1).

$$a_{s,min} = \frac{k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct}}{\sigma_s}$$

Equation 2.68

where

- k_c Coefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm
- k Coefficient which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces
- $f_{ct,eff}$ Mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur
- A_{ct} Area of concrete within tensile zone (i.e. part of the cross-section which is calculated to be in tension just before formation of the first crack)
- σ_s Absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack

The maximum bar diameter ϕ_s^* is determined according to EN 1992-1-1, clause 7.3.3 (2) depending on the actually provided diameter ϕ_s from the converted equation (7.6N).

$$\phi_s = \phi_s^* \cdot \frac{f_{ct,eff}}{2.9} \cdot \frac{k_c \cdot h_{cr}}{2 \cdot (h - d)}$$

Equation 2.69

where

- ϕ_s Adjusted maximum bar diameter
- ϕ_s^* Maximum bar diameter according to EN 1992-1-1, Table 7.2 (see Figure 2.96)
- h Overall depth of cross-section
- h_{cr} Depth of the tensile zone immediately prior to cracking, considering the characteristic values of prestress and axial forces under the quasi-permanent combination of actions
- d Effective depth to centroid of outside reinforcement

Table 7.2N Maximum bar diameters ϕ_s for crack control¹

Steel stress ² [MPa]	Maximum bar size [mm]		
	$w_k = 0,4 \text{ mm}$	$w_k = 0,3 \text{ mm}$	$w_k = 0,2 \text{ mm}$
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

Figure 2.96: Maximum bar diameter of reinforcing bars according to EN 1992-1-1, clause 7.3.3



For the example, it is necessary to exclude the determination of the minimum reinforcement at the bottom surface of the plate. To do this, we go to the *Limit of Crack Width* tab of the 1.3 *Surfaces* window. In the *Further Settings for Min. Reinforcements Due to Restraint* dialog box, clear the selection of the check boxes for the *Bottom (+z) reinforcement*.

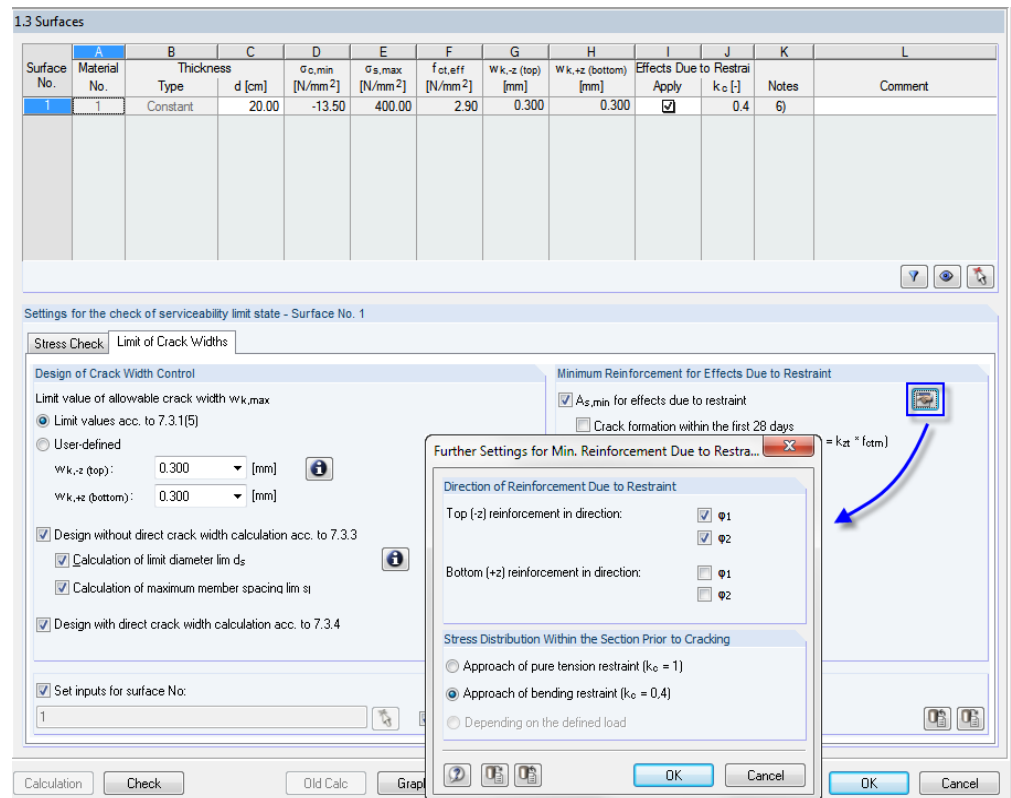


Figure 2.97: Dialog box *Further Settings for Min. Reinforcement Due to Restraint* in window 1.3 *Surfaces*

The maximum reinforcement diameter $d_{s,-z,\phi_1^*}$ for the reinforcement direction ϕ_1 at the bottom surface of the plate is determined according to Equation 2.69.

$$d_{s,-z,\phi_1^*} = 12 \cdot \frac{2.9}{2.9} \cdot \frac{2 \cdot (200 - 170)}{0.4 \cdot 100} = 18.00 \text{ mm}$$

Internal Forces of Linear Statics				
Check if internal forces cause cracks				
Check Existing Longitudinal Reinforcement				
Design Internal Forces in Serviceability Limit State				
Determination of minimum reinforcement				
Calculation Parameter for All Directions				
Maximum allowable crack width at the bottom (+z) surface	$w_{k,-z,limit}$	0.300	mm	
Maximum allowable crack width at the top (-z) surface	$w_{k,-z,limit}$	0.300	mm	
Factor for Unevenly Distributed Tensile Stress	k	1.000		
Basic tensile strength of concrete for Table 7.2	$f_{ct,0}$	2.90	N/mm ²	
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²	
Area of the Tension Zone	A_{ct}	1000.00	cm ²	
Width of the Element	b_w	1.000	m	
Depth of Structural Member	h	0.200	m	
Bottom surface (+z)				
Top surface (-z)				
Minimum Reinforcement into Direction 1	$a_{s,min,-z,1}$	5.02	cm ² /m	
Factor of Stress Distribution Prior to Initial Crack Formation	k_c	0.400		
Governing Design Forces				
Factor for Unevenly Distributed Tensile Stress	k	1.000		
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²	
Area of the Tension Zone	$A_{ct,-z,\phi 1}$	1000.00	cm ²	
Allowable stress in the reinforcement acc. to Table 7.2	$\sigma_{s,-z,\phi 1}$	231.11	N/mm ²	
Limit Diameter of Reinforcement	$d_{s,-z,\phi 1}^*$	1.80	cm	
First Calculated Value of Limit Diameter	$d_{s,calc1}^*$	1.80	cm	
Existing Diameter of Reinforcement	$d_{s,min,-z,\phi 1}$	1.20	cm	
Effective Depth	$d_{-z,\phi 1}$	0.170	m	
Minimum Reinforcement into Direction 2	$a_{s,min,-z,2}$	5.83	cm ² /m	

Figure 2.98: Maximum bar diameter for the reinforcement direction ϕ_1

Similarly, we obtain for the reinforcement direction ϕ the maximum bar diameter $d_{s,-z,\phi 2}^*$:

$$d_{s,-z,\phi 2}^* = 12 \cdot \frac{2.9}{2.9} \cdot \frac{2 \cdot (200 - 158)}{0.4 \cdot 100} = 25.20 \text{ mm}$$

Internal Forces of Linear Statics				
Check if internal forces cause cracks				
Check Existing Longitudinal Reinforcement				
Design Internal Forces in Serviceability Limit State				
Determination of minimum reinforcement				
Calculation Parameter for All Directions				
Bottom surface (+z)				
Top surface (-z)				
Minimum Reinforcement into Direction 1	$a_{s,min,-z,1}$	5.02	cm ² /m	
Minimum Reinforcement into Direction 2	$a_{s,min,-z,2}$	5.83	cm ² /m	
Factor of Stress Distribution Prior to Initial Crack Formation	k_c	0.400		
Governing Design Forces				
Factor for Unevenly Distributed Tensile Stress	k	1.000		
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²	
Area of the Tension Zone	$A_{ct,-z,\phi 2}$	1000.00	cm ²	
Allowable stress in the reinforcement acc. to Table 7.2	$\sigma_{s,-z,\phi 2}$	198.86	N/mm ²	
Limit Diameter of Reinforcement	$d_{s,-z,\phi 2}^*$	2.52	cm	
First Calculated Value of Limit Diameter	$d_{s,calc1}^*$	2.52	cm	
Existing Diameter of Reinforcement	$d_{s,min,-z,\phi 2}$	1.20	cm	
Effective Depth	$d_{-z,\phi 2}$	0.158	m	

Figure 2.99: Maximum bar diameter for reinforcement direction ϕ_2

In the 1.3 *Surfaces* window, the allowable crack width $w_{k,max}$ is predefined as 0.3 mm (see Figure 2.97). With the maximum bar diameters $d_{s,-z,\phi 1}^* = 18.00$ mm and $d_{s,-z,\phi 2}^* = 25.20$ mm, we can interpolate from EN 1992-1-1, Table 7.2N (see Figure 2.96) the allowable stress σ_s .

$$\sigma_{s,-z,\phi 1} = 240 + \frac{280 - 240}{16 - 25} \cdot (18.00 - 16) = 231.11 \text{ N/mm}^2$$

$$\sigma_{s,-z,\phi 2} = 200 + \frac{200 - 160}{25 - 32} \cdot (25.20 - 25) = 198.86 \text{ N/mm}^2$$

Those allowable steel stresses are also shown in Figure 2.98 and Figure 2.99

The steel stress in the direction ϕ_2 is governing.

The surface of the concrete compression zone in the cross-section is determined as follows:

$$A_{ct} = b \cdot \frac{h}{2} = 100 \cdot \frac{20}{2} = 1000 \text{ cm}^2$$

According to the Equation 2.68 on page 81, we obtain for the reinforcement direction 2 the following minimum reinforcement:

$$a_{s,min,\phi 2} = \frac{0.4 \cdot 1.0 \cdot 2.9 \cdot 1000}{198.86} = 5.83 \text{ cm}^2 / \text{m}$$

For this reinforcement direction, the applied reinforcement is greater than the minimum reinforcement. Thus we obtain the following design criterion:

$$\frac{a_{s,min,-z,2}}{a_{s,exist,-z,2}} = \frac{5.83}{11.31} = 0.516$$

☑ Design Internal Forces in Serviceability Limit State				
☑ Determination of minimum reinforcement				
☑ Existing Longitudinal Reinforcement				
☑ Check				
Minimum Reinforcement at the Top (-z) Surface in Direction 2	$a_{s,min,-z,2}$	5.83	cm ² /m	
Existing Reinforcement at the Top (-z) Surface in Direction 2	$a_{s,exist,-z,2}$	11.31	cm ² /m	
Criterion of Check	Criterion	0.516		

Figure 2.100: Design criterion for minimum reinforcement

2.6.4.10 Checking the Rebar Diameter

The maximum bar diameter of the rebars, max d_s , is determined according to EN 1992-1-1, Equation (7.6N) (see Equation 2.69, page 81).

At the top surface of the plate, the program determines the maximum bar diameter $d_{s,-z,\phi 1}^*$ of the first reinforcement direction, depending on the stress in this direction. In the check of the limitation of the steel stress, this stress was computed with $\sigma_{s,-z,\phi 1} = 208.18 \text{ N/mm}^2$. Together with the selected crack width $w_k = 0.3 \text{ mm}$, we obtain by interpolation in Table 7.2N the following maximum bar diameter $d_{s,-z,\phi 1}^*$:

$$d_{s,-z,\phi 1}^* = 25 + \frac{25 - 16}{200 - 240} \cdot (208.18 - 200) = 23.16 \text{ mm}$$

☑ Internal Forces of Linear Statics				
☑ Check if internal forces cause cracks				
☑ Check Existing Longitudinal Reinforcement				
☑ Design Internal Forces in Serviceability Limit State				
☑ Determination of the maximum steelbar diameter				
☐ Calculation Parameter for All Directions				
Maximum allowable crack width at the bottom (+z) surface	$w_{k,-z,limit}$	0.300	mm	
Maximum allowable crack width at the top (-z) surface acc.	$w_{k,-z,limit}$	0.300	mm	
☐ Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²	
Mean Axial Tensile Strength	f_{ctm}	2.90	N/mm ²	
Depth of Structural Member	h	0.200	m	
Reference tensile strength acc. to Table 7.2	$f_{ct,0}$	3.00	N/mm ²	
Width of the Element	b_w	100.00	cm	
☑ Bottom surface (+z)				
☑ Top surface (-z)				
☐ Maximum Steelbar Diameter into Reinforcement Direction	$d_{s,max,-z,\phi 1}$	1.54	cm	
☐ Limit Diameter of Reinforcement	$d_{s,-z,\phi 1}^*$	2.32	cm	
Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\phi 1}$	208.18	N/mm ²	
☐ First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.54	cm	
Effective Depth	$d_{-z,\phi 1}$	0.170	m	
Factor of Stress Distribution Prior to Initial Crack Form	k_c	0.400		
Depth of Tension Zone	h_{cr}	0.100	m	
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²	

Figure 2.101: Maximum bar diameter in reinforcement direction 1

Similarly, the maximum bar diameter for the reinforcement direction 2 $d_{s,-z,\phi 2}^*$ is determined from the tension stress $\sigma_{s,-z,\phi 2} = 167.09 \text{ N/mm}^2$ and the crack width $w_k = 0.3 \text{ mm}$:

$$d_{s,-z,\phi 2}^* = 32 + \frac{32 - 25}{160 - 200} \cdot (167.09 - 160) = 30.76 \text{ mm}$$

Internal Forces of Linear Statics			
Check if internal forces cause cracks			
Check Existing Longitudinal Reinforcement			
Design Internal Forces in Serviceability Limit State			
Determination of the maximum steelbar diameter			
Calculation Parameter for All Directions			
Bottom surface (+z)			
Top surface (-z)			
Maximum Steelbar Diameter into Reinforcement Direction 1	$d_{s,max,-z,\phi 1}$	1.54	cm
Maximum Steelbar Diameter into Reinforcement Direction 2	$d_{s,max,-z,\phi 2}$	1.46	cm
Limit Diameter of Reinforcement	$d_{s,-z,\phi 2}^*$	3.08	cm
Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\phi 2}$	167.09	N/mm ²
First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.46	cm
Existing Longitudinal Reinforcement	$a_{s,exist,-z,2}$	0.158	m
Effective Depth	$d_{-z,\phi 2}$	0.400	
Factor of Stress Distribution Prior to Initial Crack Formation	k_c	0.100	m
Factor for Unevenly Distributed Tensile Stress	k	2.90	N/mm ²

Figure 2.102: Maximum bar diameter in reinforcement direction 2

With the maximum bar diameters d_s^* for the two reinforcement directions and the respective steel stresses, the following maximum bar diameters d_s are determined:

$$d_{s,max,-z,\phi 1} = 23.16 \cdot \frac{2.9}{2.9} \cdot \frac{0.4 \cdot 100}{2 \cdot (200 - 170)} = 15.44 \text{ mm}$$

$$d_{s,max,-z,\phi 2} = 30.76 \cdot \frac{2.9}{2.9} \cdot \frac{0.4 \cdot 100}{2 \cdot (200 - 158)} = 14.65 \text{ mm}$$

Internal Forces of Linear Statics			
Check if internal forces cause cracks			
Check Existing Longitudinal Reinforcement			
Design Internal Forces in Serviceability Limit State			
Determination of the maximum steelbar diameter			
Calculation Parameter for All Directions			
Bottom surface (+z)			
Top surface (-z)			
Maximum Steelbar Diameter into Reinforcement Direction 1	$d_{s,max,-z,\phi 1}$	1.54	cm
Limit Diameter of Reinforcement	$d_{s,-z,\phi 1}^*$	2.32	cm
First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.54	cm
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²
Maximum Steelbar Diameter into Reinforcement Direction 2	$d_{s,max,-z,\phi 2}$	1.46	cm
Limit Diameter of Reinforcement	$d_{s,-z,\phi 2}^*$	3.08	cm
First Calculated Value of Maximum Steelbar Diameter	$d_{s,max,calc1}$	1.46	cm
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²

Figure 2.103: Maximum bar diameter

For both reinforcement directions, the bar diameter $d_s = 12 \text{ mm}$ is specified, respectively. Thus, we obtain the design criterion for the governing reinforcement direction ϕ_1 :

$$\frac{d_{s,exist,-z,\phi 2}}{\max d_{s,-z,\phi 2}} = \frac{12.0}{14.65} = 0.819$$

Design Internal Forces in Serviceability Limit State			
Determination of the maximum steelbar diameter			
Existing Steelbar Diameter			
Check			
Existing steelbar diameter at the top of the surface in direction 2	$d_{s,exist,-z,\phi 2}$	1.20	cm
Maximum Steelbar Diameter at the Top Surface into Direction 2	$d_{s,max,-z,\phi 2}$	1.46	cm
Criterion of Check	Criterion	0.819	

Figure 2.104: Design criterion for bar diameter

2.6.4.11 Design of Rebar Spacing



In the *Longitudinal Reinforcement* tab of the 1.4 *Reinforcement* window, we clicked the [Rebars] button, and specified a rebar spacing of $a = 100$ mm for both reinforcement directions in the *Import Reinforcement Area Due to Rebar* dialog box (see Figure 2.84, page 74).

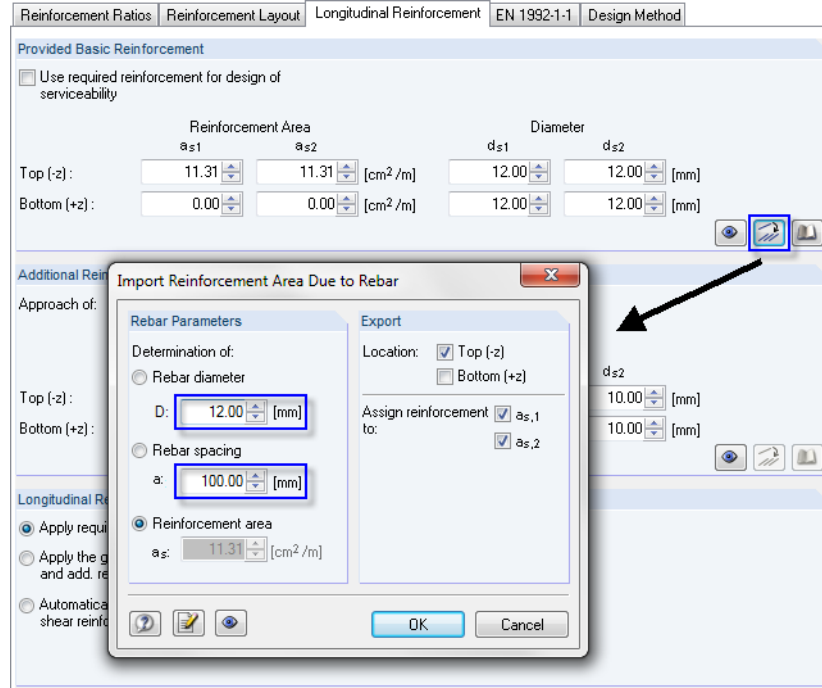


Figure 2.105: Dialog box *Import Reinforcement Area due to Rebar*

The maximum rebar spacing $s_{l,-z,\phi 1}$ is determined by interpolation according to EN 1992-1-1, Table 7.3N for the existing tension stress $\sigma_{s,-z,\phi 1} = 208.18$ N/mm² and the crack width $w_k = 0.3$ mm.

Table 7.3N Maximum bar spacing for crack control¹

Steel stress ² [MPa]	Maximum bar spacing [mm]		
	$w_k=0.4$ mm	$w_k=0.3$ mm	$w_k=0.2$ mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

Figure 2.106: Maximum values for rebar spacing according to EN 1992-1-1, Table 7.3N

$$\max s_{l,-z,\phi 1} = 250 + \frac{250 - 200}{200 - 240} \cdot (208.18 - 200) = 239.8 \text{ mm}$$

Similarly, the maximum bar spacing for the direction ϕ_2 is determined from the existing tension stress $\sigma_{s,-z,\phi 2} = 167.09$ N/mm² as $\max s_{l,-z,\phi 2} = 291.1$ mm.

Internal Forces of Linear Statics			
Check if internal forces cause cracks			
Check Existing Longitudinal Reinforcement			
Design Internal Forces in Serviceability Limit State			
Determination of maximum steelbar distance			
Calculation Parameter for All Directions			
Maximum allowable crack width at the bottom (+z) surface acc.	$w_{k,-z,limit}$	0.300	mm
Maximum allowable crack width at the top (-z) surface acc. to 1	$w_{k,-z,limit}$	0.300	mm
Bottom surface (+z)			
Top surface (-z)			
Maximum Steelbar Distance in Direction 1	$s_{l,max,-z,\phi 1}$	0.240	m
Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\phi 1}$	208.18	N/mm ²
Maximum Steelbar Distance in Direction 2	$s_{l,max,-z,\phi 2}$	0.291	m
Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\phi 2}$	167.09	N/mm ²

Figure 2.107: Maximum rebar spacing in both directions of reinforcement

For the direction of reinforcement ϕ_1 , the existing rebar spacing $s_{l,exist,-z,\phi 1} = 100$ mm is smaller than the maximum allowable rebar spacing $\max s_{l,max,-z,\phi 11} = 240$ mm.

Therefore, the following design criterion is determined for the reinforcement direction ϕ_1 :

$$\frac{s_{l,exist,-z,\phi 1}}{s_{l,max,-z,\phi 1}} = \frac{0.100}{0.240} = 0.417$$

Design Internal Forces in Serviceability Limit State			
Determination of maximum steelbar distance			
Existing Steelbar Distance			
Check			
Existing steelbar spacing at the top (-z) surface in direction 1	$s_{l,exist,-z,\phi 1}$	0.100	m
Maximum steelbar spacing at the top (-z) surface in direction 1	$s_{l,max,-z,\phi 1}$	0.240	m
Criterion of Check	Criterion	0.417	

Figure 2.108: Design criterion for rebar spacing

2.6.4.12 Check of Crack Width

The calculation value w_k of the crack width is determined according to EN 1992-1-1, clause 7.3.4, equation (7.8).

$$w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm})$$

Equation 2.70

where

$s_{r,max}$	Maximum crack spacing in final crack state (see Equation 2.71 or Equation 2.72)
ϵ_{sm}	Mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening (only the additional tensile strain beyond zero strain in the concrete is considered)
ϵ_{cm}	Mean strain in the concrete between cracks

Maximum crack spacing $s_{r,max}$

If the spacing of the bonded reinforcement is not greater than $5 \cdot (c + \phi/2)$ in the tension zone, the maximum crack spacing for the final crack state may be determined according to EN 1992-1-1, Equation (7.11):

$$s_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p,eff}}$$

Equation 2.71

If the rebar spacing in the bonded reinforcement exceeds $5 \cdot (c + \phi / 2)$, or if no bonded reinforcement is available within the tension zone, the limit for the crack width may be determined with the following crack spacing:

$$s_{r,max} = 1.3 \cdot (h - x)$$

Equation 2.72

Therefore, the depth of the neutral axis x in state II is to be calculated for the check of the crack width. It is determined with the depth of the structural member related to the depth of the neutral axis ξ .

$$x = \xi \cdot h = \frac{0.5 + \alpha_e \cdot \frac{a_{s,exist}}{b \cdot h} \cdot \frac{d}{h}}{1.0 + \alpha_e \cdot \frac{a_{s,exist}}{b \cdot h}} \cdot h$$

Equation 2.73

Internal Forces of Linear Statics				
Check if internal forces cause cracks				
Check Existing Longitudinal Reinforcement				
Design Internal Forces in Serviceability Limit State				
Check Steel Stress				
Check Final Crack Spacing				
Bottom surface (+z)				
Top surface (-z)				
Maximum Crack Spacing into Reinforcement Direction 1	$s_{r,max,-z,\phi 1}$	0.177	m	
Limiting Spacing of Bonded Reinforcement	$s_{l,limit,-z,\phi 1}$	0.150	m	
Concrete Cover	$c-z,\phi 1$	2.40	cm	
Existing Steelbar Diameter	$d_{s,exist,-z,\phi 1}$	0.012	m	
Existing Steelbar Distance	$s_{l,exist,-z,\phi 1}$	0.100	m	
Existing Reinforcement in Direction 1	$a_{s,max,-z,1}$	11.31	cm ² /m	
Existing Steelbar Diameter	$d_{s,max,-z,\phi 1}$	0.012	m	
Existing bar spacing is not larger than limiting spacing => Formula (7.11)				
Concrete Cover	$c-z,\phi 1$	2.40	cm	
Existing Steelbar Diameter	$d_{s,max,-z,\phi 1}$	0.012	m	
Coefficient for consideration of bond properties	k_1	0.800		
Coefficient for consideration of the strain distribution	k_2	0.500		
Parameter in National Annex	k_3	3.400		
Parameter in National Annex	k_4	0.425		
Effective Reinforcement Ratio	$\rho_{eff,-z,\phi 1}$	0.021		
Existing Reinforcement in Direction 1	$a_{s,max,-z,1}$	11.31	cm ² /m	
Concrete area in which the reinforcement is effective	$a_{o,eff,-z,\phi 1}$	527.05	cm ²	
Width of the Element	b_w	100.00	cm	
Depth of the area in which the reinforcement is effective	$h_{eff,-z,\phi 1}$	0.053	m	
First calculated depth of the area in which the reinforcement is effective	$h_{eff,calc1}$	0.075	m	
Concrete Cover to Rebar Centroid	$d'-z,\phi 1$	3.00	cm	
Second calculated depth of the area in which the reinforcement is effective	$h_{eff,calc2}$	0.053	m	
Depth of Structural Member	h	0.200	m	
Maximum Crack Spacing into Reinforcement Direction 2	$s_{r,max,-z,\phi 2}$	0.218	m	
Maximum Final Crack Spacing acc. (7.15)	$s_{r,max,-z,res}$	0.137	m	
Angle Between Reinforcement Direction and Crack Opening	$\phi-z$	40.254	°	

Figure 2.109: Maximum crack spacing in reinforcement direction 1

Furthermore, the maximum crack width is analyzed according to EN 1992-1-1, equation (7.15):

$$s_{r,max} = \frac{1}{\frac{\cos \theta}{s_{r,max,x}} + \frac{\sin \theta}{s_{r,max,y}}}$$

Equation 2.74

where

θ Angle between the reinforcement in x-direction and the principal tension stress
 $s_{r,max,x}, s_{r,max,y}$ Maximum crack spacing in y- or z-direction

This equation is important if in the dialog box *Settings for Analytical Method of Serviceability Limit State Designs* (see Figure 2.89, page 77) the first method *By assuming an identical deformation*

ratio of the longitudinal reinforcement has been selected to determine the design internal forces in the serviceability limit state. In the third method (*By taking into account the deformation ratio of the longitudinal reinforcement*), on the other hand, the direction of the concrete strut is determined according to BAUMANN. The limit angle of 15° is ignored since the crack width in this area is not governing.

☒ Internal Forces of Linear Statics			
☒ Check if internal forces cause cracks			
☒ Check Existing Longitudinal Reinforcement			
☒ Design Internal Forces in Serviceability Limit State			
☒ Check Steel Stress			
☒ Check Final Crack Spacing			
☐ Bottom surface (+z)			
Concrete does not crack on this side.			
☐ Top surface (-z)			
☒ Maximum Crack Spacing into Reinforcement Direction 1	$s_{r,max,-z,\Phi 1}$	0.177	m
☒ Maximum Crack Spacing into Reinforcement Direction 2	$s_{r,max,-z,\Phi 2}$	0.218	m
☐ Maximum Final Crack Spacing acc. (7.15)	$s_{r,max,-z,res}$	0.137	m
Angle Between Reinforcement Direction and Crack Opening		$\Phi-z$	40.254 °

Figure 2.110: Maximum crack spacing for both reinforcement directions

Difference of mean strain ($\varepsilon_{sm} - \varepsilon_{cm}$)

For the calculation value of the crack width w_k according to Equation 2.70 on page 87, we now need to determine the factor ($\varepsilon_{sm} - \varepsilon_{cm}$) for each direction of the resulting strain.

The difference of mean strain for concrete and reinforcing steel is determined in accordance with EN 1992-1-1, 7.3.4, Eq. (7.9):

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \cdot \frac{f_{ct,eff}}{\rho_{eff}} \cdot (1 + \alpha_e \cdot \rho_{eff})}{E_s} \geq 0.6 \cdot \frac{\sigma_s}{E_s}$$

Equation 2.75

The maximum mean strain ($\varepsilon_{sm} - \varepsilon_{cm}$)- z_{res} is obtained as the resulting mean strain of the individual reinforcement directions and is 1.291 ‰.

☒ Internal Forces of Linear Statics				
☒ Check if internal forces cause cracks				
☒ Check Existing Longitudinal Reinforcement				
☒ Design Internal Forces in Serviceability Limit State				
☒ Check Steel Stress				
☒ Check Final Crack Spacing				
☒ Determination of difference in the main strain				
☐ Calculation Parameter for All Directions				
Axial Tensile Strength of Concrete	$f_{ct,eff}$	2.90	N/mm ²	
☐ Ratio of Modulus of Elasticity	α_e	6.061		
Modulus of Elasticity of Reinforcement	E_s	200000.00	N/mm ²	
Mean Secant Modulus of Elasticity	E_{cm}	33000.00	N/mm ²	
Factor to Consider the Load Duration	k_t	0.400		
☒ Bottom surface (+z)				
☐ Top surface (-z)				
☐ Difference in the main strain into the direction 1	$(\varepsilon_{sm} - \varepsilon_{cm})-z,\Phi 1$	0.735	‰	
☒ First Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc1}$	0.735	‰	
Second Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc2}$	0.625	‰	
☒ Steel Stress into Reinforcement Direction 1	$\sigma_{s,-z,\Phi 1}$	208.18	N/mm ²	
☐ Difference in the main strain into the direction 2	$(\varepsilon_{sm} - \varepsilon_{cm})-z,\Phi 2$	0.527	‰	
☒ First Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc1}$	0.527	‰	
Second Calculated Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})_{calc2}$	0.501	‰	
☒ Steel Stress into Reinforcement Direction 2	$\sigma_{s,-z,\Phi 2}$	167.09	N/mm ²	
☐ Resulting Difference in the Main Strain	$(\varepsilon_{sm} - \varepsilon_{cm})-z,res$	1.291	‰	
☐ Differential Angle According to Baumann				
1st Differential Angle	$\alpha_{m,-z}$	30.000	°	
2nd Differential Angle	$\beta_{m,-z}$	120.000	°	
Concrete Strut Direction acc. to Baumann	$\gamma_{strut,Bau,exist,3}$	79.746	°	

Figure 2.111: Difference of the mean strain for both reinforcement directions

To make the expression clearer, we introduce symbols for the sought mean strain ($\epsilon_{sm} - \epsilon_{cm}$): ϵ for the side length in the direction of the reinforcement, d for the partial length of the concrete struts, l for the perpendicular to the concrete strut, and ϵ .

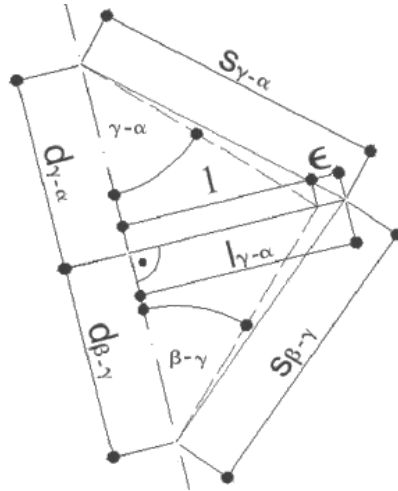


Figure 2.112: Mean strain ϵ

The partial length $d_{\gamma-\alpha}$ is determined for a selected inclination of the concrete strut as follows:

$$d_{\gamma-\alpha} = \frac{1}{\tan(\gamma-\alpha)}$$

The length is unitless (the perpendicular to the concrete strut is included without unit).

Then, the length $s_{\gamma-\alpha}$ is determined.

$$s_{\gamma-\alpha} = \frac{1 + \epsilon_{\alpha}}{\sin(\gamma-\alpha)}$$

If the reinforcement direction θ_1 forms the smallest differential angle with the principal moment m_1 , then for ϵ_{α} we have to insert the previously determined difference of the mean strains $(\epsilon_{sm} - \epsilon_{cm})_{\theta_1}$ of concrete and reinforcing steel:

$$s_{\gamma-\alpha} = \frac{1 + (\epsilon_{sm} - \epsilon_{cm})_{\theta_1}}{\sin(\gamma-\alpha)}$$

If the reinforcement direction θ_1 forms the smallest differential angle with the principal moment m_1 , then for ϵ_{α} we have to insert the previously determined difference of the mean strains $(\epsilon_{sm} - \epsilon_{cm})_{\theta_2}$ of concrete and reinforcing steel:

With the Pythagorean theorem, we can determine the value $l_{\gamma-\alpha}$ from the lengths $d_{\gamma-\alpha}$ and $s_{\gamma-\alpha}$:

$$l_{\gamma-\alpha} = \sqrt{s_{\gamma-\alpha}^2 - d_{\gamma-\alpha}^2}$$

Since all formulas are based on an initial length of 1.0 units of length, the strain ϵ is determined as follows:

$$\epsilon = l_{\gamma-\alpha} - 1.0$$

This strain $\epsilon = (\epsilon_{sm} - \epsilon_{cm})$ is checked again by means of the intermediate angle $(\beta - \gamma)$.

For the determination of the SLS design internal forces according to the method *By assuming an identical deformation ratio of the longitudinal reinforcement*, the deformation ratio of the reinforcements can significantly deviate from the assumed geometric deformation ratio. To determine the resulting deformation ratio correctly, the program uses the strain of the reinforcement that is closer to the main action.

Crack width w_k

The calculated value of the crack width w_k is determined according to Equation 2.70, page 87.

☐ Determination of calculated value of crack width			
☐ Bottom surface (+z)	Ratio	0.000	
Concrete does not crack on this side.			
☐ Top surface (-z)	Ratio	0.591	
☐ Calculated Value of Crack Width into Reinforcement Direction 1	$w_{k,-z,\phi 1}$	0.130	mm
Maximum Crack Spacing into Reinforcement Direction 1	$s_{r,max,-z,\phi 1}$	0.177	m
Difference in the meanstrain into the direction 1	$(\epsilon_{sm} - \epsilon_{cm})_{-z,\phi 1}$	0.735	‰
☐ Calculated Value of Crack Width into Reinforcement Direction 2	$w_{k,-z,\phi 2}$	0.115	mm
Maximum Crack Spacing into Reinforcement Direction 2	$s_{r,max,-z,\phi 2}$	0.218	m
Difference in the meanstrain into the direction 2	$(\epsilon_{sm} - \epsilon_{cm})_{-z,\phi 2}$	0.527	‰
☐ Crack width into the resulting difference in the meanstrain	$w_{k,-z,res}$	0.177	mm
Maximum Final Crack Spacing acc. (7.15)	$s_{r,max,-z,res}$	0.137	m
Resulting Difference in the Mean Strain	$(\epsilon_{sm} - \epsilon_{cm})_{-z,res}$	1.291	‰

Figure 2.113: Calculation value of crack width

In the 1.3 *Surfaces* window, we specified the maximum allowable crack width $\max w_k = 0.3$ mm. Thus, we obtain the design criterion for the governing resulting direction.

☐ Design Internal Forces in Serviceability Limit State			
☐ Check Steel Stress			
☐ Check Final Crack Spacing			
☐ Determination of difference in the meanstrain			
☐ Determination of calculated value of crack width			
☐ Check			
Crack width at the top (-z) surface in direction of the resulting strain	$w_{k,-z,res}$	0.177	mm
Maximum allowable crack width at the top (-z) surface acc. to user input	$w_{k,-z,limit}$	0.300	mm
Criterion of Check	Criterion	0.591	

Figure 2.114: Design criterion for crack width

2.6.5 Governing Effects of Actions

In RFEM, it is possible to define the various loadings in the individual load cases (LC). Load cases can be superimposed in load combinations (CO) and result combinations (RC). The difference between both types of combinations is described in chapters 5.5 and 5.6 of the RFEM manual.

Load cases and load combinations yield only one set of internal forces, respectively. In a result combination, however, there can be up to 16 sets of internal forces, depending on the type of model:

- For the types of model 2D - XZ ($u_x/u_z/\phi_y$) and 2D - XY ($u_x/u_y/\phi_z$) (wall), we obtain only the axial forces n_x , n_y , and n_{xy} in the surfaces. Their combination yields six sets of internal forces, of which one of these axial forces shows its maximum or minimum value, respectively.
- For the type of model 2D - XY ($u_z/\phi_x/\phi_y$) (plate), the maximum and minimum values of the moments m_x , m_y , and m_{xy} as well as their shear forces v_x and v_y are determined. Thus, we obtain ten sets of internal forces.
- The type of model 3D contains all the axial forces, moments, and shear forces mentioned above and, therefore, yields 16 sets of internal forces.

The analysis core for the serviceability limit state designs proceeds with the internal forces of the selected load cases and load combinations sequentially, that is, row by row. The same is true for the sets of internal forces of a result combination. This shows that the design of a result combination is much more time-consuming.

In most checks for the individual reinforcement directions, the internal forces or sets of internal forces result in an action-effect. The program determines the greatest action-effect of all reinforcement directions. If the resistance for the individual reinforcement directions is different, the program searches for the reinforcement direction that yields the greatest quotient from action-effect over resistance.

2.7 Deformation analysis with RF-CONCRETE Deflect



For the deformation analysis, you need a license of the **RF-CONCRETE Deflect** add-on module.

2.7.1 Basic Material and Geometric Assumptions

For the deformation analysis with RF-CONCRETE Deflect, a linear-elastic compression and tension behavior of reinforcing steel is assumed. Here, a linear-elastic compression behavior and a linear-elastic behavior is applied until the tension strength is reached. Such assumptions are sufficiently exact for the serviceability limit state. If the provided stress exceeds the concrete strength, damage develops according to EN 1992-1-1, clause 7.3.4.

The calculation uses a simple isotropic model of fracture mechanics that is defined individually in the two directions of reinforcement. From an engineering point of view, in accordance with EN 1992-1-1, a material stiffness matrix is computed by interpolation between the uncracked (state I) and cracked state (state II) according to clause 7.4.3, Equation (7.18). Thus, the reinforced concrete is modeled as an orthotropic material. All laws of the damage development can take into account the tension stiffening effect and simple long-term effects (shrinkage and creep).

The computation of the material stiffness matrices is implemented for the types of model *2D-XY* ($u_z/\varphi_x/\varphi_y$) and *3D*. For the *3D* type of model, the influence of the eccentricities of the ideal centroids (see below) is additionally considered in the stiffness matrix.

2.7.2 Design Internal Forces

As described above, the calculation of stiffnesses is based on linear-elastic assumptions. The internal forces are transformed in the orthogonal direction of reinforcements ϕ and to the two surfaces s (top and bottom). The obtained internal forces – bending moments $m_{\phi,s}$ and axial forces $n_{\phi,s}$ (torsion moments are eliminated by a transformation in the directions of reinforcement) – depend on the

- (a) type of model
- (b) method of calculation
- (c) classification criterion.

2.7.3 Critical Surface

For the determination of the critical surface, each reinforcement direction ϕ is considered separately. The state of stress is analyzed on both surfaces s – bottom surface (in the direction of the local $+z$ -axis) and top surface (in the direction of the local $-z$ -axis). The surface with the greater tension in concrete is classified as governing. The internal forces on the critical surfaces are termed n_ϕ and m_ϕ .

The axial force $n_{\phi,s}$ transformed in the direction of reinforcement ϕ has the same value for both surfaces s ($n_\phi = n_{\phi,top} = n_{\phi,bottom}$). Therefore, the axial forces are not relevant for the determination of the critical surface; only the moments are considered in order to find the governing surface. The signs for the bending moments $m_{\phi,s}$ are determined regarding whether the moments cause tension or compression on the corresponding surface, respectively. Therefore, the critical surface is that with the greater bending moment (that is, the surface that is more subjected to tension).

For the calculation of the stiffness, only the internal forces n_ϕ and m_ϕ on the critical surface are taken into account. Until now, the term "bottom surface" referred to the local $+z$ -axis. In the following, however, "bottom surface" refers to the critical side of the surface.

2.7.4 Cross-Section Properties

The cross-section properties are determined for both reinforcement directions and for both cross-section states c (cracked / uncracked). For state I (uncracked cross-section), a linear-elastic behavior of concrete in tension is applied. For state II (cracked cross-section), the tensile strength of concrete is not applied.

If there are no axial forces n_ϕ (for example in the type of model $2D - XY (u_z/\varphi_x/\varphi_y)$), this part of the computation is independent of internal forces and a direct calculation of the cross-section properties is possible. In the other cases, the depth of the neutral axis is calculated by means of an iterative method of calculation, the so-called "binary method." For numerical reasons, the program uses the minimum value for the reinforcement ratio $\rho_{\min} = 10^{-4}$ in every iteration step. That is, if there is no reinforcement, a virtual minimum reinforcement area is applied. Such a small value has no appreciable influence on the results (stiffnesses).

The calculated ideal cross-section properties (related to the concrete cross-section) in a reinforcement direction ϕ and the crack state c are the:

- (a) moment of inertia to the ideal center of gravity $I_{\phi,c}$
- (b) moment of inertia to the geometric center of the cross-section $I_{0,\phi,c}$
- (c) cross-section area $A_{\phi,c}$
- (d) eccentricity of the ideal center of gravity $e_{\phi,c}$.

2.7.5 Long-Term Effects

Shrinkage and creep are time-dependent properties of concrete. According to EN 1992-1-1, the long-term effects are to be considered separately.

2.7.5.1 Creep

The creep effects are considered by reduction of the concrete modulus of elasticity E , using the effective creep coefficient φ_{eff} according to EN 1992-1-1, Equation (7.20):

$$E_{\text{cd,eff}} = \frac{E_{\text{cd}}}{1 + \varphi_{\text{eff}}}$$

Equation 2.76

2.7.5.2 Shrinkage

In the deflection calculation according to EN 1992-1-1, there are two topics that are influenced by shrinkage effects.

Reduction of material stiffness

The material stiffness in each reinforcement direction ϕ is reduced by a so-called coefficient of shrinkage influence $k_{\text{sh},\phi,c}$. For both crack states c (cracked / uncracked), the axial forces $n_{\text{sh},\phi,c}$ and bending moments $m_{\text{sh},\phi,c}$ are determined from the free shrinkage strain ϵ_{sh} :

$$n_{\text{sh},\phi} = -\epsilon_{\text{sh}} \cdot E_s (a_{s1} + a_{s2})$$

$$m_{\text{sh},\phi} = n_{\text{sh}} \cdot e_{\text{sh}}$$

Equation 2.77

where

- $n_{\text{sh},\phi}$ Additional axial force from shrinkage in reinforcement direction ϕ
- $m_{\text{sh},\phi}$ Additional moment from shrinkage in center of gravity of the ideal cross-section in the direction of reinforcement ϕ
- a_{s1} Bottom surface of reinforcement
- a_{s2} Top surface of reinforcement

- E_s Modulus of elasticity of the reinforcing steel
- ϵ_{sh} Shrinkage strain
- e_{sh} Eccentricity of the shrinkage forces (state I and state II) from the center of gravity of the ideal cross-section

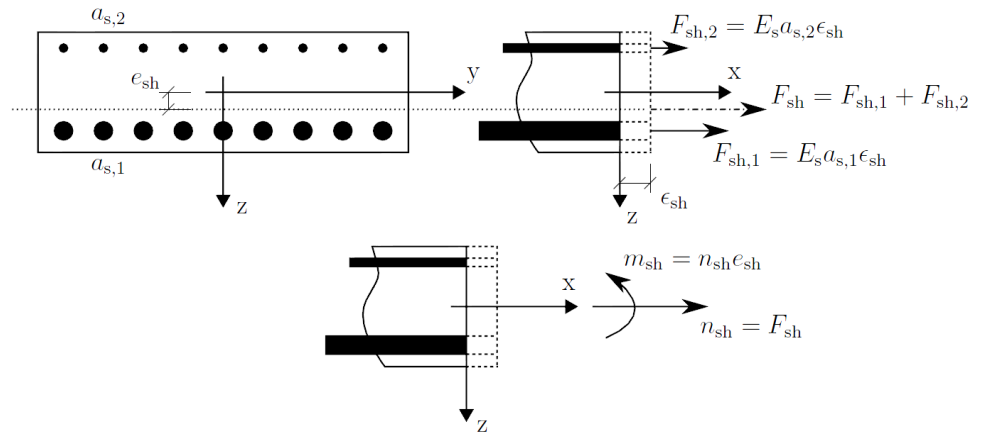


Figure 2.115: Internal forces $n_{sh,\phi}$ and $m_{sh,\phi}$

With these internal forces from shrinkage, the additional curvature induced by shrinkage $\kappa_{sh,\phi,c}$ is calculated in the analyzed point – without influence of the surrounding model. Then, the new coefficient of shrinkage $k_{sh,\phi,c}$ is calculated according to:

$$k_{sh,\phi,c} = \frac{\kappa_{sh,\phi,c} + \kappa_{\phi}}{\kappa_{\phi}}$$

Equation 2.78

where

- κ_{ϕ} Curvature induced by external loading without the shrinkage influence in the direction of reinforcement ϕ
- $\kappa_{sh,\phi,c}$ Curvature induced by shrinkage (and reinforcement arrangement) without influence of creep in reinforcement direction ϕ

The coefficient $k_{sh,\phi,c}$ is limited to the interval $k_{sh,\phi,c} \in (1, 100)$: Therefore, $k_{sh,\phi,c}$ must not reduce the stiffness by more than 100 times (for numerical and physical reasons). Furthermore, the minimum value $k_{sh,\phi,c} = 1$ means that it is not possible to consider an influence of shrinkage if the influence of shrinkage has an opposite orientation compared to the loading-induced curvature κ_d .

The shrinkage influence on the membrane stiffness is not considered.

Calculation of Distribution Coefficient

The second influence of shrinkage is an influence on the calculation of the distribution coefficient (damage parameter) ζ according to EN 1992-1-1, clause 7.4.3, Equation (7.18). The following chapter describes the distribution coefficient in detail.

2.7.6 Distribution Coefficient

The calculation of the distribution coefficient ζ_d is shown for reinforcement direction ϕ . First, the program calculates the maximum concrete tension stress $\sigma_{\max,\phi}$ under the assumption of a linear-elastic material behavior:

$$\sigma_{\max,\phi} = \frac{n_\phi + n_{sh,\phi}}{A_{\phi,I}} + \frac{m_\phi - n_\phi \cdot \left(x_{\phi,I} - \frac{h}{2}\right) + m_{sh,\phi,I}}{I_{\phi,I}} \cdot (h - x_{\phi,I})$$

Equation 2.79

where

n_ϕ	Axial force from external loading in reinforcement direction ϕ
$n_{sh,\phi}$	Additional axial force from shrinkage in reinforcement direction ϕ
m_ϕ	Moment from external loading in reinforcement direction ϕ
$m_{sh,\phi,I}$	Additional moment from shrinkage in reinforcement direction ϕ in state I
$x_{\phi,I}$	Depth of neutral axis in uncracked state in reinforcement direction ϕ
h	Depth of the cross-section
$A_{\phi,I}$	Ideal cross-section area in state I in reinforcement direction ϕ
$I_{\phi,I}$	Ideal moment of inertia in state I in reinforcement direction ϕ

The influence of shrinkage force on the maximum tension stress $\sigma_{\max,\phi}$ is considered with the additional internal forces from shrinkage.

The calculation of the distribution coefficient ζ_ϕ depends on whether the tension stiffening according to EN 1992-1-1 is taken into account in the deformation calculation.

Distribution coefficient ζ_ϕ taking into account tension stiffening

- for $\sigma_{\max,\phi} > f_{ctm}$:

$$\zeta_\phi = 1 - \beta \cdot \left(\frac{f_{ctm}}{\sigma_{\max,\phi}} \right)^n$$

- for $\sigma_{\max,\phi} \leq f_{ctm}$:

$$\zeta_\phi = 0$$

Equation 2.80

where

β	Parameter to taken into account the load duration
f_{ctm}	Mean tensile strength
n	2 for EN 1992-1-1

Coefficient of distribution ζ_ϕ without consideration of tension stiffening

- for $\sigma_{\max,\phi} > f_{ctm}$:

$$\zeta_\phi = 1$$

- for $\sigma_{\max,\phi} \leq f_{ctm}$:

$$\zeta_\phi = 0$$

Equation 2.81

2.7.7 Cross-Section Properties for Deformation Analysis

In the material stiffness matrix D for the deformation analysis, the program requires the cross-section properties available in every direction of reinforcement depending on the cracked state. These are the following:

- (a) moment of inertia to the ideal center of gravity I_ϕ
- (b) moment of inertia to the geometric center of cross-section $I_{0,c}$
- (c) ideal cross-section area A_ϕ
- (d) eccentricity of the ideal center of gravity e_ϕ to the geometric center.

A mean strain ε_ϕ and a mean curvature κ_ϕ are computed as interpolation between a cracked and an uncracked state according to EN 1992-1-1, Equation (7.18):

$$\varepsilon_\phi = \zeta_\phi \cdot \varepsilon_{\phi,II} + (1 - \zeta_\phi) \cdot \varepsilon_{\phi,I}$$

$$\kappa_\phi = \zeta_\phi \cdot \kappa_{\phi,II} + (1 - \zeta_\phi) \cdot \kappa_{\phi,I}$$

Equation 2.82

The strain in the cracked state c (state I and II) are computed according to the following equations:

$$\varepsilon_{\phi,c} = \frac{n_\phi}{E \cdot A_{\phi,c}}$$

$$\kappa_{\phi,c} = \kappa_{sh,\phi,c} \cdot \frac{m_\phi - n_\phi \cdot e_{\phi,c}}{E \cdot I_{\phi,c}}$$

Equation 2.83

The influence of the shrinkage is considered by using factor $k_{sh,\phi,c}$.

If there are no axial forces n_ϕ (for example in the type of model 2D - XY ($u_z/\varphi_x/\varphi_y$)), only those ideal cross-section properties are relevant that relate to the ideal center of the cross-section:

$$A_\phi = \frac{A_{\phi,I} \cdot A_{\phi,II}}{\zeta_\phi \cdot A_{\phi,I} \cdot k_{sh,\phi,II} + (1 - \zeta_\phi) \cdot A_{\phi,II} \cdot k_{sh,\phi,I}}$$

$$I_\phi = \frac{I_{\phi,I} \cdot I_{\phi,II}}{\zeta_\phi \cdot I_{\phi,I} \cdot k_{sh,\phi,II} + (1 - \zeta_\phi) \cdot I_{\phi,II} \cdot k_{sh,\phi,I}}$$

Equation 2.84

If axial forces are available, the cross-section properties are related to the geometric center of cross-section:

$$A_\phi = \frac{n_\phi}{A \cdot \varepsilon_\phi} \quad \text{where} \quad \varepsilon_\phi = \frac{m_\phi - \kappa_\phi \cdot E \cdot I_\phi}{n_\phi}$$

$$I_{\phi,0} = I_\phi + A_\phi \cdot e_\phi^2 \quad \text{where} \quad I_\phi \text{ according to Equation 2.84}$$

Equation 2.85

In the calculation of the cross-section properties, the initial value of the poisson's ratio ν_{init} is reduced according to the following equation:

$$\nu = (1 - \max_{\phi \in \{1,2\}} (\zeta_\phi)) \cdot \nu_{init}$$

Equation 2.86

2.7.8 Material Stiffness Matrix D

Bending stiffness - plates and shells

The bending stiffnesses in the directions of reinforcement ϕ are determined as follows:

$$D_{d,d} = I_{0,d} \cdot E / (1 - \nu^2) \quad \text{where } d = \{1,2\}$$

$$D_{d,d} = I_d \cdot E / (1 - \nu^2) \quad \text{where } d = \{1,2\}$$

The non-diagonal component of the material stiffness matrix has the same expression for plates and shells:

$$D_{1,2} = D_{2,1} = \nu \cdot \sqrt{(D_{1,1} \cdot D_{2,2})}$$

For shells, the differences in the bending stiffnesses due to the moments of inertia are compensated by the eccentricity components in the material stiffness matrix.

Torsion stiffness of plates and shells

The elements of the stiffness matrix are calculated for plates and shells as follows:

$$D_{3,3} = (1-\nu)/2 \cdot \sqrt{(D_{1,1} \cdot D_{2,2})}$$

Shear stiffness of plates and shells

The elements of the stiffness matrix for shear are not reduced for the deformation analysis. They are determined from the shear modulus G of the ideal cross-section and the cross-section height h . The expression for shells and plates is the same:

$$D_{3+d,3+d} = 5/6 \cdot Gh \quad \text{where } d = \{1,2\}$$

Membrane stiffness of shells

The membrane stiffnesses in the reinforcement directions ϕ are determined as follows:

$$D_{d+5,d+5} = E \cdot A_d / (1 - \nu^2) \quad \text{where } d = \{1,2\}$$

The non-diagonal part of the material stiffness matrix is determined from:

$$D_{6,7} = D_{7,6} = \nu \cdot \sqrt{(D_{6,6} \cdot D_{7,7})}$$

The part of the shear stiffness component is:

$$D_{8,8} = G \cdot h$$

Eccentricity – shells

The elements of the stiffness matrix for the eccentricity of the centroid (ideal cross-section) in the reinforcement direction ϕ are determined as follows:

$$D_{d,6} = D_{6,d} = D_{d+5,d+5} \cdot e_d \quad \text{where } d = \{1,2\}$$

The non-diagonal component of the material stiffness matrix is determined from:

$$D_{1,7} = D_{7,1} = \nu/2 \cdot (e_{\phi 1} + e_{\phi 2}) \cdot \sqrt{(D_{6,6} \cdot D_{7,7})}$$

The eccentricity components for torsion are determined as follows:

$$D_{3,8} = D_{8,3} = 1/2 \cdot G \cdot h \cdot (e_{\phi 1} + e_{\phi 2})$$

2.7.9 Positive Definite Test

The positive definite of the material stiffness matrix D is tested by a modified (with respect to the blocks of zeros) SYLVESTER'S criterion. If the stiffness matrix D is not positive definite, the non-diagonal components of the material stiffness matrix are zeroed consecutively. In the extreme case, only the positive components from the diagonal remain.

2.7.10 Example

The determination of the material stiffness matrix D is illustrated by a simple example. The model contains only one finite element and the surface is reinforced only on one side (top). For simplicity's sake, the manual calculation is carried out in the reinforcement direction ϕ_1 .

2.7.10.1 Geometry

The model with the dimensions 1×1 m and the thickness 0.20 m is fixed at one side. The free side is subjected to the bending moment $m_x = -30$ kNm/m and the axial force $n_x = -100$ kN/m. The automatic self-weight is not taken into account.

The longitudinal reinforcement in ϕ_1 is 1000 mm².

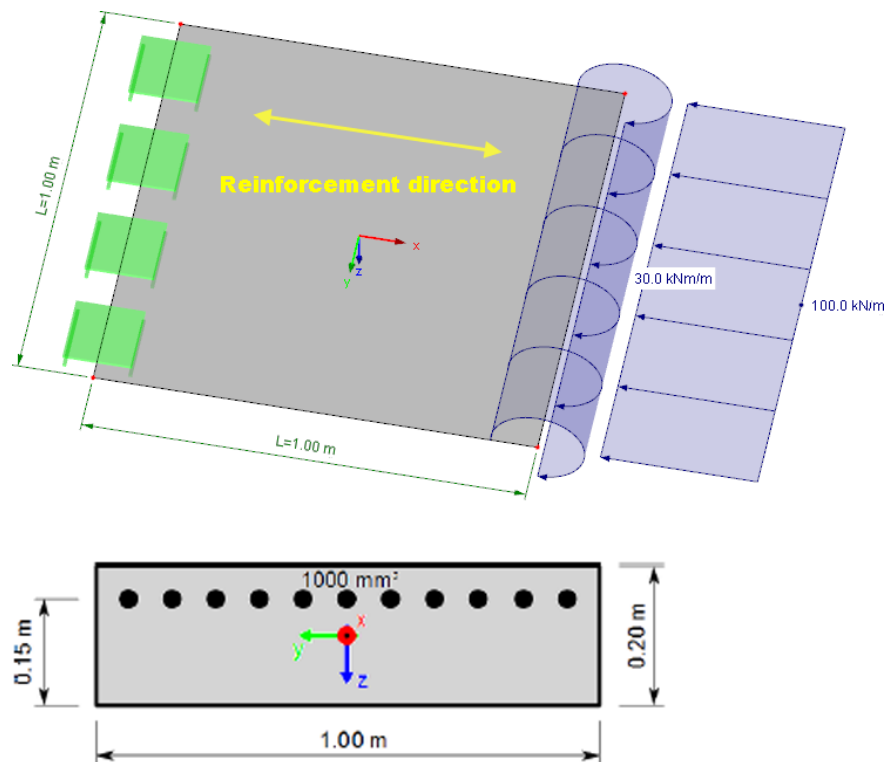


Figure 2.116: Model with loading and reinforcement

2.7.10.2 Materials

The following table shows the material properties.

☑ Internal Forces of Linear Statics				
☑ Principal Internal Forces				
☑ Check if internal forces cause cracks				
☑ Design Internal Forces				
☑ Selection of Internal Design Forces				
☑ Determination of Critical Surfaces				
☑ Material Data for Stiffness Calculation				
☐ Concrete				
Modulus of Elasticity	E_b	33000.0	MPa	
Shear Modulus	G_c	11800.0	MPa	
Poisson's Ratio	ν	0.20		
Mean Axial Tensile Strength	f_{ctm}	2.9	MPa	
Creep coefficient	φ	2.00		
Coefficient of Torsion	Φ	1.00		
☐ Reinforcement				
Modulus of Elasticity	E_s	200000.0	MPa	

Figure 2.117: Material data for stiffness calculation

2.7.10.3 Selection of the Design Internal Forces

First, the internal forces are transformed in the first direction of reinforcement ϕ_1 . The bending moments have different value for the bottom (+z) and the top (-z) surface; the axial forces have the same signs after the transformation.

$$m_{\phi 1, +z} = -30 \text{ kN}$$

$$m_{\phi 1, -z} = 30 \text{ kN}$$

$$n_{\phi 1, +z} = n_{\phi 1, -z} = -100 \text{ kN}$$

Internal Forces of Linear Statics				
Principal Internal Forces				
Check if internal forces cause cracks				
Design Internal Forces				
Selection of Internal Design Forces				
into Reinforcement Direction 1				
Design Axial Force	$n_{\phi 1}$	-100.0	kN/m	
Design Moment				
Design Moment at the Bottom Surface (+z)	$m_{+z, \phi 1}$	-30.0	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z, \phi 1}$	30.0	kNm/m	
into Reinforcement Direction 2				
Design Axial Force	$n_{\phi 2}$	0.0	kN/m	
Design Moment				
Design Moment at the Bottom Surface (+z)	$m_{+z, \phi 2}$	-1.1	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z, \phi 2}$	1.1	kNm/m	

Figure 2.118: Selection of the design internal forces

2.7.10.4 Determination of Critical Surface

The top surface (-z) is determined to be the critical surface. For the further calculation, only the bending moment and the axial force of this surface are considered.

$$m_{\phi 1} = m_{\phi 1, -z} = 30 \text{ kN}$$

$$n_{\phi 1} = n_{\phi 1, +z} = n_{\phi 1, -z} = -100 \text{ kN}$$

Internal Forces of Linear Statics				
Principal Internal Forces				
Check if internal forces cause cracks				
Selection of Internal Design Forces				
Determination of Critical Surfaces				
into Reinforcement Direction 1				
Critical Surface				
	topen (-z)			
Design Moment at the Bottom Surface (+z)	$m_{+z, \phi 1}$	-30.0	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z, \phi 1}$	30.0	kNm/m	
into Reinforcement Direction 2				
Critical Surface				
	oben (-z)			
Design Moment at the Bottom Surface (+z)	$m_{+z, \phi 2}$	-1.1	kNm/m	
Design Moment at the Top Surface (-z)	$m_{-z, \phi 2}$	1.1	kNm/m	

Figure 2.119: Determination of critical surface

2.7.10.5 Cross-Section Properties (Cracked and Uncracked State)

The cross-section properties depend on the governing surface and the direction of reinforcement ϕ_1 . The minimum values are used for the surfaces of reinforcement a_{s2,ϕ_1} , a_{s1,ϕ_2} , and a_{s2,ϕ_2} .

The following cross-section properties for the uncracked and cracked state are to be calculated in order to be able to assemble the stiffness matrix of the material **D**.

Center of gravity

The distance of the center of gravity of the ideal cross-section from the concrete surface in compression is directly determined for the uncracked state.

$$z_{l,\phi_1} = \frac{\frac{b \cdot h^2}{2} + \alpha \cdot (a_{s1,\phi_1} \cdot d_{1,\phi_1} + a_{s2,\phi_1} \cdot d_{2,\phi_1})}{b \cdot h + \alpha \cdot (a_{s1,\phi_1} + a_{s2,\phi_1})}$$

$$= \frac{\frac{1000 \cdot 200^2}{2} + 6.061 \cdot (1000 \cdot 150 + 15 \cdot 50)}{1000 \cdot 200 + 6.061 \cdot (1000 + 15)} = 101.4 \text{ mm}$$

For the cracked state, the depth χ_{ll,ϕ_1} of the zone in compression must be calculated iteratively. The distance of the center of gravity of the ideal cross-section from the surface in compression is calculated for the cracked state.

Ideal cross-section area $A_{c,d}$

The effective cross-section area in the uncracked state without the influence of creep is:

$$A_{l,\phi_1} = b \cdot h + \alpha \cdot (a_{s1,\phi_1} + a_{s2,\phi_1}) = 1000 \cdot 200 + 6.061 \cdot (1000 + 15) = 2061.5 \text{ cm}^2$$

The effective cross-section area in the cracked state is determined with the influence of creep.

$$A_{ll,\phi_1} = b \cdot \chi_{ll,\phi_1} + \alpha \cdot (a_{s1,\phi_1} + a_{s2,\phi_1}) = 1000 \cdot 68.3 + 18.182 \cdot (1000 + 15) = 867.19 \text{ cm}^2$$

The coefficient α is the ratio of the moduli of elasticity of steel and concrete with or without the influence of creep.

Ideal moment of inertia to ideal center of gravity $I_{c,d}$

The effective moment of inertia to the ideal center of gravity in the uncracked state without influence of creep is:

$$I_{l,\phi_1} = 1/12 \cdot b \cdot h^3 + b \cdot h \cdot (z_{l,\phi_1} - h/2)^2 + \alpha \cdot a_{s1,\phi_1} \cdot (d_{1,\phi_1} - z_{l,\phi_1})^2 + \alpha \cdot a_{s2,\phi_1} \cdot (z_{l,\phi_1} - d_{2,\phi_1})^2$$

$$= 1/12 \cdot 1000 \cdot 200^3 + 1000 \cdot 200 \cdot (101.4 - 200/2)^2 + 6.061 \cdot 1000 \cdot (150 - 101.4)^2 + 6.061 \cdot 15 \cdot (101.4 - 50)^2$$

$$= 68,161.30 \text{ cm}^4$$

The effective moment of inertia to the ideal center of gravity in the cracked state is determined with the influence of creep.

$$I_{ll,\phi_1} = 1/12 \cdot \chi_{ll,\phi_1}^3 + b \cdot \chi_{ll,\phi_1} \cdot (z_{ll,\phi_1} - \chi_{ll,\phi_1}/2)^2 + \alpha \cdot a_{s1,\phi_1} \cdot (d_{1,\phi_1} - z_{ll,\phi_1})^2 + \alpha \cdot a_{s2,\phi_1} \cdot (z_{ll,\phi_1} - d_{2,\phi_1})^2$$

$$= 1/12 \cdot 1000 \cdot 68.3^3 + 1000 \cdot 68.3 \cdot (58.5 - 68.3/2)^2 + 18.182 \cdot 1000 \cdot (150 - 58.5)^2 + 18.182 \cdot 15 \cdot (58.5 - 50)^2$$

$$= 21,928.70 \text{ cm}^4$$

Ideal moment of inertia to geometric center of cross-section $I_{0,c,d}$

The ideal moment of inertia to the geometric center of the cross-section in the uncracked state without influence of creep is:

$$I_{0,l,\phi_1} = 1/12 \cdot b \cdot h^3 + \alpha \cdot a_{s1,\phi_1} \cdot (d_{1,\phi_1} - h/2)^2 + \alpha \cdot a_{s2,\phi_1} \cdot (h/2 - d_{2,\phi_1})^2$$

$$= 1/12 \cdot 1000 \cdot 200^3 + 6.061 \cdot 200 \cdot (150 - 200/2)^2 + 6.061 \cdot 15 \cdot (200/2 - 50)^2$$

$$= 68,204.50 \text{ cm}^4$$

The ideal moment of inertia to the geometric center of the cross-section in the cracked state is determined from the influence of creep.

$$\begin{aligned}
 I_{0,II,\phi 1} &= 1/12 \cdot \chi_{II,\phi 1}^3 + b b \cdot \chi_{II,\phi 1} \cdot (h/2 - \chi_{II,\phi 1}/2)^2 + \alpha \cdot a_{s1,\phi 1} \cdot (d_{1,\phi 1} - h/2)^2 + \alpha \cdot a_{s2,\phi 1} \cdot (h/2 - d_{2,\phi 1})^2 \\
 &= 1/12 \cdot 1000 \cdot 68.3^3 + 1000 \cdot 68.3 \cdot (200/2 - 68.3/2)^2 + 18.182 \cdot 1000 \cdot (150 - 200/2)^2 + 18.182 \cdot 15 \cdot (200/2 - 50)^2 \\
 &= 36,881.50 \text{ cm}^4
 \end{aligned}$$

Eccentricity of the centroid $e_{c,d}$

The eccentricity of the ideal center of gravity is determined as follows:

$$e_{c,\phi 1} = z_{c,\phi 1} - h/2$$

- Uncracked state:

$$e_{\phi 1,I} = 101.4 - 200/2 = 1.4 \text{ mm}$$

- Cracked state:

$$e_{\phi 1,II} = 58.5 - 200/2 = -41.5 \text{ mm}$$

Internal Forces of Linear Statics			
Principal Internal Forces			
Check if internal forces cause cracks			
Design Internal Forces			
Selection of Internal Design Forces			
Determination of Critical Surfaces			
Material Data for Stiffness Calculation			
Cross-Sectional Properties (Uncracked and Cracked State)			
into Reinforcement Direction 1			
Geometry			
Cross-Sectional Height	h	200.0	mm
Effective Height	d _{1,φ1}	150.0	mm
Reinforcement area	a _{s,1,φ1}	10.00	cm ²
Effective Height	d _{2,φ1}	50.0	mm
Reinforcement area	a _{s,2,φ1}	0.15	cm ²
Uncracked State (State I)			
Depth of the Concrete Compression Zone	x _{I,φ1}	101.4	mm
Ideal Cross-Sectional Area	A _{I,φ1}	2061.52	cm ²
Ideal Moment of Inertia to Ideal Center of Gravity	I _{I,φ1}	68161.30	cm ⁴
Ideal Moment of Inertia to Geometrical Center of Cross-Section	I _{0,I,φ1}	68204.50	cm ⁴
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	e _{I,φ1}	1.4	mm
Cracked State (State II)			
Depth of the Concrete Compression Zone	x _{II,φ1}	68.3	mm
Ideal Cross-Sectional Area	A _{II,φ1}	867.19	cm ²
Ideal Moment of Inertia to Ideal Center of Gravity	I _{II,φ1}	21928.70	cm ⁴
Ideal Moment of Inertia to Geometrical Center of Cross-Section	I _{0,II,φ1}	36881.50	cm ⁴
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	e _{II,φ1}	-41.5	mm

Figure 2.120: Cross-sectional properties in reinforcement direction 1

□ Cross-Sectional Properties (Uncracked and Cracked State)			
□ into Reinforcement Direction 1			
□ into Reinforcement Direction 2			
□ Geometry			
Cross-Sectional Height	h	200.0	mm
Effective Height	d _{1,φ2}	139.0	mm
Reinforcement area	a _{s,1,φ2}	0.14	cm ²
Effective Height	d _{2,φ2}	60.0	mm
Reinforcement area	a _{s,2,φ2}	0.14	cm ²
□ Uncracked State (State I)			
Depth of the Concrete Compression Zone	x _{I,φ2}	100.0	mm
Ideal Cross-Sectional Area	A _{I,φ2}	2001.68	cm ²
Ideal Moment of Inertia to Ideal Center of Gravity	I _{I,φ2}	66692.90	cm ⁴
Ideal Moment of Inertia to Geometrical Center of Cross-Section	I _{0,I,φ2}	66692.90	cm ⁴
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	e _{I,φ2}	0.0	mm
□ Cracked State (State II)			
Depth of the Concrete Compression Zone	x _{II,φ2}	9.5	mm
Ideal Cross-Sectional Area	A _{II,φ2}	100.42	cm ²
Ideal Moment of Inertia to Ideal Center of Gravity	I _{II,φ2}	516.86	cm ⁴
Ideal Moment of Inertia to Geometrical Center of Cross-Section	I _{0,II,φ2}	8734.84	cm ⁴
Eccentricity of Center of Gravity (Positive Value in Direction of Critical	e _{II,φ2}	-90.5	mm

Figure 2.121: Cross-sectional properties in reinforcement direction 2

2.7.10.6 Consideration of Shrinkage

The influence of shrinkage is directly introduced in the calculation with the defined value of the free shrinkage ϵ_{sh} . Thus, the influence of the structural restraints or redistributions of the shrinkage forces is not taken into account.

In our example, the shrinkage strain is applied with the following values:

$$\epsilon_{sh} = -0.5 \cdot 10^{-3}$$

The free shrinkage strain causes additional forces in the cross-section:

$$\begin{aligned} n_{sh,\phi 1} &= -E_s \cdot \epsilon_{sh} \cdot (a_{s1,\phi 1} + a_{s2,\phi 1}) \\ &= -200 \cdot 10^9 \cdot (-0.5 \cdot 10^{-3}) \cdot (1000 + 15) \cdot 10^{-6} \\ &= 101.5 \text{ kN/m} \end{aligned}$$

The forces act for both crack states c (cracked or uncracked) with the eccentricity to the center of gravity of the ideal cross-section:

$$e_{sh,c,\phi 1} = \frac{a_{s1,\phi 1} \cdot d_{1,\phi 1} + a_{s2,\phi 1} \cdot d_{2,\phi 1}}{a_{s1,\phi 1} + a_{s2,\phi 1}} - z_{c,\phi 1}$$

- Uncracked state:

$$e_{sh,c,\phi 1} = \frac{1000 \cdot 150 + 15 \cdot 50}{1000 + 15} - 101.4 = 47.1 \text{ mm}$$

- Cracked state:

$$e_{sh,c,\phi 1} = \frac{1000 \cdot 150 + 15 \cdot 50}{1000 + 15} - 58.5 = 90.0 \text{ mm}$$

The bending moment due to the axial force $n_{sh,\phi 1}$ for both states c is:

$$m_{sh,c,\phi 1} = n_{sh,\phi 1} \cdot e_{sh,c,\phi 1}$$

- Uncracked state:

$$m_{sh,I,\phi 1} = 101.5 \cdot 10^3 \cdot 0.047 = 4.8 \text{ kNm/m}$$

- Cracked state:

$$m_{sh,II,\phi 1} = 101.5 \cdot 10^3 \cdot 0.090 = 9.1 \text{ kNm/m}$$

In determining the coefficient $k_{sh,c,d}$ for both states, we have to distinguish:

- for $m_{\phi 1} \neq 0$:

$$k_{sh,c,\phi 1} = \frac{m_{sh,c,\phi 1} + m_{\phi 1} - n_{\phi 1} \cdot e_{c,\phi 1}}{m_{\phi 1} - n_{\phi 1} \cdot e_{c,\phi 1}}$$

- for $m_{\phi 1} = 0$:

$$k_{sh,c,\phi 1} = 1 \quad \text{where } k_{sh,c,\phi 1} \in \{1, 100\}$$

The following is valid for this example: $m_{\phi 1} \neq 0$

- Uncracked state:

$$k_{sh,I,\phi 1} = \frac{4.771 \cdot 10^3 + 30 \cdot 10^3 - (-100 \cdot 10^3) \cdot 1.4 \cdot 10^{-3}}{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot 1.4 \cdot 10^{-3}} = 1.159$$

- Cracked state:

$$k_{sh,II,\phi 1} = \frac{9.135 \cdot 10^3 + 30 \cdot 10^3 - (-100 \cdot 10^3) \cdot (-41.5 \cdot 10^{-3})}{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot (-41.5 \cdot 10^{-3})} = 1.354$$

Shrinkage Influence				
into Reinforcement Direction 1				
Uncracked State (State I)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,I,\phi 1}$	101.5	kN/m	
Moment Induced by Shrinkage	$m_{sh,I,\phi 1}$	4.8	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,I,\phi 1}$	1.159		
Cracked State (State II)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,II,\phi 1}$	101.5	kN/m	
Moment Induced by Shrinkage	$m_{sh,II,\phi 1}$	9.1	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,II,\phi 1}$	1.354		
into Reinforcement Direction 2				
Uncracked State (State I)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,I,\phi 2}$	2.8	kN/m	
Moment Induced by Shrinkage	$m_{sh,I,\phi 2}$	0.0	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,I,\phi 2}$	1.000		
Cracked State (State II)				
Free Shrinkage	ε_{sh}	-0.001		
Axial Force Induced by Shrinkage	$n_{sh,II,\phi 2}$	2.8	kN/m	
Moment Induced by Shrinkage	$m_{sh,II,\phi 2}$	0.3	kNm/m	
Coefficient of Shrinkage Influence	$k_{sh,II,\phi 2}$	1.238		

Figure 2.122: Consideration of shrinkage

2.7.10.7 Calculation of Distribution Coefficient (Damage Parameter)

The maximum stress in the uncracked state is:

$$\begin{aligned}
 \sigma_{\max,\phi 1} &= \frac{n_{\phi 1} + n_{sh,\phi 1}}{A_{\phi 1,I}} + \frac{m_{\phi 1} - n_{\phi 1} \cdot (\chi_{l,\phi 1} - \frac{h}{2}) + m_{sh,I,\phi 1}}{I_{l,\phi 1}} \cdot (h - \chi_{l,\phi 1}) \\
 &= \frac{-100 \cdot 10^3 + 101,5 \cdot 10^3}{0,206} + \frac{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot (0,101 - \frac{0,200}{2}) + 4,778 \cdot 10^3}{6,816 \cdot 10^{-4}} \cdot (0,200 - 0,101) \\
 &= 5,1 \text{ MPa}
 \end{aligned}$$

We assume a long-term loading:

$$\beta_{\phi 1} = 0.5$$

Taking into account *Tension Stiffening*, the distribution coefficient is calculated according to the following equation:

- for $\sigma_{\max,\phi 1} > f_{ctm}$:

$$\zeta_d = 1 - \beta_{\phi 1} \cdot \left(\frac{f_{ctm}}{\sigma_{\max,\phi 1}} \right)^2$$

- for $\sigma_{\max,\phi 1} \leq f_{ctm}$:

$$\zeta_{sh,c,\phi 1} = 0$$

In the example, the maximum tension stress in concrete is greater than the concrete tension strength.

$$\sigma_{\max,\phi 1} > f_{ctm}$$

$$5.1 > 2.9$$

Thus, the distribution coefficient is:

$$\zeta_{\phi 1} = 1 - \beta_{\phi 1} \cdot \left(\frac{f_{ctm}}{\sigma_{\max,\phi 1}} \right)^2 = 1 - 0,5 \cdot \left(\frac{2,9}{5,1} \right)^2 = 0,835$$

☐ Cross-Sectional Properties (Uncracked and Cracked State)				
☐ Shrinkage Influence				
☐ Damage Parameter Calculation				
☐ Into Reinforcement Direction 1				
☐ Damage Parameter	$\zeta_{\phi 1}$	0.835		
Tension Stiffening		Yes		
Maximum Tensile Stress in Concrete	$\sigma_{c,l,\phi 1}$	5.1	MPa	
Mean Axial Tensile Strength	f_{ctm}	2.9	MPa	
Coefficient of Duration of Load	$\beta_{\phi 1}$	0.500		
☐ Into Reinforcement Direction 2				
☐ Damage Parameter	$\zeta_{\phi 2}$	0.000		
Tension Stiffening		Yes		
Maximum Tensile Stress in Concrete	$\sigma_{c,l,\phi 2}$	0.2	MPa	
Mean Axial Tensile Strength	f_{ctm}	2.9	MPa	
Coefficient of Duration of Load	$\beta_{\phi 2}$	0.500		
☐ Final Cross-Sectional Properties				

Figure 2.123: Calculation of the distribution coefficient

2.7.10.8 Final Cross-Section Properties

The curvature for both states c (cracked / uncracked) is calculated as follows:

$$\kappa_{c,\phi 1} = k_{sh,c,\phi 1} \cdot \frac{m_{\phi 1} - n_{\phi 1} \cdot e_{c,\phi 1}}{E \cdot I_{c,\phi 1}}$$

- Uncracked state:

$$\kappa_{I,\phi 1} = 1,158 \cdot \frac{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot 1,4 \cdot 10^{-3}}{11 \cdot 10^9 \cdot 6,816 \cdot 10^{-4}} = 4,655 \cdot 10^{-3}$$

- Cracked state:

$$\kappa_{II,\phi 1} = 1,353 \cdot \frac{30 \cdot 10^3 - (-100 \cdot 10^3) \cdot (-41,5 \cdot 10^{-3})}{11 \cdot 10^9 \cdot 2,193 \cdot 10^{-4}} = 14,499 \cdot 10^{-3}$$

The strain for both states is determined as follows:

$$\varepsilon_{c,\phi 1} = \frac{n_{\phi 1}}{E \cdot A_{c,\phi 1}}$$

- Uncracked state:

$$\varepsilon_{I,\phi 1} = \frac{-100 \cdot 10^3}{11 \cdot 10^9 \cdot 0,206} = -4,413 \cdot 10^{-5}$$

- Cracked state:

$$\varepsilon_{II,\phi 1} = \frac{-100 \cdot 10^3}{11 \cdot 10^9 \cdot 0,087} = -10,449 \cdot 10^{-5}$$

Thus, it is possible to determine the mean strain.

$$\begin{aligned} \varepsilon_{\phi 1} &= \zeta_{\phi 1} \cdot \varepsilon_{II,\phi 1} + (1 - \zeta_{\phi 1}) \cdot \varepsilon_{I,\phi 1} \\ &= 0.835 \cdot (-10,449 \cdot 10^{-5}) + (1 - 0.835) \cdot (-4,413 \cdot 10^{-5}) = -9,459 \cdot 10^{-5} \end{aligned}$$

The mean curvature is determined as follows:

$$\begin{aligned} \kappa_{\phi 1} &= \zeta_{\phi 1} \cdot \kappa_{II,\phi 1} + (1 - \zeta_{\phi 1}) \cdot \kappa_{I,\phi 1} \\ &= 0.835 \cdot 14,499 \cdot 10^{-3} + (1 - 0.835) \cdot 4,655 \cdot 10^{-3} = 12,885 \cdot 10^{-3} \end{aligned}$$

With the mean curvature and the longitudinal strain, it is possible to compute the final cross-section properties taking into account shrinkage, creep, and *tension stiffening*.

Ideal cross-sectional area

$$A_{\phi 1} = \frac{n_{\phi 1}}{E \cdot \varepsilon_{\phi 1}} = \frac{-100 \cdot 10^3}{11 \cdot 10^9 \cdot (-9,459 \cdot 10^{-5})} = 958,59 \text{ cm}^2$$

The ideal moment of inertia to the ideal center of the cross-section

$$I_{\phi 1} = \frac{I_{l,\phi 1} \cdot I_{ll,\phi 1}}{\zeta_{\phi 1} \cdot I_{l,\phi 1} \cdot k_{sh,ll,\phi 1} + (1 - \zeta_{\phi 1}) \cdot I_{ll,\phi 1} \cdot k_{sh,l,\phi 1}}$$

$$= \frac{6,816 \cdot 10^{-4} \cdot 2,193 \cdot 10^{-4}}{0,836 \cdot 6,816 \cdot 10^{-4} \cdot 1,353 + (1 - 0,836) \cdot 2,193 \cdot 10^{-4} \cdot 1,158}$$

$$= 18391,50 \text{ cm}^2$$

Eccentricity of centroid

$$e_{\phi 1} = \frac{m_{\phi 1} - \kappa_{\phi 1} \cdot E \cdot I_{\phi 1}}{n_{\phi 1}} = \frac{30 \cdot 10^3 - 12,855 \cdot 10^{-3} \cdot 11 \cdot 10^9 \cdot 1,839 \cdot 10^{-4}}{-100 \cdot 10^3} = -39 \text{ mm}$$

Ideal moment of inertia to the geometric center of cross-section

$$I_{0,\phi 1} = I_{\phi 1} + A_{\phi 1} \cdot e_{\phi 1}^2 = 1.839 \cdot 10^4 + 0.096 \cdot (-0.0393)^2 = 33,207.10 \text{ cm}^4$$

The poisson's ratio is determined as follows:

$$\nu = (1 - \max_{d \in \{1,2\}} (\zeta_d)) \cdot \nu_{init} = (1 - \max(0, 0.836)) \cdot 0.2 = 0,0328$$

☒ Internal Forces of Linear Statics
☒ Principal Internal Forces
☒ Check if internal forces cause cracks
☒ Design Internal Forces
☒ Selection of Internal Design Forces
☒ Determination of Critical Surfaces
☒ Material Data for Stiffness Calculation
☒ Cross-Sectional Properties (Uncracked and Cracked State)
☒ Shrinkage Influence
☒ Damage Parameter Calculation
☒ Final Cross-Sectional Properties
☒ into Reinforcement Direction 1
Ideal Cross-Sectional Area
Ideal Moment of Inertia to Ideal Center of Gravity
Ideal Moment of Inertia to Geometrical Center of Cross-Section
Eccentricity of Center of Gravity (Positive Value in Direction of Critical
☒ into Reinforcement Direction 2
Ideal Cross-Sectional Area
Ideal Moment of Inertia to Ideal Center of Gravity
Ideal Moment of Inertia to Geometrical Center of Cross-Section
Eccentricity of Center of Gravity (Positive Value in Direction of Critical

Figure 2.124: Final cross-section properties

2.7.10.9 Stiffness Matrix of the Material

Flexural resistance

$$D_{1.1} = \frac{I_{0,\phi 1} \cdot E}{1 - \nu^2} = \frac{3,322 \cdot 10^{-4} \cdot 11 \cdot 10^9}{1 - 0,0328^2} = 3656,74 \text{ kNm}$$

$$D_{1.2} = D_{2.1} = 0,0328 \cdot \sqrt{(3,656 \cdot 10^6 \cdot 7,344 \cdot 10^6)} = 170,58 \text{ kNm}$$

Torsional stiffness

$$D_{3.3} = \frac{1 - \nu}{2} \cdot \sqrt{D_{1.1} \cdot D_{2.2}} = \frac{1 - 0,0328}{2} \cdot \sqrt{(3,656 \cdot 10^6 \cdot 7,344 \cdot 10^6)} = 2505,84 \text{ kNm}$$

Shear stiffness

$$D_{4.4} = D_{5.5} = \frac{5}{6} \cdot G \cdot h = \frac{5}{6} \cdot 11,8 \cdot 10^9 \cdot 0,2 = 1966670 \text{ kNm/m}$$

Membrane stiffness

$$D_{6.6} = \frac{E \cdot A_{\phi 1}}{1 - \nu^2} = \frac{11 \cdot 10^9 \cdot 0,096}{1 - 0,0328^2} = 1055590 \text{ kNm/m}$$

$$D_{6.7} = D_{7.6} = \nu \cdot \sqrt{(D_{6.6} \cdot D_{7.7})} = 0,0328 \cdot \sqrt{(1055,590 \cdot 10^6 \cdot 2204,240 \cdot 10^6)} = 50210,6 \text{ kNm}$$

$$D_{8.8} = G \cdot h = 11,8 \cdot 10^9 \cdot 0,2 = 2,360,000 \text{ kNm/m}$$

Eccentricity

$$D_{1.6} D_{6.1} = D_{6.1} \cdot e_{\phi 1} = 1055,590 \cdot 10^9 \cdot 0,0393 = 41499,2 = \text{kNm/m}$$

$$D_{2.7} = D_{7.2} = D_{7.7} \cdot e_{\phi 2} = 0$$

$$D_{1.7} = D_{7.1} = \frac{\nu}{2} \cdot (e_{\phi 1} + e_{\phi 2}) \cdot \sqrt{D_{6.6} \cdot D_{7.7}} = \frac{0,0328}{2} \cdot (0,0393 + 0) \cdot \sqrt{1055,590 \cdot 10^6 \cdot 2505,84 \cdot 10^6} = 987,0 \text{ kNm}$$

$$D_{3.8} = D_{8.3} = \frac{1}{2} \cdot G \cdot h \cdot (e_{\phi 1} + e_{\phi 2}) = \frac{1}{2} \cdot 11,8 \cdot 10^9 \cdot 0,2 \cdot (0,0393 + 0) = 46390,2 \text{ kNm}$$

⊞ Cross-Sectional Properties (Uncracked and Cracked State)				
⊞ Shrinkage Influence				
⊞ Damage Parameter Calculation				
⊞ Final Cross-Sectional Properties				
⊞ Material Stiffness Matrix				
⊞ Bending Stiffness				
into Reinforcement Direction 1	D ₁₁	3656.74	kNm	
into Reinforcement Direction 2	D ₂₂	7344.18	kNm	
transversal influence	D ₁₂	170.58	kNm	
torsion	D ₃₃	2505.84	kNm	
⊞ Shear Stiffness				
into Reinforcement Direction 1	D ₄₄	1966670.0	kN/m	
into Reinforcement Direction 2	D ₅₅	1966670.0	kN/m	
⊞ Membrane Stiffness				
into Reinforcement Direction 1	D ₆₆	1055590.0	kN/m	
into Reinforcement Direction 2	D ₇₇	2204240.0	kN/m	
transversal influence	D ₆₇	50210.6	kN/m	
torsion	D ₈₈	2360000.0	kN/m	
⊞ Eccentricity				
into Reinforcement Direction 1	D ₁₆	41499.2	kNm/m	
into Reinforcement Direction 2	D ₂₇	0.0	kNm/m	
transversal influence	D ₁₇	987.0	kNm/m	
torsion	D ₃₈	46390.2	kNm/m	

Figure 2.125: Stiffness matrix of the material

2.8 Nonlinear Method

2.8.1 General

The serviceability limit state (SLS) design is generally divided the three following groups:

- Stress limitation (EN 1992-1-1, clause 7.2)
- Crack control (EN 1992-1-1, clause 7.3)
- Deflection control (EN 1992-1-1, clause 7.4)

The design of concrete structures is usually based on linear structural analyses: To determine the reinforcement, including the serviceability limit state design, the internal forces are determined linearly; the cross-section analysis is performed subsequently. However, this procedure takes into account the cracking typical for reinforced concrete with the corresponding nonlinear material rules of reinforced concrete only on the cross-section level.

By including the nonlinear behavior of reinforced concrete in the determination of internal forces, we obtain realistic states of stress and, therefore, distributions of internal forces that in statically indeterminate systems differ significantly – due to stiffness redistributions – from the linearly determined internal forces. For the serviceability limit state design, this means that we must take into account the nonlinear material behavior of reinforced concrete to obtain a realistic calculation of deformations, stresses, and crack widths.

If the cracking is not taken into account in the deformation calculation, the occurring deformations will be underestimated. By considering creeping and shrinkage, the deformation may be 3 to 8 times larger, depending on the stress and boundary conditions. The add-on module **RF-CONCRETE NL** allows for the realistic calculation of the deformations, crack widths, and stresses of reinforced concrete surfaces by considering the nonlinear material behavior in the determination of internal forces.

2.8.2 Equations and Methods of Approximations

2.8.2.1 Theoretical Approaches

"Nonlinear calculation" refers to the determination of internal forces and deformations taking into account the nonlinear behavior of internal forces and deformations (physical).

Surface models can be described as two-dimensional structures with the following functions of state: surface loads, deformations, internal forces, and strains in the centroids of the surface areas. However, material properties that vary over the depth of the area must be taken into account for the nonlinear reinforced concrete model. Hence, it becomes necessary to extend the 2D model by additionally taking into account the depth of the cross-section. The cross-section of the reinforced concrete is subdivided into a number of reinforcing steel and concrete layers (see Figure 2.126).

Based on the strains in the centroids of the areas, under the assumption of the Bernoulli hypothesis, we obtain the strains for each layer. After applying the corresponding reinforcing steel and concrete laws to the strains, we obtain the stresses. The resulting stresses per layer can be integrated into internal forces of the total cross-section.

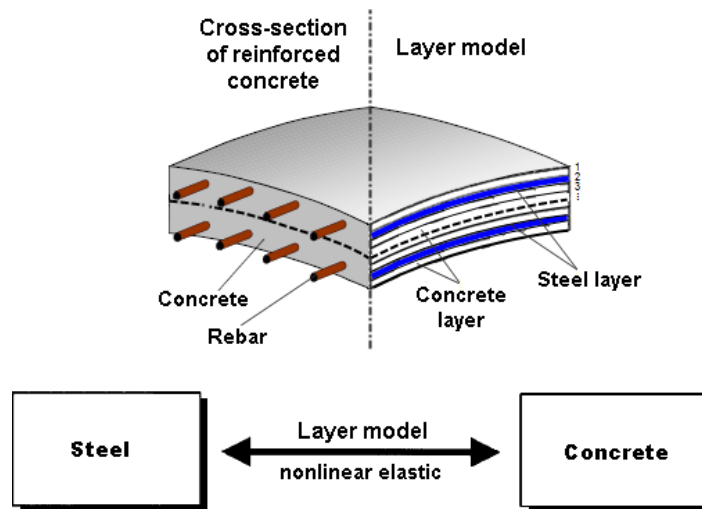


Figure 2.126: Layer model for reinforced concrete surfaces

If the tension strength of concrete is reached in a point of the structure, a discontinuity occurs in the form of a crack. If taken strictly, this would require an adjustment of the discretization (*remeshing*) to include every crack in the calculation in its actual position and extension. For several cracks, this method would result in a high numerical effort, because every crack would increase the number of elements. Therefore, occurring cracks are "smeared" within an element and the stiffness-reducing influences of the cracks are taken into account by adjusting the material rule in the calculation.

If the first principal stress in a concrete layer reaches the concrete tensile strength, a crack is formed perpendicular to the first principal direction of stress. This principal direction can change if the load changes. Here we can assume that a forming crack does not change its position and orientation (the so-called *fixed crack model*) or that the crack always runs orthogonally to the variable principal direction (*rotating crack model*). RF-CONCRETE NL uses the *rotating crack model*.

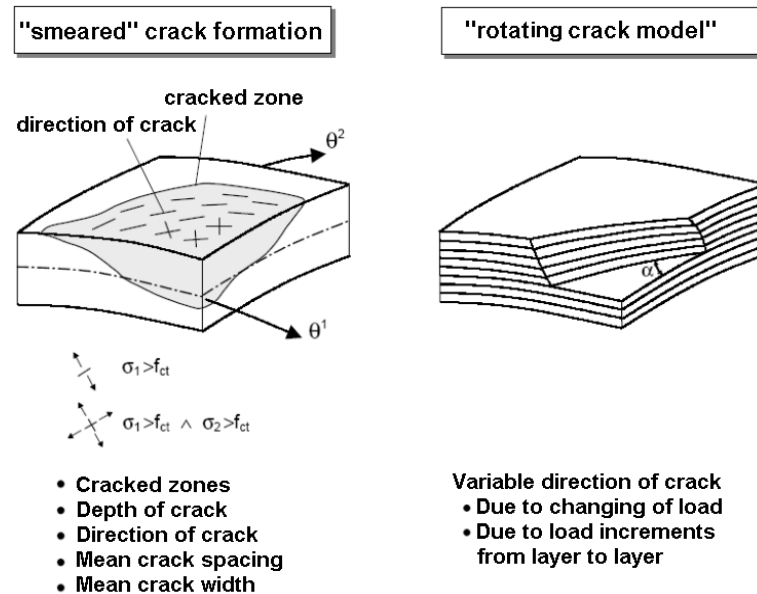


Figure 2.127: Crack models in reinforced concrete surface elements

2.8.2.2 Flowchart

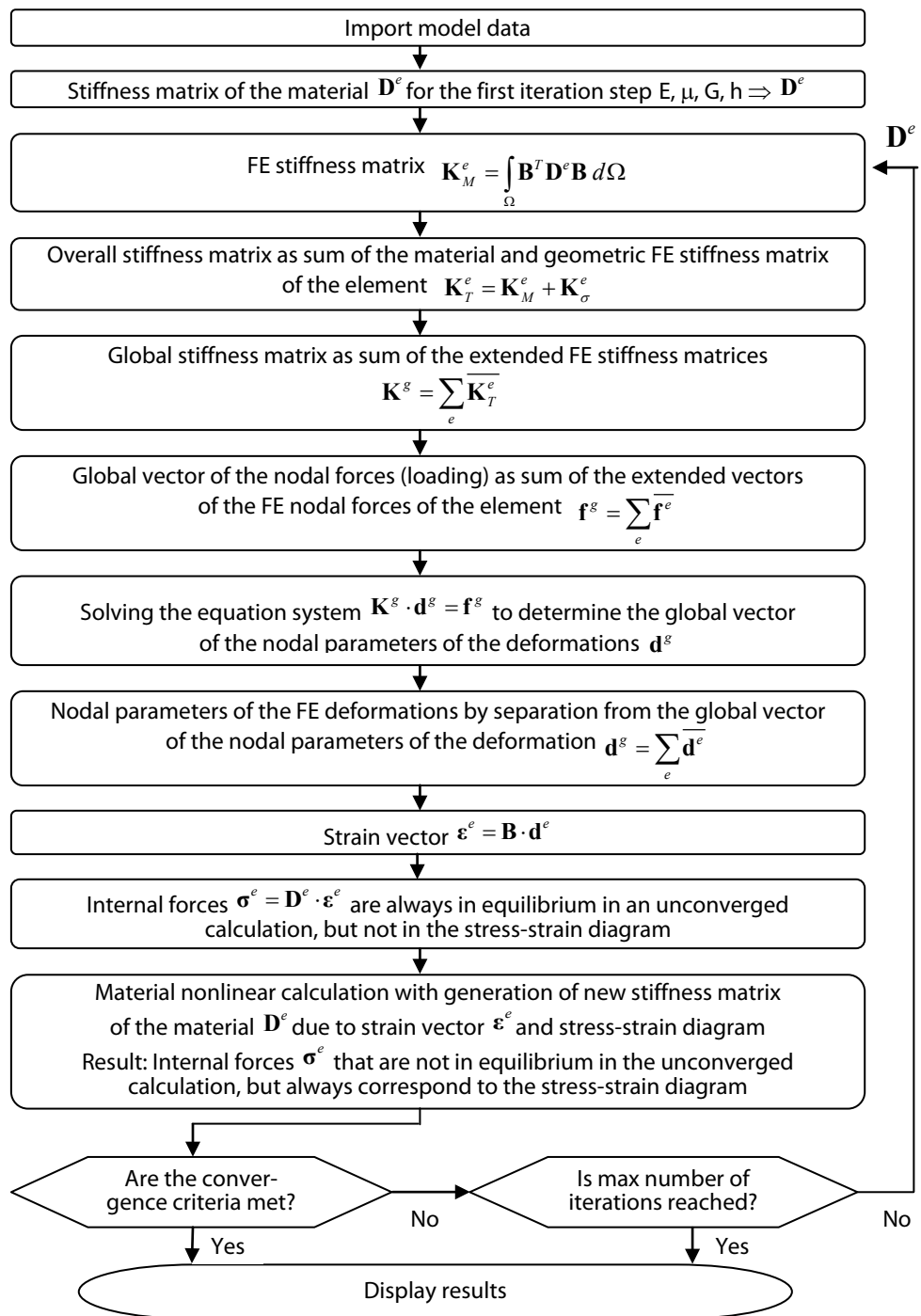


Figure 2.128: Flowchart

where

$\mathbf{D}^e$	Stiffness matrix of the material, constitutive matrix
$\mathbf{B}$	Matrix due to the geometry and the basic type of the FE function $\boldsymbol{\varepsilon}^e = \mathbf{B} \cdot \mathbf{d}^e$
$\mathbf{K}_M^e$	Material FE stiffness matrix
$\mathbf{K}_\sigma^e$	Geometric FE stiffness matrix
$\mathbf{K}_T^e$	FE overall stiffness matrix
$\mathbf{K}^g$	Global stiffness matrix of the entire model

f^e	FE nodal forces vector
f^g	Global nodal forces vector (loading on the entire model), vector of the right sides
d^e	Nodal parameter vector of the FE deformation
d^g	Global nodal parameter vector of the deformation, vector of the unknown
ε^e	Strain vector
σ^e	Vector of the internal forces

2.8.2.3 Method for Solving Nonlinear Equations

The application of the FE method to solve nonlinear differential equations results in algebraic equations that can be expressed in the following form:

$$K(d) \cdot d = f$$

Equation 2.87

where

K	Stiffness matrix of the model
d	Vector of the unknown (usually of the nodal parameters of the deformation)
f	Vector of the right sides (usually of the nodal forces)

The matrix K is the function of d , and can therefore not be evaluated without knowledge of the vector of the system root d . Since this nonlinear system cannot be solved directly, iteration methods are used that are aimed at progressively increasing the precision of the solution.

RF-CONCRETE NL uses the iteration method according to PICARD. This method is also known as the *Direct Iteration Method* or *Secant Modulus Method*.

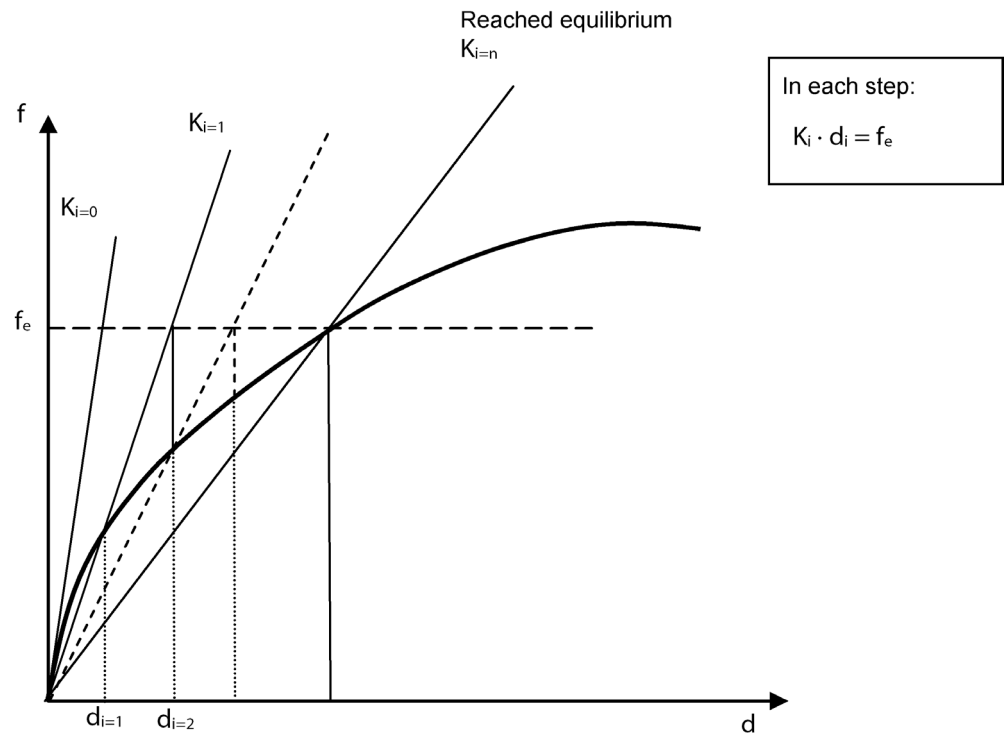


Figure 2.129: Direct iteration method

2.8.2.4 Convergence Criteria

In solving the nonlinear equations, two convergence criteria are considered. The iteration step is deemed to be completed when one convergence criterion is satisfied. The first convergence criterion observes how the diagonal components of the stiffness matrix of the material change. A convergence is reached when the stiffness matrix of the material has stabilized for all finite elements.

$$D_{\text{tot}}^{i-1} = \begin{bmatrix} D_j^{i-1} & & \\ & \dots & \\ & & D_n^{i-1} \end{bmatrix} \quad D_{\text{tot}}^i = \begin{bmatrix} D_j^i & & \\ & \dots & \\ & & D_n^i \end{bmatrix} \quad \frac{\sum_{j=1}^n |D_j^i - D_j^{i-1}|}{\sum_{j=1}^n D_j^{i-1}} \leq \varepsilon$$

Equation 2.88

where

- D_{tot}^{i-1} Stiffness matrix of the material from the previous iteration step.
- D_{tot}^i Stiffness matrix of the material in the current iteration step
- ε Desired precision (for RFEM precision 1: $\varepsilon = 0.05 \%$)

The second convergence criterion observes how the size of the maximum deformation changes. At the same time, the program controls whether the place of the maximum deformation within the structure has changed. Since the deformation usually converges faster than the stiffness matrix, the deformation criterion is activated only after 50 iteration steps (for RFEM precision 1).

$$\frac{d_{\text{max}}^i - d_{\text{max}}^{i-1}}{d_{\text{max}}^{i-1}} < \varepsilon \quad \text{and} \quad N_{\text{max}}^i = N_{\text{max}}^{i-1}$$

Equation 2.89

where

- d_{max}^{i-1} Maximum nodal displacement from the previous iteration step
- d_{max}^i Maximum nodal displacement of the structure in the current iteration step
- ε Desired precision (for RFEM precision 1, the following applies: $\varepsilon = 0.05 \%$)
- N_{max}^{i-1} Number of the node with maximum displacement from previous iteration step
- N_{max}^i Number of the node with maximum displacement from current iteration step



The precision of the convergence criterion for the nonlinear calculation and the point in time defining after which iteration step the deformation criterion is additionally considered are controlled in the *Global Calculation Parameters* tab of the *Calculation Parameters* dialog box in RFEM. You can also open this dialog box in the 1.1 *General Data* window of RF-CONCRETE Surfaces:

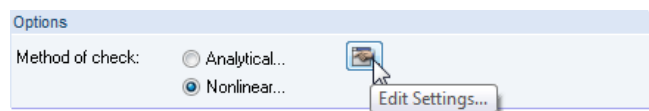


Figure 2.130: [Edit Settings] button in the *Serviceability Limit State* tab of the 1.1 *General Data* window

The *Settings for Nonlinear Calculation* appears (see the following figure).



Then we click [Details] to open the RFEM dialog box *Calculation Parameters*.

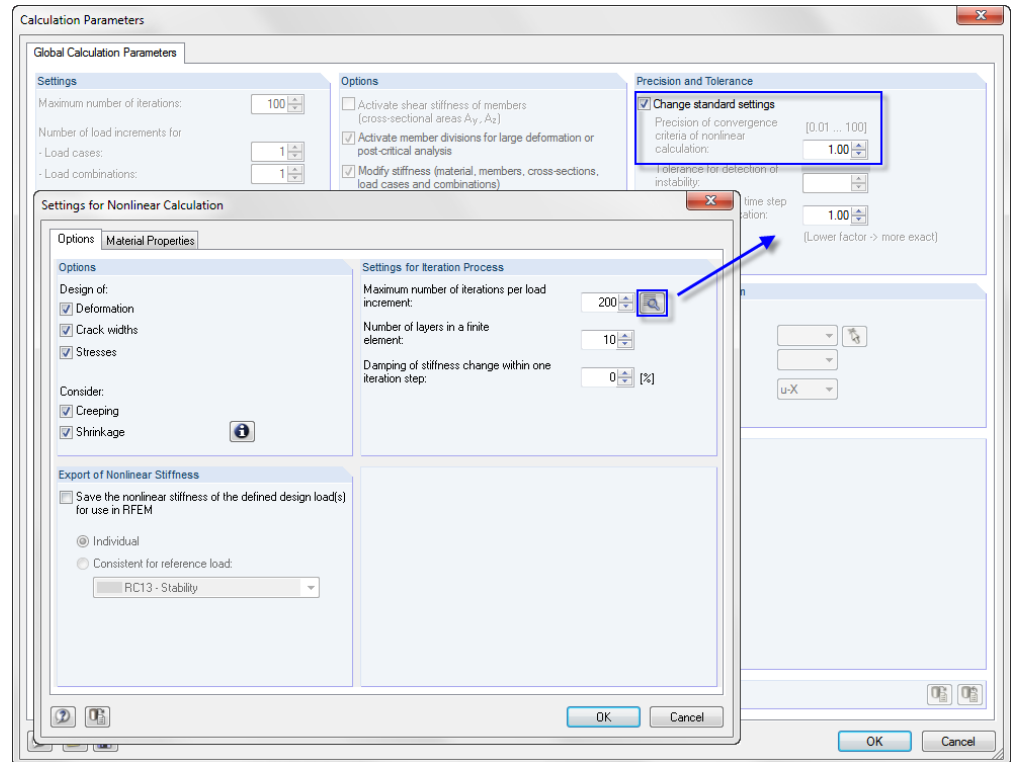


Figure 2.131: Dialog box *Settings for Nonlinear Calculation* with access to the convergence criterion of RFEM

The value for the *Precision of convergence criteria* ("RFEM precision") in the RFEM dialog box influences the break-off limit ε for the physically nonlinear calculation and the iteration step n_i from which on the deformation criterion is additionally considered:

$$\varepsilon = \text{"RFEM precision"} \cdot 0.05 \%$$

$$n_i = 1/\text{"RFEM precision"} \cdot 50$$

The default value of the RFEM precision is 1. Thus, the precision of the convergence criterion for the physical nonlinearity is $\varepsilon = 0.05 \%$ and the additional consideration of the deformation criterion starts after the 50th iteration step. For a higher precision, it is necessary to reduce the value of the RFEM precision. Thus, ε becomes smaller and the deformation criterion is considered later.

2.8.3 Material Properties

2.8.3.1 Concrete in Compression

In the serviceability limit state design, the mean strengths of the materials are used for the calculation. In the compressed area, it is possible to choose between a parabola and a parabola-rectangle distribution of the stress-strain relationship.

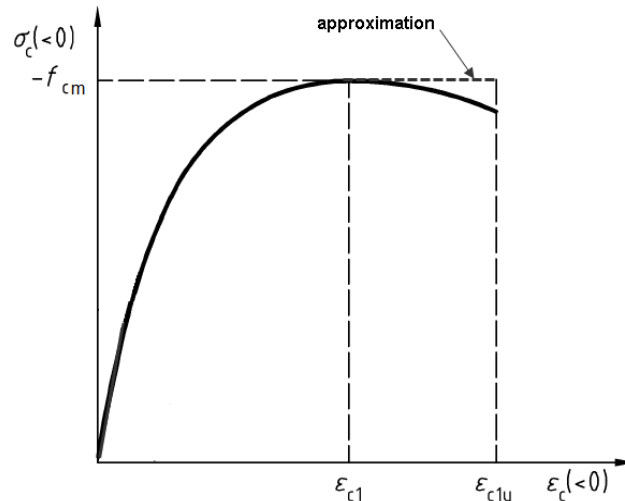


Figure 2.132: Stress-strain diagram for concrete in compression



The relevant settings are specified in the *Material Properties* tab of the *Settings for Nonlinear Calculation* dialog box (see Figure 3.11, page 135). You can open the dialog box in the *Serviceability Limit State* tab of the 1.1 *General Data* window by clicking the button shown on the left (see Figure 2.130).

2.8.3.2 Concrete in Tension

There are several options for the stress-strain relationship of the concrete in tension.

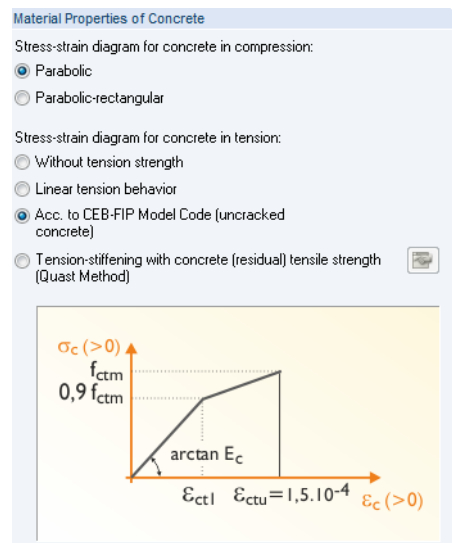


Figure 2.133: Stress-strain diagram for concrete in tension

The tensile strength of the concrete can, for example, be considered according to the specifications of the CEB-FIP Model Code 90:1993. Until the tensile strength of concrete f_{ctm} is reached, the program assumes the distribution shown in Figure 2.134 to reach the crack strain of 0.15 ‰.

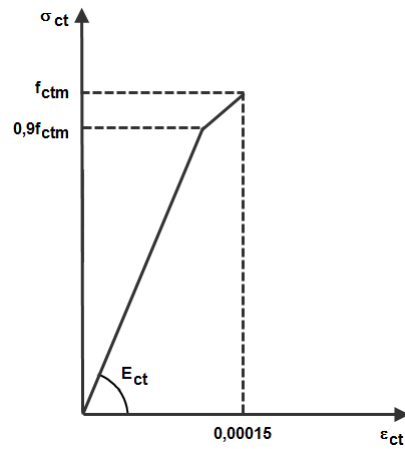


Figure 2.134: Stress-strain relationship for concrete in tension according to CEB-FIP Model Code 90:1993

Alternatively, you can use *tension stiffening*. This procedure is described in the following chapter.

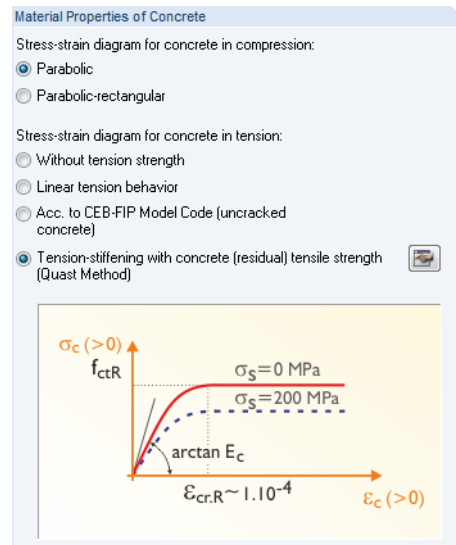


Figure 2.135: Stress-strain diagram for *tension stiffening*

2.8.3.3 Tension Stiffening: Stiffening Effect of Concrete in Tension

In cracked parts of the reinforced concrete, the tensile forces in the crack are resisted by the reinforcement alone. Between two cracks, however, tension forces are transferred to the concrete by the (displaceable) bond. Thus, the concrete is contributing to the resistance of internal tension forces. This leads to an increased stiffness of the component. The effect of *tension stiffening* thus refers to the stiffening contribution of concrete in tension between cracks.

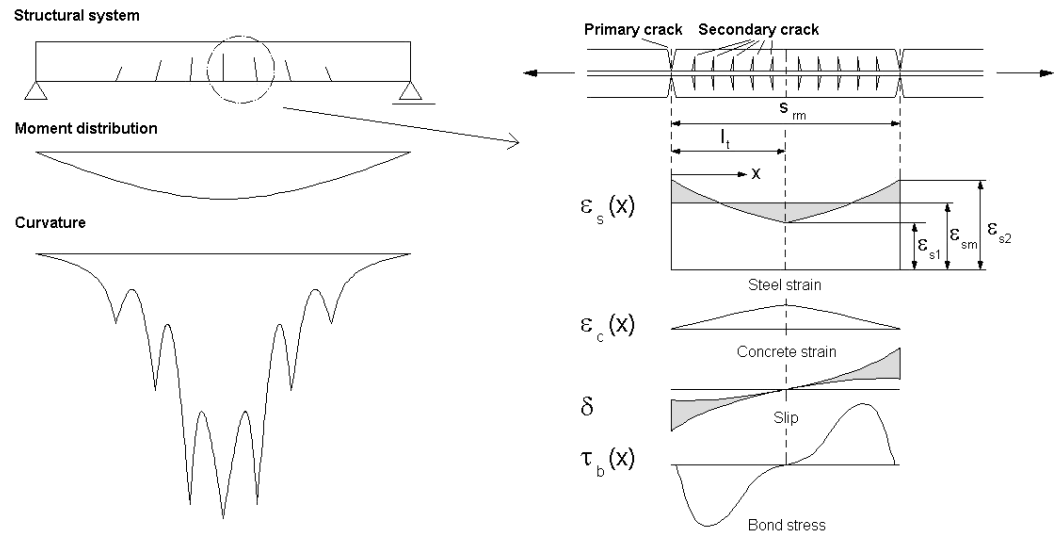


Figure 2.136: Stress and strain behavior between two primary cracks

The increase of the structural component stiffness due to tension stiffening can be considered in two ways:

- It is possible to include in the concrete's stress-strain diagram a constant residual tension stress which remains after the crack formation. The residual tension stress is much smaller than the tensile strength of the concrete. Alternatively, it is also possible to introduce modified stress-strain relations for the tension zone that take into account the contribution of concrete in tension between cracks in the form of a decreasing graph after the tensile strength is reached.
- Another approach is to modify the "pure" stress-strain diagram of reinforcing steel. In this case, a reduced steel strain ϵ_{sm} is applied in the relevant cross-section. This strain results from ϵ_{s2} and a reduction term due to the tension stiffening.

For considering *tension stiffening*, RF-CONCRETE NL uses the approach of modeling the concrete tensile strength according to QUASt [16]. This model is based on a defined stress-strain relationship of the concrete in tension (parabola-rectangle diagram). The basic assumptions of QUASt's approach can be summarized as follows:

- The full contribution of the concrete to tension until reaching the crack strain ϵ_{cr} or calculational concrete tensile strength $f_{ct,R}$
- Reduced stiffening contribution of concrete in tension according to the existing concrete strain
- No application of *tension stiffening* after the start of yielding of the governing rebar

To sum up, this means that the tensile strength $f_{ct,R}$ that is used for the calculation is **not** a fixed value but refers to the existing strain in the governing steel (tension) fiber. The maximum tensile strength $f_{ct,R}$ decreases linearly to zero, starting at the defined crack strain ϵ_{cr} until reaching the yield strain of the reinforcing steel in the governing steel fiber. This is achieved by means

of the stress-strain relationship of the concrete in tension (parabola-rectangle diagram) (see Figure 2.137) and the tension stiffening parameter (reduction factor) VMB.

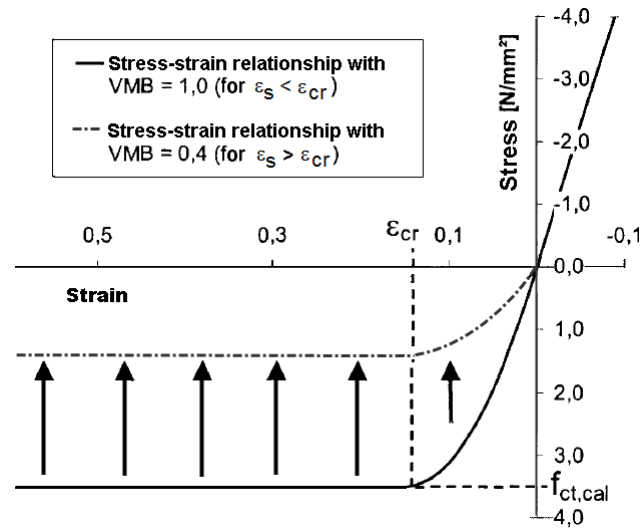


Figure 2.137: Stress-strain relationship of concrete in tension with parameter VMB = 0.4

The stress-strain relationship can be described with the following equations.

$$\sigma_c = \text{VMB} \cdot \left[f_{ct,R} \cdot \left(1 - \left(1 - \frac{\varepsilon}{\varepsilon_{cr}} \right)^{n_{PR}} \right) \right] \quad \text{for } 0 < \varepsilon \leq \varepsilon_{cr}$$

$$\sigma_c = \text{VMB} \cdot f_{ct,R} \quad \text{for } \varepsilon > \varepsilon_{cr} \text{ (constant distribution)}$$

Equation 2.90

The curvature of the parabola in the first section can be controlled by the exponent n_{PR} . The exponent is to be adjusted in such a way that the transition from compression to tension zone is achieved as much as possible with the same modulus of elasticity.

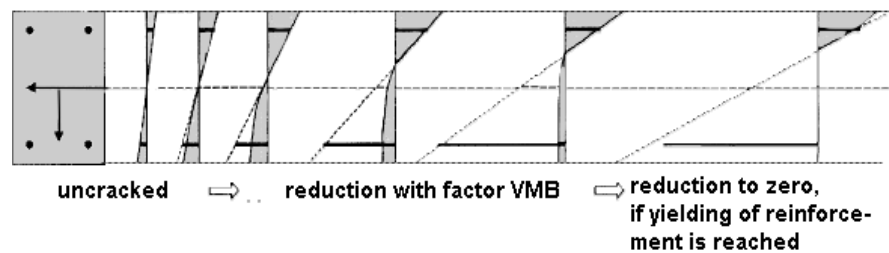


Figure 2.138: Stress conditions for increasing effect of tension stiffening

To determine the reduction parameter VMB, the strain is used at the most tensioned steel fiber. The position of the reference point is shown in Figure 2.139.

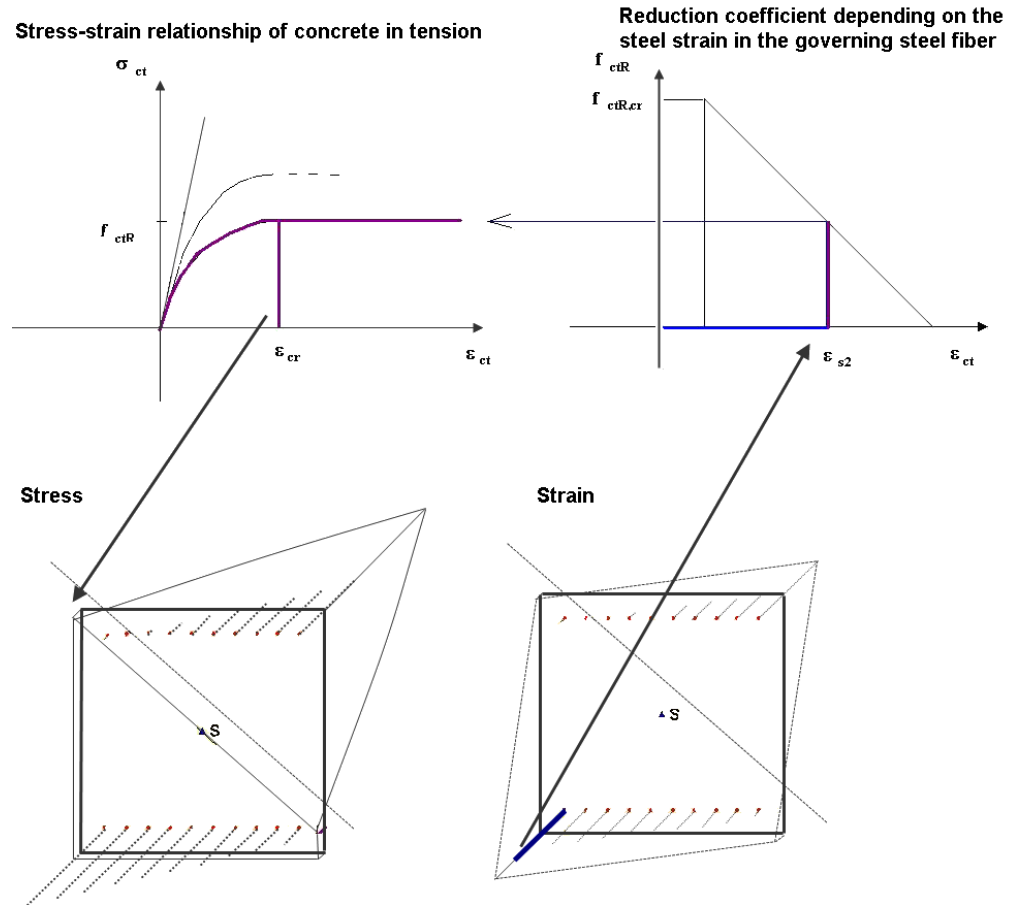


Figure 2.139: Determination of residual tensile strength for *Tension Stiffening* according to QUASt [16]

The reduction parameter VMB decreases with increasing steel strain. In the diagram for parameter VMB (see Figure 2.140), we can see that the parameter VMB is reduced to zero exactly at the point when the yielding of the reinforcement starts.

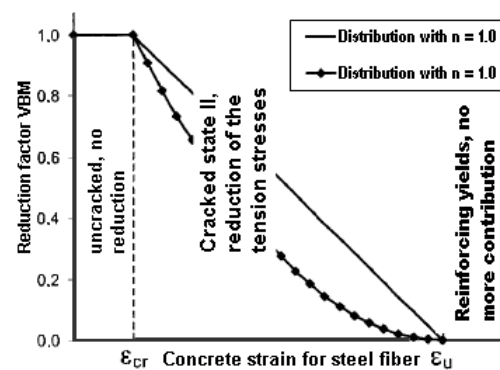


Figure 2.140: Reduction parameter VMB

The distribution for the reduction parameter VMB in state II ($\varepsilon > \varepsilon_{cr}$) can be controlled by means of the exponent n_{VMB} . The experiential values according to PFEIFFER [11] are the values $n_{VMB} = 1$ (linear) to $n_{VMB} = 2$ (parabola) for bending elements. QUASt [17] uses the exponent $n_{VMB} = 1$ (linear), thus achieving a good match for the check calculation of column tests. According to PFEIFFER [11], it is possible to represent pure tension tests showing acceptable matches by using $n_{VMB} = 2$.

The assumption of a parabola-rectangle diagram for a cracked tension zone of concrete can be regarded as a computational aid. At first glance, there are great differences compared to the experimentally determined stress-strain diagrams on the tension face of the pure concrete.

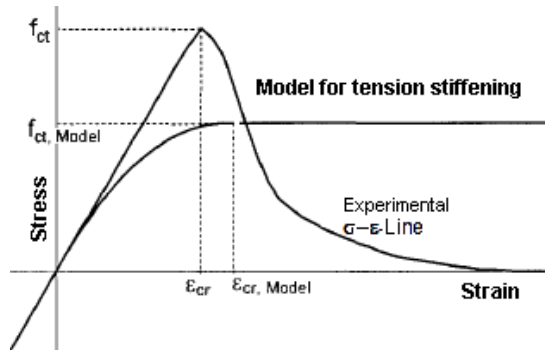


Figure 2.141: Comparison of model and laboratory test

The given stresses in the reinforced concrete cross-section in bending show that the parabola-rectangle diagram is indeed better suited to describe the mean of the strains and stresses.

In a bending beam, a concrete body forms between the cracks. It acts as a kind of wall, into which tension forces are reintroduced gradually by the reinforcement. This results in a very irregular distribution of stress and strain. For the mean, however, we can create a plane of strain with a parabola-rectangle distribution with which it is possible to consider the mean curvature.

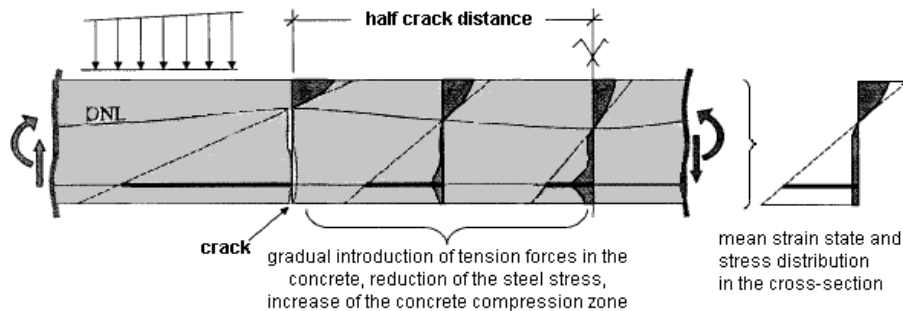


Figure 2.142: Existing state of stress when subjected to bending

QUAST suggests the following calculation value for the tension strength $f_{ct,R}$ and the crack strain $\epsilon_{cr,R}$ for his model.

$$f_{ct,R} = \left| \frac{1}{20} \cdot f_{cm} \right| \quad \epsilon_{cr,R} = \left| \frac{1}{20} \cdot \epsilon_{c1} \right|$$

Equation 2.91

The calculational value for the tension strength $f_{ct,R}$ is thus smaller than specified by the Eurocodes. This is due to the description of the stress-strain relation and the determination of the reduction parameter VMB, where the assumed tension stress and the resulting tension force after the exceeding of the tension strain are reduced only slowly. For a strain of $2 \cdot \epsilon_{cr}$, there is also an acting tension stress of about $0.95 \cdot f_{ct,R}$. Thus, in case of bending, the reduction of the stiffness can be well predicted. In case of pure tension, the values for $f_{ct,R}$ are too low. According to PFEIFFER [11], the values from EC2 should be taken for the calculation value of the tension strength.

The values recommended by QUASt [17] for $f_{ct,R} = 1/20 \cdot f_{cm}$ can be obtained by taking 60 % of the tensile strengths given in EC2. On the one hand, the cracking of the cross-section is predicted too early when applying $f_{ct,R} = 0.6 \cdot f_{ctm}$. On the other hand, this already takes into account a reduction of the tensile strength under permanent load (ca. 70 %) or a temporarily higher load (for example, the short-term application of the rare combination of action) that results in a damaged tension zone.

The individual calculation values for the tensile zone of the concrete can be described as follows:

$f_{ct,R} = 0,60 \cdot f_{ct,standard}$	Calculational tensile strength
$v = \left \frac{f_{cm}}{f_{ct,standard}} \right $	Ratio, auxiliary value
$\varepsilon_{cr,R} = \left \frac{\varepsilon_{c1}}{v} \right $	Calculational crack strain
$n_{PR} = 1.1 \cdot E_{cm} \cdot \frac{\varepsilon_{c1}}{f_{cm}}$	Exponent for general parabola (for Equation 2.90)

Equation 2.92

2.8.3.4 Reinforcing Steel

For the serviceability limit state design, the mean strengths of the materials are used in the calculation. The mean reinforcing steel strengths were published by the JCSS in *Probabilistic Model Code*. This code specifies the mean value of the yield strength with $f_{ym} = 1.1 \cdot f_{yk}$.

RF-CONCRETE NL uses a bilinear distribution for the stress-strain relationship of reinforcing steel.

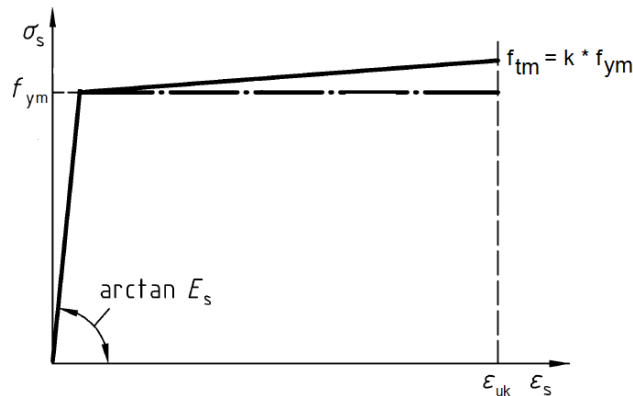


Figure 2.143: Stress-strain relationship of reinforcing steel

You can choose if the graph of plasticity is horizontal or increases up to f_{tm} . The settings are specified in the *Settings for Nonlinear Calculation* dialog box (see Figure 3.11, page 135). To open this dialog box, click the button shown on the left that can be found in the *Serviceability Limit State* tab of the 1.1 *General Data* window.

2.8.4 Creep and Shrinkage

2.8.4.1 Consideration of Creep

Creep describes the time-dependent deformation of concrete under loading within a particular period of time. The essential influence values are similar to those of shrinkage (see chapter 2.8.4.2). Additionally, the so-called "creep-producing stress" has a considerable effect on the creep deformation.

Special attention must be paid to the load duration, the point of time of load application, and the extent of the action-effects. Creep is taken into account by the creep coefficient $\varphi(t, t_0)$ at the time t .

In RF-CONCRETE Surfaces, the specifications for the determination of the creep coefficient are set in the 1.3 *Surfaces* window. In this tab, you specify the concrete age at the considered point of time and at the beginning of loading, the relative air humidity, as well as the type of cement. Based on these conditions, the program determines the creep coefficient φ .

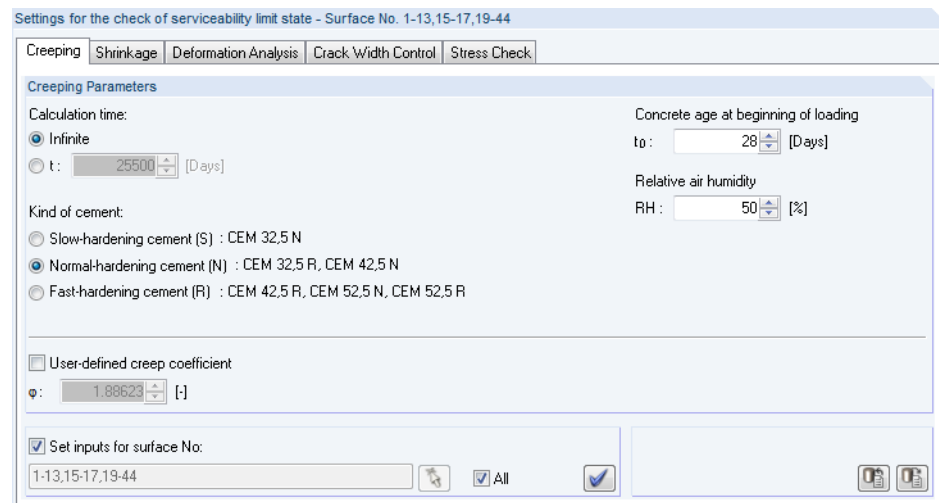


Figure 2.144: Window 1.3 *Surfaces*, tab *Creeping*

We now will briefly look at the determination of the creep coefficient φ according to EN 1992-1-1, clause 3.1.4. Using the following formulas requires that the creep-producing stress σ_c of the acting permanent load does not exceed the following value.

$$\sigma_c \leq 0.45 \cdot f_{ckj}$$

Equation 2.93

where

f_{ckj} Compressive cylinder strength of concrete at point of time when creep-producing stress is applied

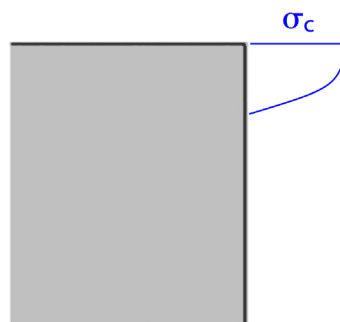


Figure 2.145: Creep-producing stress σ_c

Under the assumption of a linear creep behavior ($\sigma_c < 0.45 \cdot f_{ck}$), the creep of the concrete can be determined by reducing the concrete's modulus of elasticity.

$$E_{c,eff} = \frac{E_{cm}}{1.0 + \varphi(t, t_0)}$$

Equation 2.94

where

E_{cm}	Mean modulus of elasticity according to EN 1992-1-1, Table 3.1
$\varphi(t, t_0)$	Creep coefficient
t	Age of concrete in days at relevant point of time
t_0	Age of concrete in days when load application starts

According to EN 1992-1-1, clause 3.1.4, the creep coefficient $\varphi(t, t_0)$ at the analyzed point of time t can be calculated as follows.

$$\varphi(t, t_0) = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \cdot \beta_c(t, t_0)$$

Equation 2.95

where

$$\varphi_{RH} = \left[1 + \frac{1 - \frac{RH}{100}}{0.10 \cdot \sqrt[3]{h_0}} \cdot \alpha_1 \right] \cdot \alpha_2$$

RH Relative air humidity [%]

$$h_0 = \frac{2 \cdot A_c}{u}$$

Notional size of the member [mm] (for surfaces: $h_0 = h$)

A_c Cross-sectional area

u Perimeter of the member

$$\alpha_1 = \left(\frac{35}{f_{cm}} \right)^{0.7}$$

Adjustment factor

f_{cm} Mean compressive strength of concrete

$$\alpha_2 = \left(\frac{35}{f_{cm}} \right)^{0.2}$$

Adjustment factor

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$$

Factor to consider effect of concrete compressive strength

$$\beta(t_0) = \frac{1}{0.1 + t_{0,eff}^{0.2}}$$

Factor to allow for the effect of concrete age

$$t_{0,eff} = t_0 \left[1 + \frac{9}{2 + t_0^{1.2}} \right]^\alpha \geq 0.5 \cdot d$$

$$\beta_c(t, t_0) = \left[\frac{t - t_0}{\beta_H + t - t_0} \right]^{0.3}$$

Coefficient for taking into account the loading duration

t

Age of concrete in days at the moment considered

t_0

Age of concrete at loading start in days

$$\beta_H = 1.5 \cdot \left[1 + (0.012 \cdot RH)^{18} \right] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3$$

RH

Relative air humidity [%]

h_0

Notional size of the structural component [mm]

$$\alpha_3 = \left(\frac{35}{f_{cm}} \right)^{0.5}$$

Adjustment factor

The influence of the selected type of cement on the concrete's creep coefficient can be taken into account by modifying the concrete's load application age t_0 with the following equation:

$$t_0 = t_{0,T} \cdot \left(1 + \frac{9}{2 + (t_{0,T})^{1.2}} \right)^{\alpha} \geq 0.5$$

Equation 2.96

where

- | | |
|-------------|---|
| $t_0 = t_T$ | Effective age of concrete at start of load application considering the influence of temperature |
| α | Exponent which depends on type of cement: |
| -1 | slow-hardening cements (S) (32.5) |
| 0 | normal- or rapid-hardening cements (N) (for example 32.5 R; 42.5) |
| 1 | rapid-hardening, high-strength cements (R) (42.5 R; 52.5) |

Consideration of Creep by Calculation

If the strains are known at the moment $t = 0$ as well as at a later moment t , it is possible to determine the creep coefficient φ for calculational consideration in the model.

$$\varphi_t = \frac{\varepsilon_t}{\varepsilon_{t=0}} - 1$$

Equation 2.97

The equation is converted to the strain at the moment t . Thus, we obtain the following relation, which is valid for uniform stresses (compare Equation 2.93):

$$\varepsilon_t = \varepsilon_{t=0} \cdot (\varphi_t + 1)$$

Equation 2.98

For stresses higher than approx. $0.4 f_{ck}$, the strains are rising disproportionately, resulting in loss of the linearly assumed reference.

The calculation in RF-CONCRETE NL uses a common solution that is reasonable for construction purposes. The stress-strain diagram of concrete is distorted by the factor $(1 + \varphi)$.

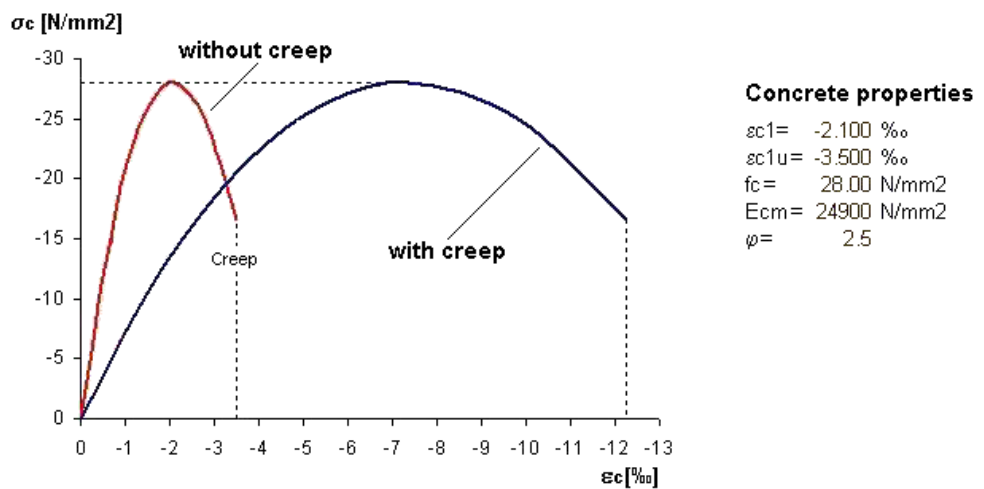


Figure 2.146: Distortion of stress-strain relationship for determination of creep effect

When taking into account creep, uniform creep-producing stresses are assumed during the period of load application (see Figure 2.146). Due to the neglect of stress redistributions, the deformation is slightly overestimated by this assumption. In this model, the stress reduction

without a change in strain (relaxation) is only partly taken into account. If we assume a linear elastic behavior, it would be possible to assume a proportionality and the horizontal distortion would reflect the relaxation at a ratio of $(1 + \varphi)$. This correlation, however, is lost for the nonlinear stress-strain relationship.

Thus, it becomes clear that this procedure is to be understood as an approximation. A reduction of stresses due to relaxation as well as nonlinear creep cannot be or can only be approximately represented.

2.8.4.2 Consideration of Shrinkage

Shrinkage describes a time-dependent change of volume without the action of external loads or temperature. The present documentation does not describe in detail the shrinkage problems and their individual types (drying shrinkage, autogenous shrinkage, plastic shrinkage, and carbonation shrinkage).

Significant influence values of shrinkage are relative humidity, effective thickness of structural components, aggregate, concrete strength, water-cement ratio, temperature, as well as the type and duration of curing. The shrinkage-determining value is represented by the total shrinkage strain $\varepsilon_{cs}(t, t_s)$ at the considered moment t .

According to EN 1992-1-1, clause 3.1.4, the total shrinkage strain ε_{cs} can be calculated from the deformation components drying shrinkage ε_{cd} and autogenous shrinkage ε_{ca} :

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} \quad [4] \text{ Eq. (3.8)}$$

Equation 2.99

The component from **drying shrinkage** ε_{cd} is determined as follows.

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \quad [4] \text{ Eq. (3.9)}$$

Equation 2.100

where

$$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04 \cdot \sqrt{h_0^3}} \quad [4] \text{ Eq. (3.10)}$$

t Age of concrete at considered time in days

t_s Age of concrete in days at the beginning of shrinkage

$h_0 = 2 \cdot A_c / u$ Notional size of the structural component [mm] (for surfaces: $h_0 = h$)

A_c Cross-sectional area

u Perimeter of cross-section

k_h Coefficient according to [4] Table 3.3 depending on the effective cross-section thickness h_0

$\varepsilon_{cd,0}$ Basic value according to [4] Table 3.2 or Annex B, Eq. (B.11):

$$\varepsilon_{cd,0} = 0.85 \cdot \left[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}}\right) \right] \cdot 10^{-6} \cdot \beta_{RH}$$

$\alpha_{ds1}, \alpha_{ds2}$ Factors to consider the type of cement (see Table 2.3)

f_{cm} Mean compressive cylinder strength of concrete in [N/mm²]

$f_{cmo} = 10 \text{ N/mm}^2$

$$\beta_{RH} = 1.55 \cdot \left[1 - \left(\frac{RH}{RH_0} \right)^3 \right]$$

RH Relative air humidity of environment in [%]

$RH_0 = 100 \%$

Cement	Class	Characteristic	α_{ds1}	α_{ds2}
32.5 N	S	Slow-hardening	3	0.13
32.5 R; 42.5 R	N	Normal- or rapid-hardening	4	0.12
42.5 R; 52.5 N/R	R	Rapid-hardening	6	0.11

Table 2.3: Coefficients α_{ds} , α_{ds1} , α_{ds2} depending on the type of cement

The **autogenous shrinkage strain** ε_{ca} is determined as follows.

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) \quad [4] \text{ Eq. (3.11)}$$

Equation 2.101

where

$$\beta_{as}(t) = 1 - e^{-0.2\sqrt{t}} \quad [4] \text{ Eq. (3.12)}$$

$$\varepsilon_{ca}(\infty) = 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6} \quad [4] \text{ Eq. (3.13)}$$

t in days

Consideration of shrinkage in RF-CONCRETE NL (with consideration of the reinforcement)

The data for the determination of shrinkage strain is entered in the 1.3 *Surfaces* window. There, you specify the age of concrete at the considered moment and at the beginning of shrinkage, the relative air humidity, and the type of cement. Based on these specifications, RF-CONCRETE NL determines the shrinkage ε .

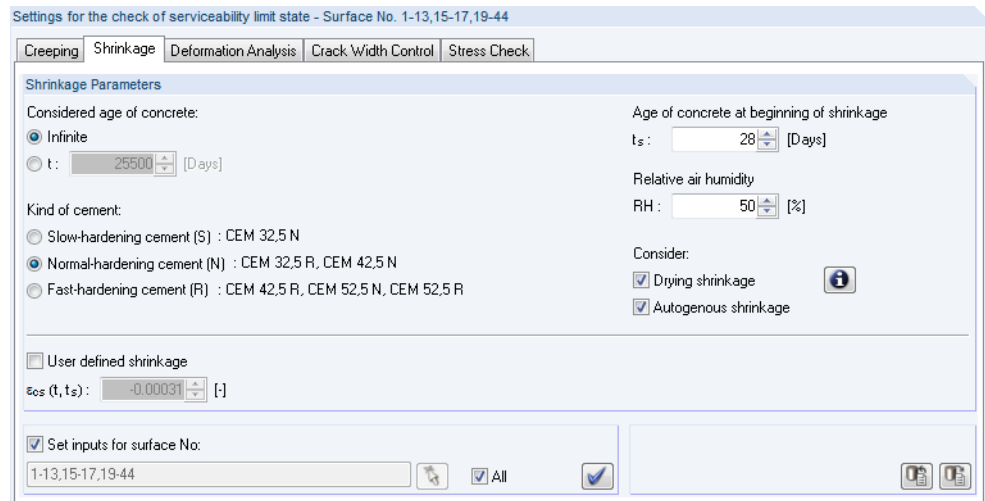


Figure 2.147: Window 1.3 *Surfaces*, tab *Shrinkage*

The shrinkage strain $\varepsilon_{cs}(t, t_s)$ can also be specified manually and independent of standards.

The shrinkage strain is only applied to the concrete layers; the reinforcement layers are not considered. Thus, there is a difference to the classical temperature loading, which also affects the reinforcement layers. The model for shrinkage used in RF-CONCRETE NL thus considers the restraint of the shrinkage strain ε_{sh} that is exerted by the reinforcement or the cross-section curvature of an unsymmetrical reinforcement. The resulting loads from the shrinkage strain are automatically applied as virtual loads to the surfaces, and calculated. Depending on the structural system, the shrinkage strain results in additional stresses (statically indeterminate system) or deformations (statically determinate system). For shrinkage, RF-CONCRETE NL takes into account the influence of the structural boundary conditions in different ways.

The loads resulting from shrinkage are automatically assigned to the loading defined for serviceability in the 1.1 *General Data* window and are therefore included in the nonlinear calculation.

The shrinkage depends on the correct distribution of the stiffness in the cross-section. For the concrete in tension, it is therefore recommended to consider *tension stiffening* (concrete tensile strength according to QUASt) and a small value for dampening.

The following 1D model illustrates how the shrinkage is considered in the program.

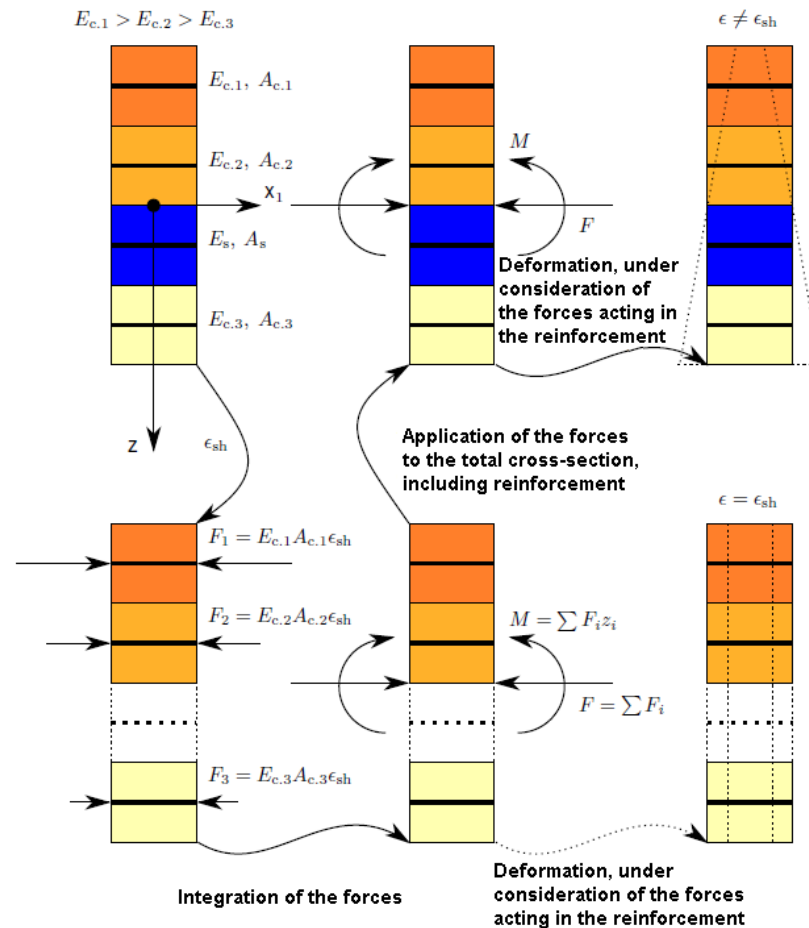


Figure 2.148: 1D example for shrinkage

As a simplification, four layers are considered: The dark orange layers represent the little damaged concrete, the light orange layers the more damaged concrete. The blue layer represents the reinforcement. Each concrete layer is characterized by the actual modulus of elasticity $E_{c,i}$ and each cross-sectional area by $A_{c,i}$. The reinforcement is characterized by the actual modulus of elasticity E_s and the cross-sectional area A_s . Each layer is described by means of the coordinate z_i .

Consideration of Shrinkage as External Load

The shrinkage strain can also be applied in RFEM as external load. In the *New Surface Load* dialog box of RFEM, you can open the *Generate Surface Load Due to Shrinkage* dialog box by clicking the button shown on the left.

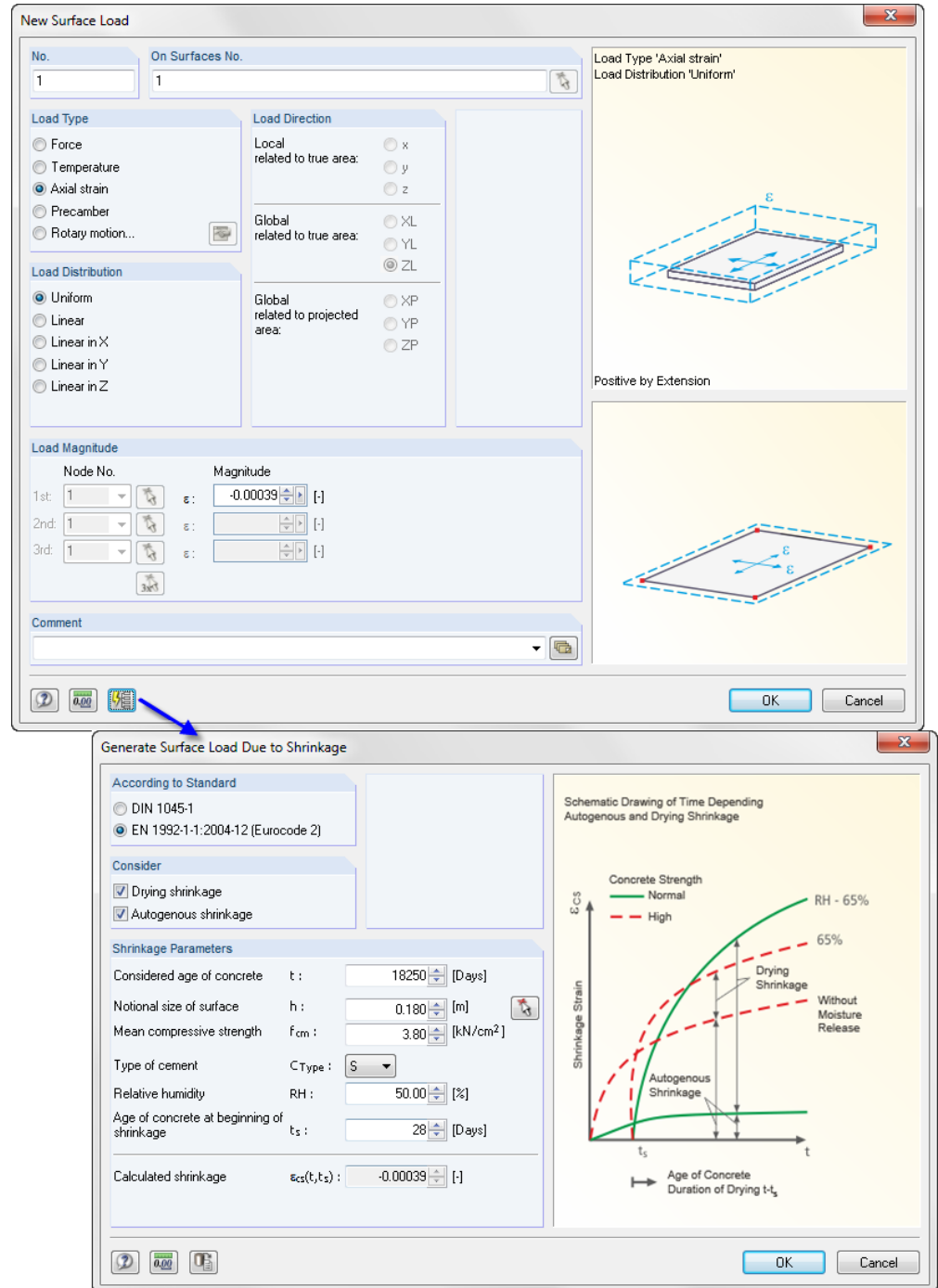


Figure 2.149: RFEM dialog box *Generate Surface Load Due to Shrinkage*

In this dialog box, you can enter the parameters to determine the shrinkage strain. By clicking [OK], you transfer the determined shrinkage as load magnitude to the initial dialog box *New Surface Load*. The load type is automatically set to *Axial strain*. Please note that the shrinkage strain acts on the entire cross-section, and that possible restraints or cross-section curvatures are not taken into account by the reinforcement.

3. Input Data

When you start the add-on module, a new window opens. In this module window, the navigator on the left lists all available windows. The drop-down list above the navigator contains the design cases (see chapter 8.1, page 190).

You define the design-relevant data in several input windows. When you open RF-CONCRETE Surfaces for the first time, the following parameters are imported automatically:

- Load cases, load combinations, and result combinations
- Materials
- Surfaces
- Internal forces (in background, if calculated)



To open a window, click the entry in the navigator. By using the buttons shown on the left, you can go the previous or next window. You can also use the function keys [F2] and [F3] to select the previous or next window.

To save the results, click [OK]. Thus you exit RF-CONCRETE Surfaces and return to the main program. By clicking [Cancel], you exit the add-on module without saving the data.

3.1 General Data

In the 1.1 *General Data* window, you specify the design standard and the actions. The tabs manage the load cases, load combinations, and result combinations for the ultimate and the serviceability limit state design.

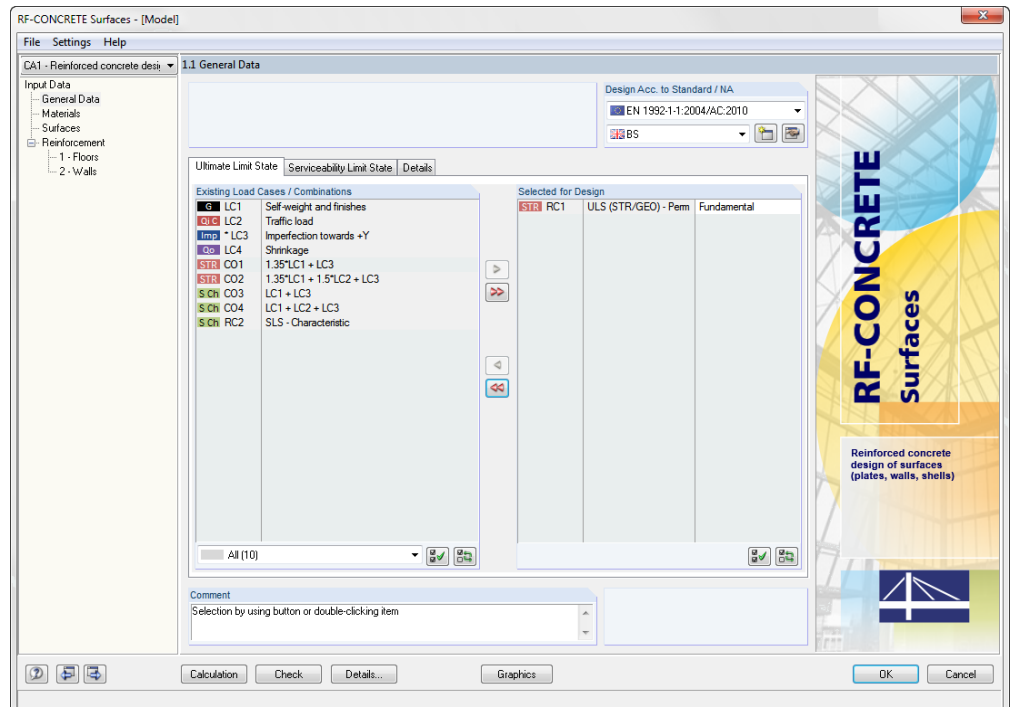


Figure 3.1: Window 1.1 *General Data*

Design Acc. to Standard / NA

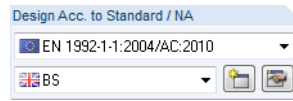


Figure 3.2: Standard and National Annex for reinforced concrete design

Standard

Specify the standard for the serviceability limit state design and the ultimate limit state design. The following standards for reinforced concrete can be selected from the list:

- EN 1992-1-1:2004/AC:2010 European Union
- DIN 1045-1:2008-08 Germany
- DIN 1045:1988-07 Germany
- ACI 318-11 United States of America
- SIA 262:2003 Switzerland
- GB 50010-2010 China

You can purchase each standard separately.

National Annex

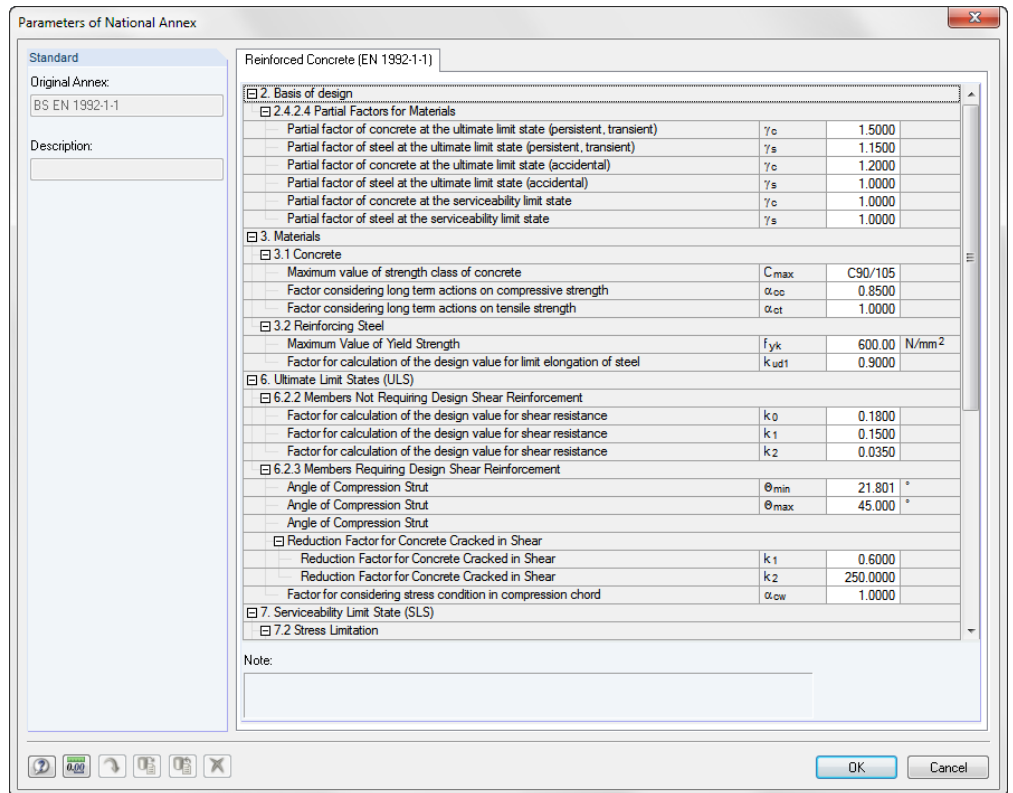
For the design according to Eurocode (EN 1992-1-1:2004/AC:2010), you have to specify the National Annex whose parameters apply for the checks.

CEN	EU
BS	United Kingdom
CSN	Czech Republic
DIN	Germany
DK	Denmark
NBN	Belgium
NEN	Netherlands
NF	France
NP	Portugal
PN	Poland
SFS	Finland
SIST	Slovenia
SS	Singapore
SS	Sweden
STN	Slovakia
UNE	Spain
UNI	Italy
ÖNORM	Austria

Figure 3.3: National Annexes for EN 1992-1-1



Click [Edit] to view the preset values.



Parameters of National Annex

Standard: BS EN 1992-1-1

Description:

Reinforced Concrete (EN 1992-1-1)

2. Basis of design

2.4.2.4 Partial Factors for Materials

Partial factor of concrete at the ultimate limit state (persistent, transient)	γ_c	1.5000
Partial factor of steel at the ultimate limit state (persistent, transient)	γ_s	1.1500
Partial factor of concrete at the ultimate limit state (accidental)	γ_c	1.2000
Partial factor of steel at the ultimate limit state (accidental)	γ_s	1.0000
Partial factor of concrete at the serviceability limit state	γ_c	1.0000
Partial factor of steel at the serviceability limit state	γ_s	1.0000

3. Materials

3.1 Concrete

Maximum value of strength class of concrete	C_{max}	C90/105
Factor considering long term actions on compressive strength	α_{cc}	0.8500
Factor considering long term actions on tensile strength	α_{ct}	1.0000

3.2 Reinforcing Steel

Maximum Value of Yield Strength	f_{yk}	600.00 N/mm ²
Factor for calculation of the design value for limit elongation of steel	k_{ud1}	0.9000

6. Ultimate Limit States (ULS)

6.2.2 Members Not Requiring Design Shear Reinforcement

Factor for calculation of the design value for shear resistance	k_0	0.1800
Factor for calculation of the design value for shear resistance	k_1	0.1500
Factor for calculation of the design value for shear resistance	k_2	0.0350

6.2.3 Members Requiring Design Shear Reinforcement

Angle of Compression Strut	θ_{min}	21.801 °
Angle of Compression Strut	θ_{max}	45.000 °

Reduction Factor for Concrete Cracked in Shear

Reduction Factor for Concrete Cracked in Shear	k_1	0.6000
Reduction Factor for Concrete Cracked in Shear	k_2	250.0000

Factor for considering stress condition in compression chord

Factor for considering stress condition in compression chord	α_{cw}	1.0000
--	---------------	--------

7. Serviceability Limit State (SLS)

7.2 Stress Limitation

Note:

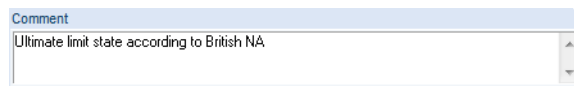
Figure 3.4: Dialog box *Parameters of National Annex*

In this dialog box, you find all design-relevant coefficients specified in the National Annexes. They are listed by the Eurocode's clause number.



If other specifications are necessary for the partial safety factors, reduction factors, the angle of the concrete strut, etc., you can adjust the parameters. To do this, click the [Create New Nation Annex (NA)] button in order to create a copy of the currently selected National Annex. In this user-defined annex, you can modify the parameters.

Comment



Comment

Ultimate limit state according to British NA

Figure 3.5: User-defined comment

In this input field, you can enter user-defined notes to describe, for example, the current design case.

3.1.1 Ultimate Limit State

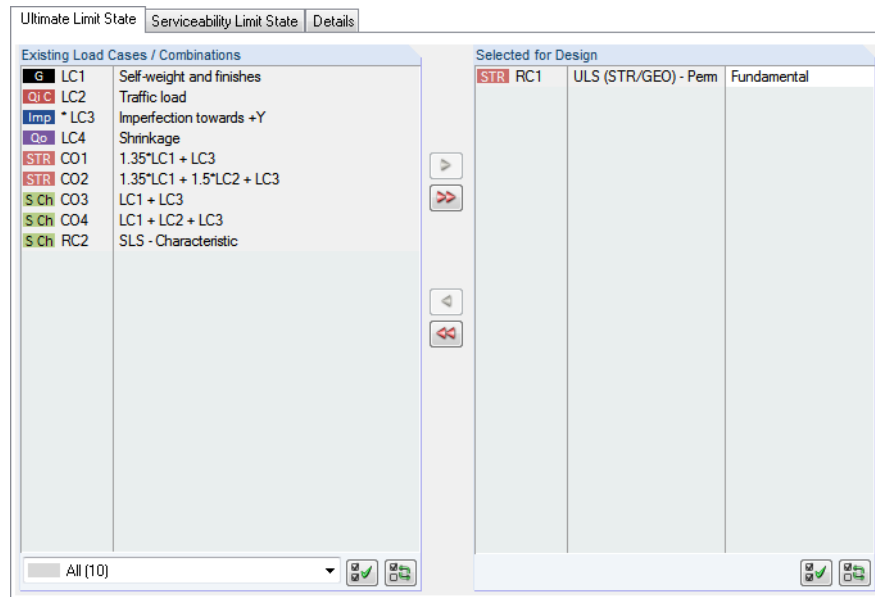


Figure 3.6: Window 1.1 General Data, tab Ultimate Limit State

Existing Load Cases / Combinations

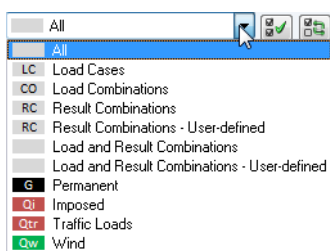
This column lists all load cases, load combinations, and result combinations created in RFEM.

Click [►] to transfer the selected entries to the list *Selected for Design* on the right. You can also double-click the items to select them for design. To transfer the complete list to the right, click [►►].

To select several load cases at the same time, click them one by one while pressing [Ctrl]. This allows you to transfer several load cases at once.

If a load case is marked by an asterisk (*), like LC3 in Figure 3.6, it is not possible to calculate it: It indicates a load case without load data or a load case that contains imperfections. If you transfer such a load case, a corresponding warning appears.

Several filter options are available below the list. These options make it easier to assign the entries sorted by load cases, combinations, or action categories. The buttons have the following functions:



	Selects all load cases in the list
	Inverts the selection of load cases

Table 3.1: Buttons in the tab Load Combinations

Selected for Design

The column on the right lists the loads cases, load combinations, and result combinations selected for the design. To remove selected items from the list, click [◀] or double-click the item. To transfer the entire list to the right, click [◀◀].

You can assign the load cases, load combinations, and result combinations to the following design situations:

- *Fundamental*
- *Accidental*

This classification influences the partial safety factors γ_c and γ_s according to EN 1992-1-1, Table 2.1 (see Figure 3.4, page 129 and Figure 3.37, page 157).

You can change the design situation by using the drop-down list. To open the list, click [▼].

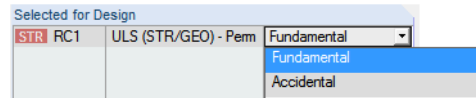


Figure 3.7: Assignment of design situation

In this dialog box section, you can also perform a multiple selection by clicking the items while pressing [Ctrl].

The analysis of an enveloping max/min result combination is faster than the analysis of all load cases and load combinations indiscriminately selected for design. In the analysis of a result combination, however, it is difficult to discern the influence of the included actions (see also chapter 4.1, page 160).

3.1.2 Serviceability Limit State

The serviceability limit state design depends on the results of the ultimate limit state design. Therefore, it is not possible to perform the serviceability limit state design alone.

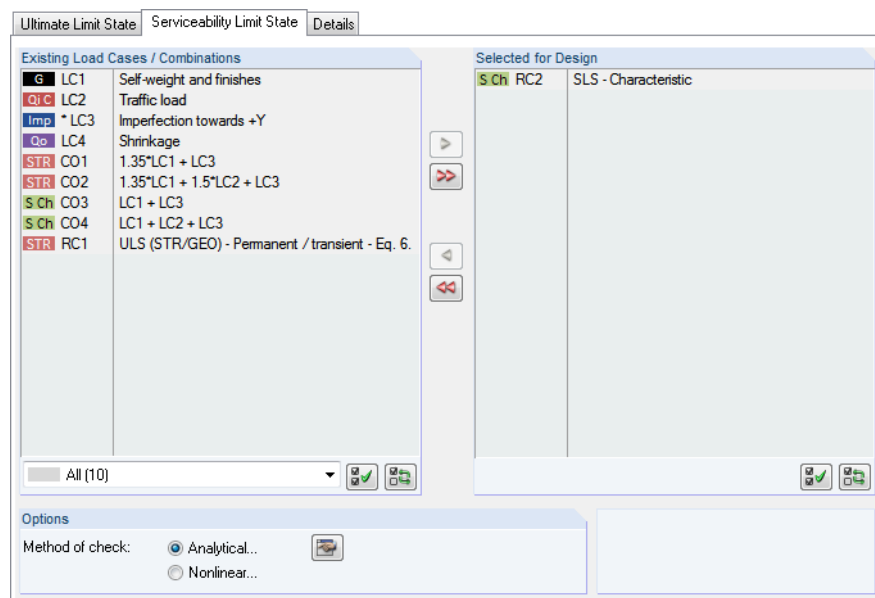


Figure 3.8: Window 1.1 General Data, tab Serviceability Limit State

Existing Load Cases / Combinations

These two sections list all load cases, load combinations, and result combinations defined in RFEM.

Usually, the actions and partial safety factors that are relevant for the serviceability limit state (SLS) design are different from the ones considered for the ultimate limit state. The corresponding combinations can be created in RFEM.

Selected for Design

Load cases, load combinations, and result combinations can be added or removed, as described in chapter 3.1.1.

Method of check

With the option buttons, you decide whether you want to carry out the check of the serviceability limit state designs according to the analytical or the nonlinear method.

Method of check: ☒ Analytical...
☐ Nonlinear...



3.1.2.1 Analytical Method

The option *Analytical* is preset. This method uses the equations given by the standards for reinforced concrete. This method is described in chapter 2.6 of this manual, page 69f.

To open the dialog box for checking and, if necessary, adjusting the design parameters, click [Settings].

Figure 3.9: Dialog box *Settings for Analytical Method of Serviceability Limit State Design*

Method

In this section, you decide which deformation ratio of the directions of reinforcement is applied for the serviceability limit state design.

By assuming an identical deformation ratio of the longitudinal reinforcement, the program assumes the same deformation ratio of the provided reinforcement. All rebars in the individual reinforcement directions are subjected to the same strain. This approach is a fast and exact procedure. The selection of the concrete strut inclination plays a significant role. This method is based on a purely geometrical division (see chapter 2.6, page 69). It is applicable if the provided reinforcement more or less corresponds to the required reinforcement.

The option *By classifying the macro element as plate or wall* offers you a simplified solution that you can use for a non-rotated, orthogonal reinforcement mesh: The program checks for each design point if the tensile stresses from axial forces or bending moments do not exceed a certain stress. The limit value of the stress is to be defined in the *Classification Criterion* section. This criterion is used to classify whether the surface is to be designed as plate (axial forces are set to zero) or wall (moments are set to zero). By neglecting minor internal force components, it is possible to use the flowchart shown in ENV 1992-1-1, Annex A 2.8 or 2.9. The design internal forces correspond to the values displayed in the RFEM table 4.16 (see RFEM manual, chapter 8.16).

If the program cannot satisfy the classification criterion for a design point of the surface, an error message appears during the calculation.

The option *By taking into account the deformation ratio of the longitudinal reinforcement* is enabled only for 2D types of model (see Figure 2.1, page 10). This method considers the effective deformation ratios due to the selected reinforcement and takes them into account for the serviceability limit state design.

Design of

In this section, you can specify whether to analyze stresses and/or cracks in the design. You must select at least one of the two check boxes.

If you select *Stresses*, the program analyzes the concrete compressive stresses σ_c and the steel stresses σ_s . If you select *Cracks*, the module checks the minimum reinforcements $a_{s,min}$, the limit diameters $\lim d_s$, the maximum crack spacings $\max s_l$, and the crack widths w_k . The settings for the individuals checks can be specified in the 1.3 *Surfaces* window (see chapter 3.3, page 141).

Furthermore, it is possible to calculate the *Deflection with RF-CONCRETE Deflect* taking into account creep, shrinkage, and tension stiffening (see chapter 2.7, page 92). To use this option, you need a license of the **RF-CONCRETE Deflect** add-on module.

Determination of Longitudinal Reinforcement

With the *Increase the required longitudinal reinforcement automatically* check box, you decide if to dimension the longitudinal reinforcement in such a way that the serviceability limit state designs are satisfied. If this check box is deactivated, the program uses only the specifications in the *Longitudinal Reinforcement* tab of window 1.4 (see chapter 3.4.3, page 153): Basic reinforcement, required reinforcement from ultimate limit state design, or basic reinforcement with provided additional reinforcement.

The dimensioning of the reinforcement for the serviceability limit state design is determined by increasing the reinforcement iteratively. As initial value for the iterations to resist the given characteristic load, the program takes the required ULS reinforcement. The dimensioning will have no results if the rebar spacing s_l of the applied reinforcement reaches the rebar diameter d_{sl} . In this case, the results windows will indicate that the respective point cannot be designed.

In the design according to EN 1992-1-1, you can select the option *Find the most economical reinforcement for crack width design*. To display information about this option (see the following figure), click [Info]. The *Information* dialog box describes in which case a check of crack width can be considered as satisfied. Moreover, clause 7.2 of EN 1992-1-1 describes under which conditions the stresses shall be limited.

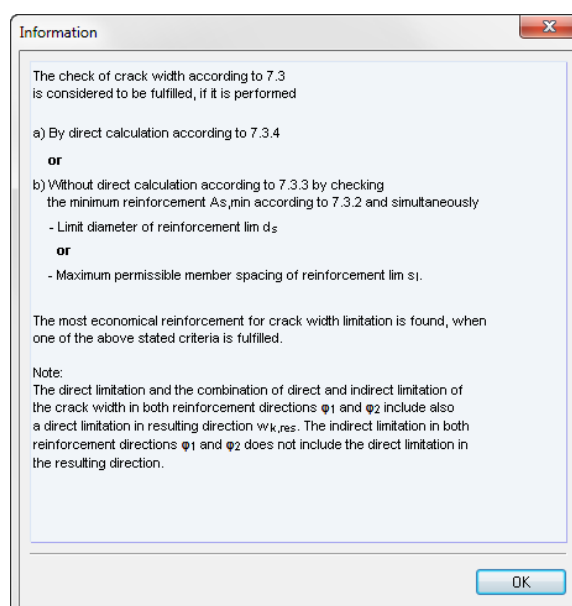


Figure 3.10: Dialog box *Information* for determination of most economical reinforcement



This means that not all design ratios shown in window 3.1 have to be less than 1 in order for the serviceability limit state design to be fulfilled.

The dimensioning of the reinforcement regarding the concrete and steel stress, the maximum diameters, and the maximum bar spacing is done separately for each reinforcement direction. However, if the resulting crack width $w_{k,res}$ is governing to satisfy the check of crack width, the reinforcement amount is increased equally for each direction.

Criteria of the check that do not have to be satisfied due to economical reasons are indicated in the results windows of the serviceability check by the message 236): "The check of the reinforcing layers need not be fulfilled for economical reasons." The governing check of crack width for the most economical reinforcement is marked with the message 235): "The check restricts increase of reinforcement for economical reasons." This message applies to the designs for $\lim d_s$, $\lim s_i$, and w_k , but not to $a_{s,min}$.

If you select the *Find the most economical reinforcement for crack width design*, you cannot specify a user-defined additional reinforcement for the SLS design in the *Longitudinal Reinforcement* tab of the 1.4 *Reinforcement* window.

Classification Criterion

This section is only available for 3D types of model. With the check boxes, you decide if minor *normal forces* and/or *moments* may be neglected in order to design surfaces in an idealized way as pure plates (selection of first check box) or wall (selection of second check box). As limit value, the mean axial tensile strength f_{ctm} is preset with 2.9 N/mm² of a concrete C30/37 for each option: It is assumed that the tensile strength of concrete compensates a crack formation due to minor tensile stresses. This is the reason why they can be neglected.

If you have selected the surface classification as plate or wall in the *Method* dialog box section on the left, you have to select at least one of the two check boxes.

3.1.2.2 Nonlinear Method

In order to perform design according to the *Nonlinear* method, a license of the add-on module **RF-CONCRETE NL** is required. This method is described in chapter 2.8, page 107. The program performs a physical and a geometrical nonlinear calculation.

The nonlinear design method assumes an interaction between model and action-effects, requiring a clear distribution of internal forces. Therefore, it is only possible to analyze load cases and load combinations, but not result combinations (RC). In a result combination, however, there are two values available for each FE node – maximum and minimum.

The internal forces according to the nonlinear design method are generally determined by second-order analysis.

To open the dialog box for checking and, if necessary, adjusting the design parameters, click [Settings]. This dialog box contains two tabs: *Options* and *Material Properties*.



Options

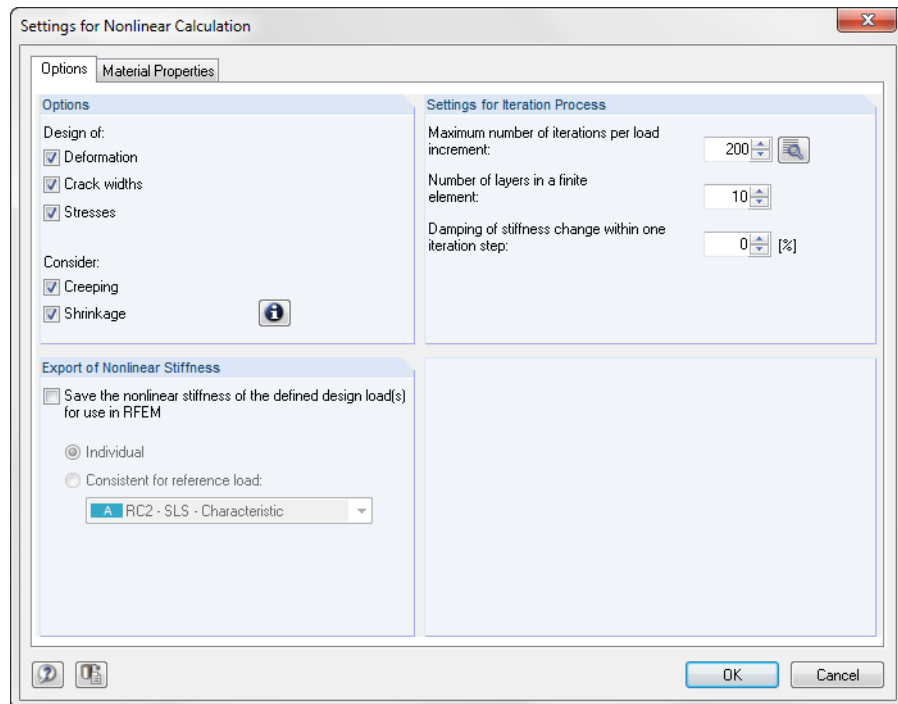


Figure 3.11: Dialog box *Settings for Nonlinear Calculation*, tab *Options*

Options

In this section, you select which serviceability limit state designs should be carried out: deformation, crack widths, stresses. You must select at least one of the three check boxes.

Furthermore, you can specify if the influence of creep and shrinkage are to be considered in the nonlinear calculation.

Detailed settings for the individual checks as well as for creep and shrinkage are defined in the 1.3 *Surfaces* window (see chapter 3.3.2, page 144).

Export of Nonlinear Stiffness

With the check box *Save the nonlinear stiffness of the defined design load(s) for use in RFEM*, you decide if the determined stiffnesses should also be available for a calculation in RFEM.

The stiffnesses can be exported *Individually* for each designed load case. In the *Load Cases and Combinations* dialog box of RFEM, you can assign the according stiffness from RF-CONCRETE Surfaces to each of these load cases. RFEM allocates the load cases automatically. If you select the *Consistent for reference load* option, you must specify the governing load case in the drop-down list below. In RFEM, you can then assign the stiffness resulting from these loads to all defined load cases.

The consideration of nonlinear stiffnesses in RFEM is described in chapter 7.3.1.3 of the RFEM manual.

Settings for Iteration Process

The settings in this section influence the process of the nonlinear design method. For more information, see chapter 2.8.2.4, page 111.



When modifying the precision of iterations, take care that the *Maximum number of iterations per load increment* is higher than the step in the calculation process from which on the deformation criterion will be additionally taken into account. To open the *Calculation Parameters* dialog box of RFEM, click [Details]. In this dialog box, you can adjust the precision of the convergence criteria for the nonlinear calculation.

In the nonlinear calculation, the surface is divided into so-called *layers* (see chapter 2.8.2.1, page 107). The recommended number of layers is 10.

Moreover, you can influence the performance of the convergence behavior by *Damping*: The damping controls the magnitude of the stiffness change in the following iteration steps. If you, for example, specify a damping of 50 %, the maximum change of the stiffness between step 2 and 3 can be 50 % of the stiffness change between step 1 and step 2.

Material Properties

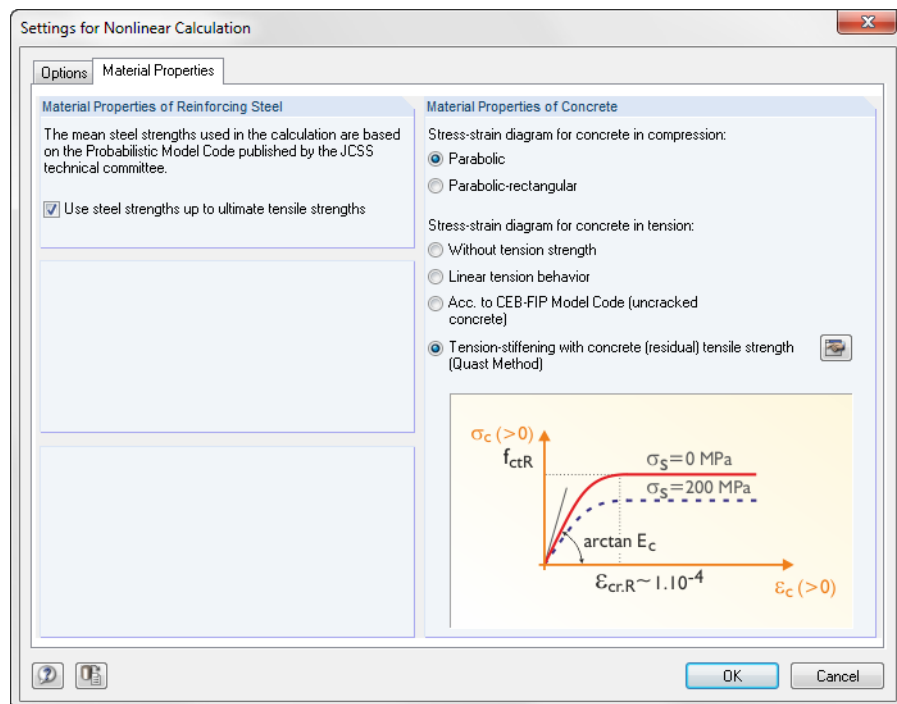


Figure 3.12: Dialog box *Settings for Nonlinear Calculation*, tab *Material Properties*

Material Properties of Reinforcing Steel

With the check box in this section, you decide if the calculation in the plastic range of the reinforcing steel's stress-strain diagram will be carried out with a rising or a horizontal graph (see chapter 2.8.3.4, page 119).

Material Properties of Concrete

In this section, you specify the stress-strain relationships of the concrete in *compression* and *tension*, respectively. The parabolic diagram for compression and tension stiffening for concrete tensile stresses are preset in this section.



For *tension stiffening* (consideration of the stiffening effect of concrete in tension), you can specify the parameters to apply the tensile strength of concrete between the cracks in a separate dialog box. To open this dialog box, click [Edit].

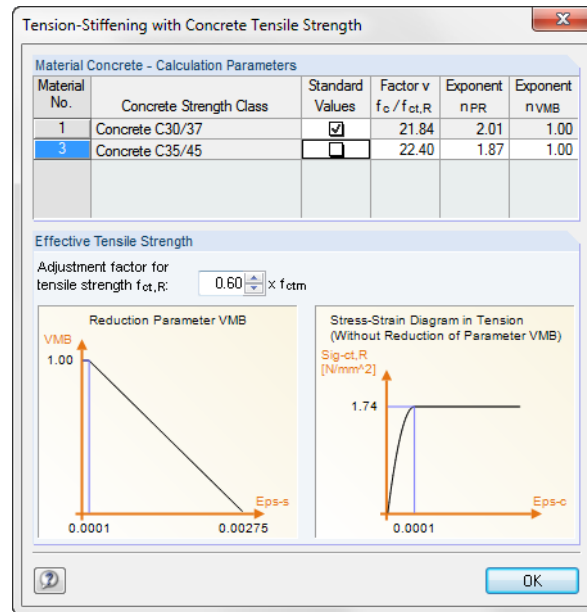


Figure 3.13: Dialog box *Tension Stiffening with Concrete Tensile Strength*

Modifications of the parameters are immediately displayed graphically in the diagrams.

For more information on *tension stiffening*, see chapter 2.8.3.3, page 115.

3.1.3 Details

This tab appears if you select load cases for the serviceability limit state design and set the standard EN 1992-1-1 or ACI 318-11. This tab is not necessary for serviceability limit state designs according to the other standards, for example DIN 1045-1, clause 11.2.4 (2), because the factor k_t is generally defined with 0.4 in Equation (136).

Ultimate Limit State			Serviceability Limit State	Details
Details for Serviceability				
No.	A Load Case / Combination Description	B Permanent Load	C Factor k_t	
LC1	Self-weight and superstructure - abutment	☒	0.400	
LC3	Active earth pressure, earth load, load on road	☐	0.600	
LC4	Earth pressure/load from traffic, locomotive - abut	☐	0.600	
LC5	Earth pressure/load from traffic, locomotive - bridge	☐	0.600	
LC7	Nosing force, abutment wall (west)	☐	0.600	
LC8	Nosing force, abutment wall (east)	☐	0.600	
LC20	Self-weight of superstructure	☐	0.600	
LC21	Full load, locomotive - abutment	☐	0.600	
LC22	Full load, locomotive - bridge	☐	0.600	
LC23	Wind from west	☐	0.600	
LC24	Wind from east	☐	0.600	
LC25	Nosing force in +Y	☐	0.600	
LC26	Nosing force in -Y	☐	0.600	
LC27	Displacement resistance in +X	☐	0.600	
LC28	Displacement resistance in -X	☐	0.600	
LC29	Bearings replacement	☐	0.600	
RC13	Stability		0.388	

Figure 3.14: Window 1.1 *General Data*, tab *Details*

In the crack width design, the program calculates the differences of the mean strains of concrete and reinforcing steel (see chapter 2.6.4.12, page 89). According to EN 1992-1-1, 7.3.4 (2), Eq. (7.9), the load duration factor k_t must be specified for these differences.

Load Case / Combination Description

This column lists all load cases, load combinations, and result combinations selected in the *Serviceability Limit State* tab for design. For load combinations or result combination, the included load cases are shown, too.

Permanent Load

This column indicates the load cases that will be applied as permanent loads. If an entry is specified as permanent load (the check box is selected), the load duration factor k_t is automatically set to 0.4 in the final column.

Factor k_t

The load duration factor k_t is used to consider the duration of the load. The factor k_t is 0.4 for long-term load actions and 0.6 for short-term actions.

For load combinations and result combinations, the mean is taken from the k_t values of the load cases contained in the CO or RC.

$$k_t = \frac{\sum_{i=1}^n \gamma_i(LC) \cdot k_{t,i}(LC)}{\sum_{i=1}^n \gamma_i(LC)}$$

Equation 3.1

3.2 Materials

The table is subdivided into two parts. The upper section lists the design-relevant concrete and steel grades. All materials of the category concrete that are used in RFEM for surfaces are already preset. In the *Material Properties* section, the properties of the current material are displayed, that is, the table row of the material currently selected in the upper section.

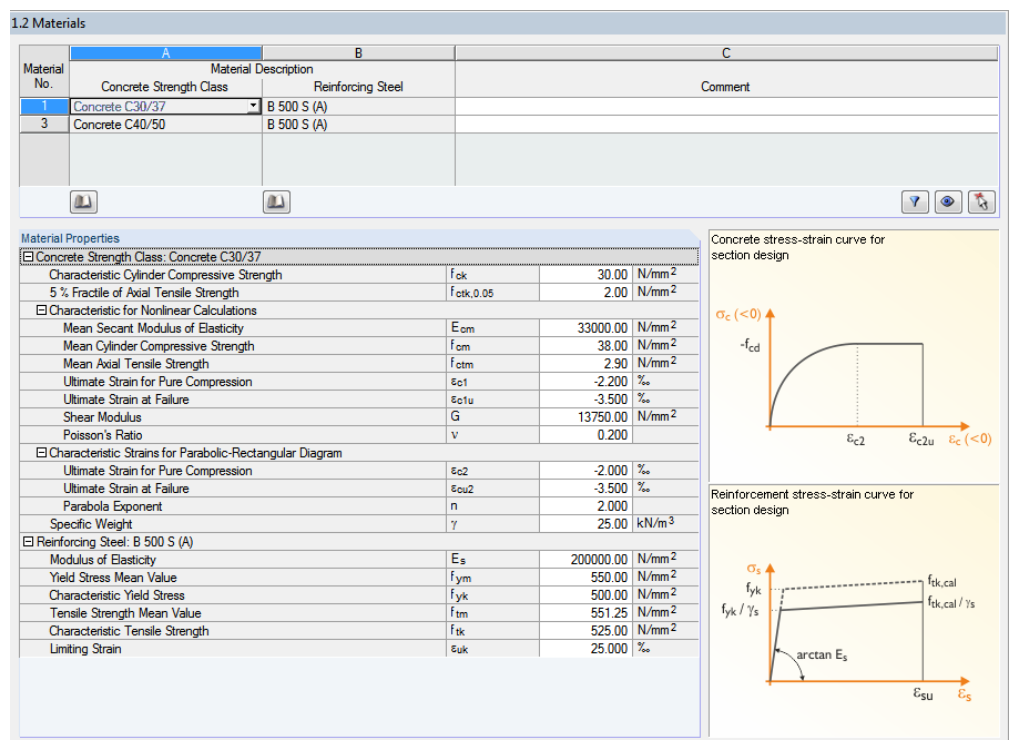


Figure 3.15: Window 1.2 Materials

The table lists only materials selected for the design. Materials that are not allowed are highlighted red; changed materials appear blue in color.

Chapter 4.3 of the RFEM manual describes the material properties that are used for the determination of the internal forces. The properties of the materials needed for the design are stored in the global material library. These values are preset for the *Concrete Strength Class* and the *Reinforcing Steel*.

To adjust the units and decimal places of material properties and stresses, select **Units and Decimal Places** in the module's "Settings" menu (see chapter 8.2, page 192).

Material Description

Concrete Strength Class

The concrete materials used in RFEM are already preset; materials that are not relevant are hidden. It is always possible to modify the strength class. To do this, click the material in column A, thus selecting the field. Then, click [▼] or press [F7] to open the list of the strength classes.

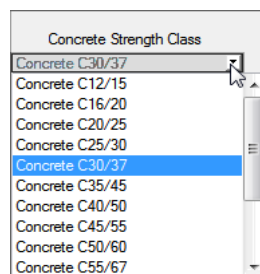


Figure 3.16: List of concrete strength classes

The list contains only strength classes complying with the design concept of the selected standard.

After the transfer, the program shows the updated design-relevant *Material Properties*.

As a matter of principle, the material properties cannot be edited in RF-CONCRETE Surfaces.

Reinforcing Steel

In this column, the program presets a steel grade that corresponds to the design concept of the selected standard.

Similarly to the concrete strength class, you can select a different reinforcing steel by using the drop-down list: Click the material in column B, thus selecting the field. Then, click [▼] or press [F7] to open the list with different reinforcing steels.

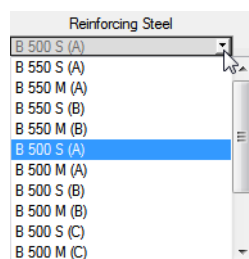


Figure 3.17: List of reinforcing steels

The list contains only steel grades that are relevant for the selected standard.

After the transfer, the program updates the *Material Properties*.

Material Library



The material library contains many materials. To open the corresponding material library, click the button shown on the left. The buttons are located below column A and B and are used to access the library for the concrete strength classes and reinforcing steels, respectively.

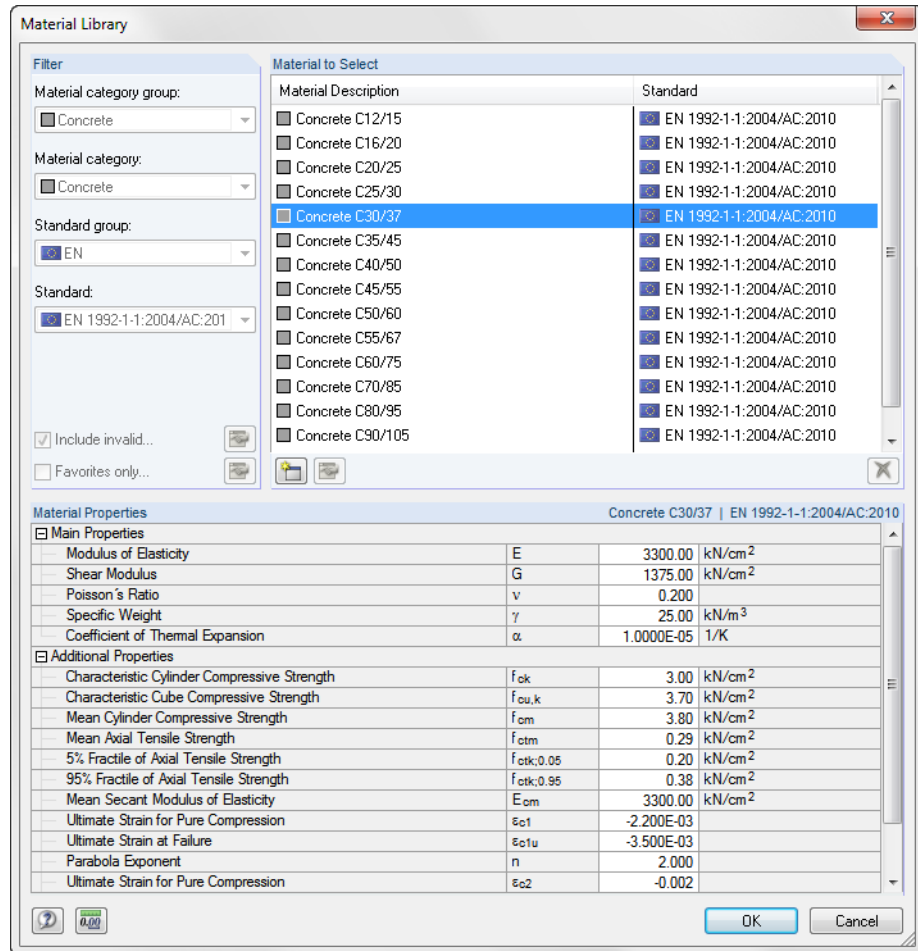


Figure 3.18: Dialog box *Material Library*

In the *Filter* section, the standard-relevant materials are already preset, thus excluding all other categories or standards. You can select the desired concrete strength class or steel grade from the *Material to Select* list; then you can check the material properties in the section below.

Click [OK] or press [↵] to transfer the selected material to window 1.2 *Materials* of RF-CONCRETE Surfaces.

Chapter 4.3 of the RFEM manual describes how to filter, add, or rearrange materials.

OK

3.3 Surfaces

This window lists the surfaces that are relevant for the design.

The makeup of the window depends on the settings in the 1.1 *General Data* window: If you design the ultimate limit state exclusively, the table lists only the surfaces with their thicknesses. If you have selected load cases for the serviceability limit state design (see Figure 3.8, page 131), however, this window shows specific setting options. They differ depending on the selected method of check.

The buttons have the following functions:

Button	Function
	Shows only surfaces that are assigned to a reinforcement group in the 1.4 <i>Reinforcement</i> window (see chapter 3.4)
	Jumps to the RFEM work window to adjust the view
	Allows you to select a surface in the RFEM work window

Table 3.2: Buttons in window 1.3 *Surfaces*

3.3.1 Analytical Method

The analytical method for the serviceability check is described in chapter 2.6, page 69.

If you use RF-CONCRETE Deflect, this window provides additional tabs and columns. They are described in chapter 3.3.2 *Nonlinear Method*.

Method of check: ☒ Analytical...
☐ Nonlinear...

1.3 Surfaces

Surface No.	Material No.	Thickness Type	d [mm]	$\sigma_{c,min}$ [N/mm ²]	$\sigma_{s,max}$ [N/mm ²]	$f_{ct,eff}$ [N/mm ²]	$w_{k,z}$ (top) [mm]	$w_{k,z}$ (bottom) [mm]	Effects Due to Restraint	Apply	k_{ct} [1]	Notes	Comment
1	1	Constant	200.0	-18.00	400.00	2.90	0.300	0.300	☒		var. 6)		Ceiling slab
2	1	Constant	200.0	-13.50	400.00	2.90	0.300	0.300	☒		var. 6)		
3	1	Constant	200.0	-13.50	400.00	2.90	0.300	0.300	☒		var. 6)		
4	1	Constant	200.0	-13.50	400.00	2.90	0.300	0.300	☒		var. 6)		Rear wall
5	1	Constant	200.0	-13.50	400.00	2.90	0.300	0.300	☒		var. 6)		Shell

Settings for the check of serviceability limit state - Surface No. 1-3

Stress Check | Limit of Crack Widths

Design of Crack Width Control

Limit value of allowable crack width $w_{k,max}$

☐ Limit values acc. to 7.3.1(5)
☒ User-defined

$w_{k,z}$ (top): 0.300 [mm]
 $w_{k,z}$ (bottom): 0.300 [mm]

☒ Design without direct crack width calculation acc. to 7.3.3
☒ Calculation of limit diameter $lim\ d_s$
☒ Calculation of maximum member spacing $lim\ s$
☒ Design with direct crack width calculation acc. to 7.3.4

☒ Set inputs for surface No.: 1-3

Minimum Reinforcement for Effects Due to Restraint

☒ $A_{s,min}$ for effects due to restraint
☐ Crack formation within the first 28 days
 (Effective tensile strength of concrete $f_{ct,eff} = k_{ct} \cdot f_{ctm}$)

Reduction factor
 k_{ct} : []

Figure 3.19: Window 1.3 *Surfaces* with settings for analytical check method, tab *Limit of Crack Widths*

Material No.

For each surface, the table shows the material numbers that are managed in the 1.2 *Materials* window.

Thickness

Type

The program is able to design constant and linearly variable thickness types as well as surfaces with orthotropic properties.

d

This column shows the surface thicknesses defined RFEM. The values can be changed for the design.



If the surface thicknesses are modified, the internal forces of RFEM are used for the design, which result from the stiffnesses of the RFEM surface thicknesses. In a statically indeterminate system, the surface thicknesses modified in RF-CONCRETE Surfaces must be also adjusted in RFEM. Thus, the distribution of internal forces is correctly considered in the design.



The other column descriptions are independent of the settings in the tabs below. However, they can be controlled in the *Settings* dialog box (see Figure 3.9, page 132); in this dialog box, you can specify whether stresses and/or cracks are to be designed.



The values in the columns are based on the entries in the tabs below. These specifications apply to all surfaces by default. It is possible, however, to assign the current specifications only to certain surfaces: Clear the selection in the *All* check box. Then, enter the numbers of the relevant surfaces or select them graphically in the RFEM work window after clicking [^]. With [x], you assign the current settings to these surfaces. However, the assignment is applicable only for the active tab, for example *Stress Check*.

The two following parameters are defined in the *Stress Check* tab (see Figure 3.20).

$\sigma_{c,min}$

This column shows the value of the minimum concrete stress for the limitation of the concrete pressure stresses (see chapter 2.6.4.7, page 77). According to EN 1992-1-1, the following applies to

- Quasi-permanent action combination if serviceability, ultimate limit state, or durability are considerably affected by creeping:

$$\sigma_c \leq 0.45 \cdot f_{ck} \quad 7.2 (3)$$

- Rare (= characteristic) action combination in exposure classes XD1 to XD3, XF1 to XF4, XS1 to XS3:

$$\sigma_c \leq 0.60 \cdot f_{ck} \quad 7.2 (2)$$

$\sigma_{s,max}$

This value represents the maximum reinforcing steel stress for the limitation of the reinforcement's tensile stresses (see chapter 2.6.4.8, page 80). According to EN 1992-1-1, the following applies to

- Rare action combinations:

$$\sigma_s \leq 0.80 \cdot f_{yk} \quad 7.2 (5)$$

- Pure restraint actions:

$$\sigma_s \leq 1.00 \cdot f_{yk} \quad 7.2 (5)$$

The remaining parameters are defined in the *Limit of Crack Widths* tab (see Figure 3.19).

$f_{ct,eff}$

This column displays the respective value of the effective concrete tensile strength. The value is required to control the rebar diameter (see chapter 2.6.4.10, page 84).

$W_{k,-z}$ (top) / $W_{k,+z}$ (bottom)

These parameters are allowable crack widths at the top and bottom sides of the surfaces (see chapter 2.6.4.12, page 87).

Restraint

If there are effects due to restraint, they shall be considered in the determination of the minimum reinforcement to limit the crack width (see chapter 2.6.4.9, page 81).



In the *Limit of Crack Widths* tab, you can click [Edit] to specify the minimum reinforcement for effects due to restraint (see Figure 2.97, page 82).

Apply

Column I or the check box in the *Limitation of Crack Widths* tab controls if there are effects due to restraint.

Symbol

In the tab *Limitation of Crack Width*, you specify whether there are inner or outer effects due to restraint. This has an influence on the factor k taking into account the nonlinearly distributed concrete tensile stresses (see Equation 2.68, page 81).

k_c

This factor takes account of the stress distribution in the tension zone (see Equation 2.68, page 81).

Notes

This column shows remarks in the form of footers that are described in detail in the status bar.

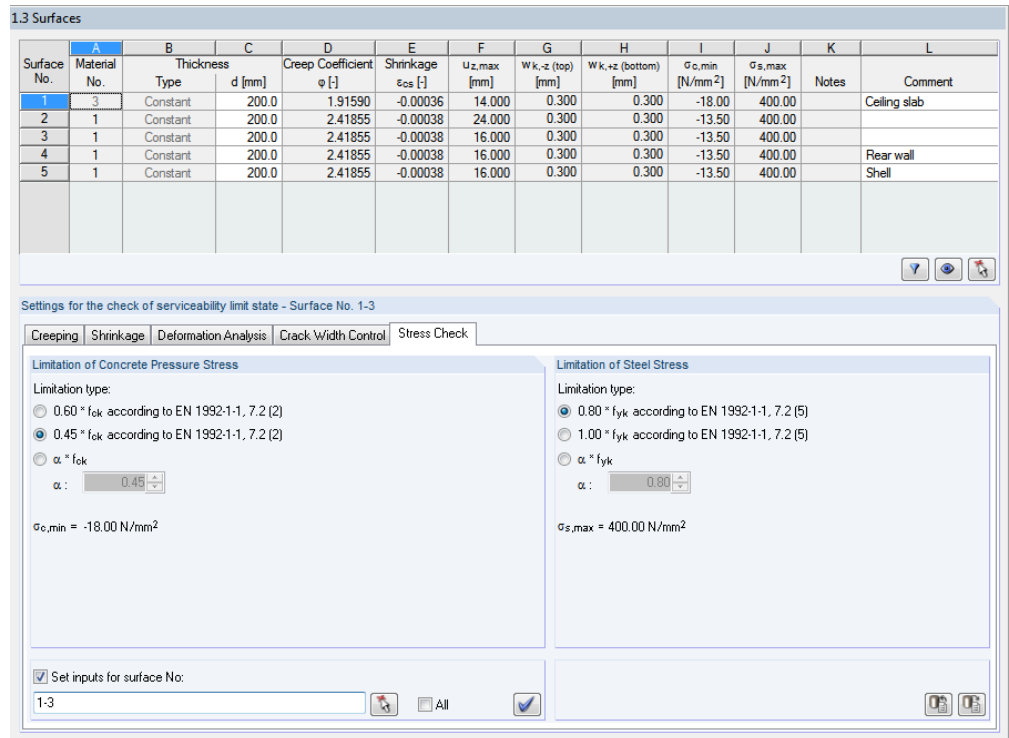
Comment

This input field can be used to enter user-defined comments.

3.3.2 Nonlinear Method

Method of check: ☐ Analytical...
☒ Nonlinear...

To design according to the *nonlinear method*, you need a license of the **RF-CONCRETE NL** add-on module. This method for the serviceability limit state design is described in detail in chapter 2.8, page 107.



Surface No.	Material No.	Thickness Type	Thickness d [mm]	Creep Coefficient φ [-]	Shrinkage ε_{cs} [-]	Uz,max [mm]	Wk,-z (top) [mm]	Wk,-z (bottom) [mm]	$\sigma_{c,min}$ [N/mm ²]	$\sigma_{s,max}$ [N/mm ²]	Notes	Comment
1	3	Constant	200.0	1.91590	-0.00036	14.000	0.300	0.300	-18.00	400.00		Ceiling slab
2	1	Constant	200.0	2.41855	-0.00038	24.000	0.300	0.300	-13.50	400.00		
3	1	Constant	200.0	2.41855	-0.00038	16.000	0.300	0.300	-13.50	400.00		
4	1	Constant	200.0	2.41855	-0.00038	16.000	0.300	0.300	-13.50	400.00		Rear wall
5	1	Constant	200.0	2.41855	-0.00038	16.000	0.300	0.300	-13.50	400.00		Shell

Settings for the check of serviceability limit state - Surface No. 1-3

Creeping | Shrinkage | Deformation Analysis | Crack Width Control | **Stress Check**

Limitation of Concrete Pressure Stress

Limitation type:

- ☐ $0.60 \cdot f_{ck}$ according to EN 1992-1-1, 7.2 (2)
- ☒ $0.45 \cdot f_{ck}$ according to EN 1992-1-1, 7.2 (2)
- ☐ $\alpha \cdot f_{ck}$

α :

$\sigma_{c,min} = -18.00 \text{ N/mm}^2$

Limitation of Steel Stress

Limitation type:

- ☒ $0.80 \cdot f_{yk}$ according to EN 1992-1-1, 7.2 (5)
- ☐ $1.00 \cdot f_{yk}$ according to EN 1992-1-1, 7.2 (5)
- ☐ $\alpha \cdot f_{yk}$

α :

$\sigma_{s,max} = 400.00 \text{ N/mm}^2$

☒ Set inputs for surface No:

Figure 3.20: Window 1.3 Surfaces with settings for nonlinear method of check, tab Stress Check

The following columns are described in the chapter above, 3.3.1 *Analytical Method*:

- Material
- Thickness
- $W_{k,-z} / W_{k,+z}$
- $\sigma_{c,min}$
- $\sigma_{s,max}$



For orthotropic surfaces, no serviceability limit state design according to the nonlinear method is possible.



The values in the columns D through J are controlled in the tabs below. The settings in these tabs are applied to all surfaces by default. It is possible, however, to assign the current specifications only to certain surfaces: Clear the selection of the *All* check box. Then, enter the number of the relevant surfaces or use [^] to select them graphically. With [✓], you assign the current settings to these surfaces. The assignment is only applicable for the current tab (for example *Stress Check*).



Creep Coefficient φ

The parameters for creep are defined in the *Creeping* tab (see Figure 2.144, page 120). Based on these conditions, the program determines the creep coefficient φ . For the notional size of the member h_0 , the program applies the surface thickness d .

The determination of the creep coefficient is described in chapter 2.8.4.1, page 120.

Shrinkage ϵ_{cs}

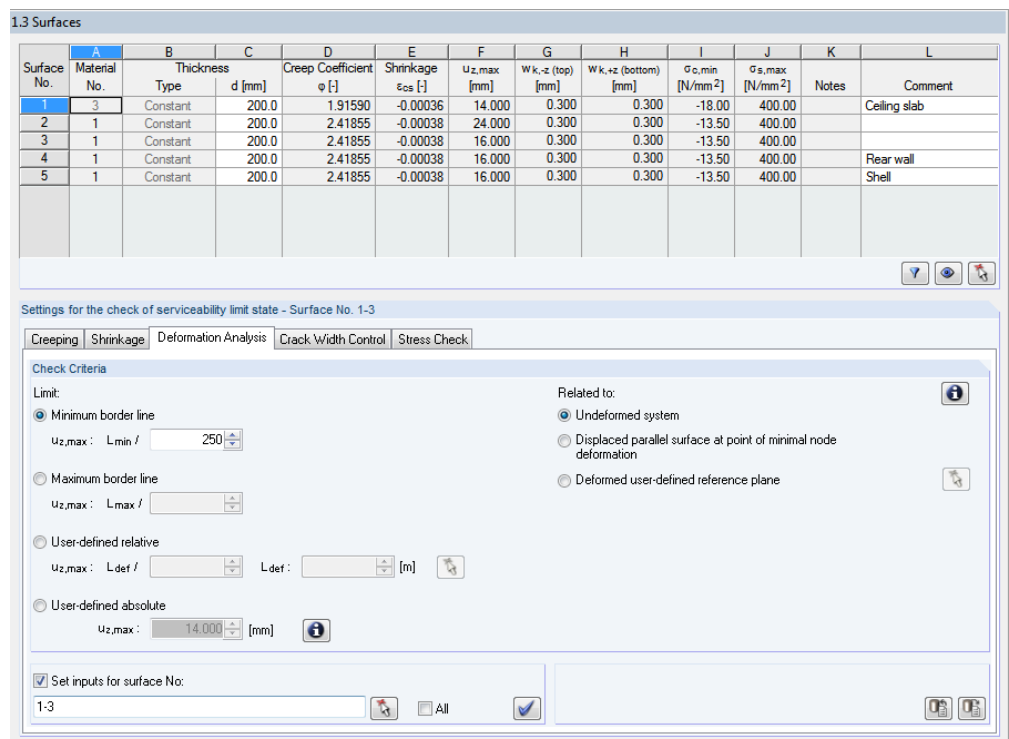
This column shows the shrinkage. The relevant parameters are defined in the *Shrinkage* tab (see Figure 2.147, page 124). Based on these boundary conditions, the program determines the appropriate shrinkage ϵ_{cs} . For the notional size of the structural component h_0 , the program assumes the surface thickness d for the calculation.

The determination of shrinkage is described in chapter 2.8.4.2, page 123. If you do not want to apply any shrinkage strain to a surface, set zero for the user-defined shrinkage strain in the *Shrinkage* tab, and then apply it to the surface.

For pure plates that are defined as the type of model 2D - XY ($u_z/\varphi_x/\varphi_y$), it is not possible to consider shrinkage: There are only degrees of freedom for bending.

$u_{z,max}$

This value represents the maximum allowable deformation that must be kept in the serviceability limit state design. The design criteria are defined in the *Deformation Analysis* tab.



1.3 Surfaces

Surface No.	Material No.	Thickness Type	Thickness d [mm]	Creep Coefficient ϕ [-]	Shrinkage ϵ_{cs} [-]	$u_{z,max}$ [mm]	$W_{k,z}$ (top) [mm]	$W_{k,z}$ (bottom) [mm]	$G_{o,min}$ [N/mm ²]	$G_{s,max}$ [N/mm ²]	Notes	Comment
1	3	Constant	200.0	1.91590	-0.00036	14.000	0.300	0.300	-18.00	400.00		Ceiling slab
2	1	Constant	200.0	2.41855	-0.00038	24.000	0.300	0.300	-13.50	400.00		
3	1	Constant	200.0	2.41855	-0.00038	16.000	0.300	0.300	-13.50	400.00		
4	1	Constant	200.0	2.41855	-0.00038	16.000	0.300	0.300	-13.50	400.00		Rear wall
5	1	Constant	200.0	2.41855	-0.00038	16.000	0.300	0.300	-13.50	400.00		Shell

Settings for the check of serviceability limit state - Surface No. 1-3

Creeping | **Shrinkage** | Deformation Analysis | Crack Width Control | Stress Check

Check Criteria

Limit:

- ☒ Minimum border line
 $u_{z,max} : L_{min} / 250$
- ☐ Maximum border line
 $u_{z,max} : L_{max} /$
- ☐ User-defined relative
 $u_{z,max} : L_{def} /$ $L_{der} :$ [m]
- ☐ User-defined absolute
 $u_{z,max} : 14.000$ [mm]

Related to:

- ☒ Undeformed system
- ☐ Displaced parallel surface at point of minimal node deformation
- ☐ Deformed user-defined reference plane

☒ Set inputs for surface No: 1-3

Figure 3.21: Window 1.3 Surfaces, tab *Deformation Analysis*

Limit

To ensure the serviceability limit state for "common structures," for example according to EN 1992-1-1, clause 7.4, the deflection in the quasi-permanent action combination must not exceed the following limit values:

- Common case:

$$u_{z,max} = \frac{l_{eff}}{250}$$

- Structural elements for which excessive deformations can result in subsequent damages:

$$u_{z,max} = \frac{l_{eff}}{500}$$

The options *Minimum border line*, *Maximum border line*, and *User-defined relative* determine which effective length l_{eff} will be applied. For the two *Border line* options, the program applies the shortest or greatest border line of the respective surface.

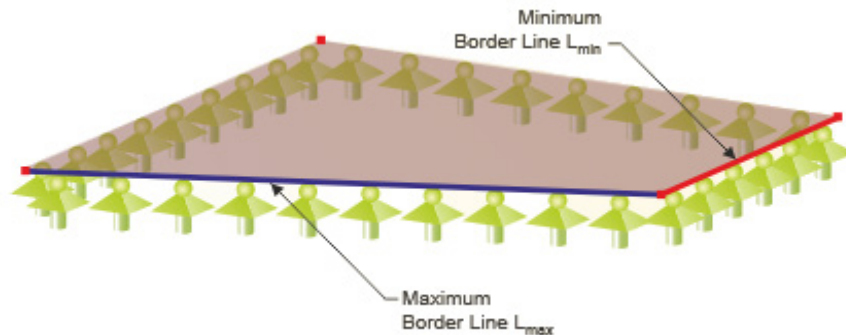


Figure 3.22: Maximum and minimum border line for determination of $u_{z,\text{max}}$



For the *User-defined relative* option, you can enter the length directly or select it graphically between two points in the RFEM model by using [↖]. For all three options, you must define a divisor by which the defined lengths will be divided.

You can also specify the allowable maximum deformation $u_{z,\text{max}}$ as *User-defined absolute*.

Related to

The deformation design criterion uses the deflection of a surface – the vertical deformation relative to the shortest line connecting the points of support. The *Deformation Analysis* tab (Figure 3.21) offers three possibilities how to calculate the local deformation $u_{z,\text{local}}$ used in the design.

- *Undeformed system:* The deformation is related to the initial structure.
- *Displaced parallel surface:* This option is recommended for an elastic support of the surface. The deformation $u_{z,\text{local}}$ is related to a virtual reference surface that is displaced parallel to the undeformed system. The displacement vector of the reference surface is as long as the minimal nodal deformation within the surface.

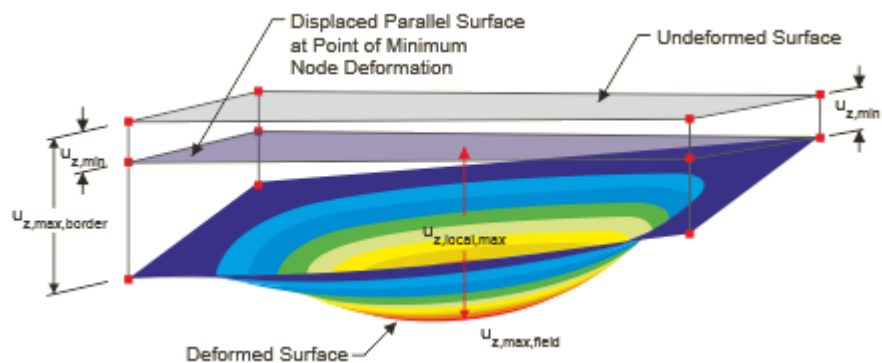


Figure 3.23: Displaced parallel surface (displacement vector: smallest nodal deformation $u_{z,\text{min}}$)

- Deformed reference plane:** If the support deformations of a surface differ considerably from each other in size and degree, you can define an inclined reference plane for the deformation $u_{z,local}$ to be checked. You must define this plane by three points of the undeformed system. The program determines the deformation of the three definition points, places the reference plane in these displaced points, and calculates the local deformation $u_{z,local}$.

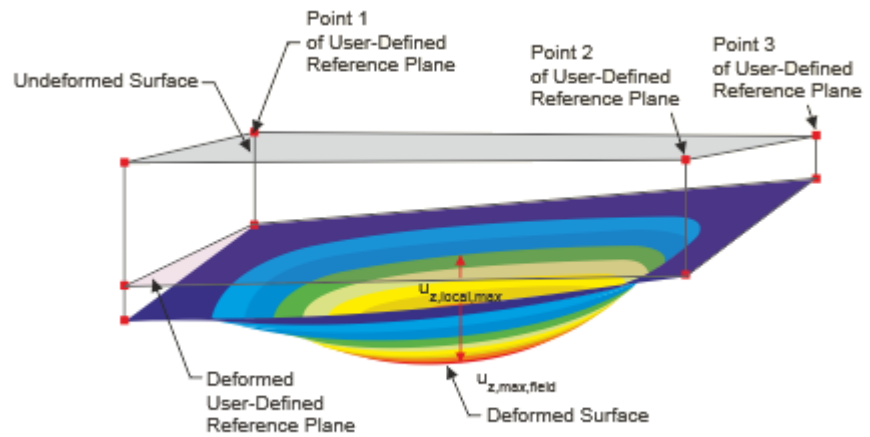


Figure 3.24: Displaced user-defined reference plane

3.4 Reinforcement

This window consists of five tabs where all reinforcement data is specified. The individual surfaces often require different settings. For this purpose, you can define so-called "reinforcement groups" in each RF-CONCRETE Surfaces case. Each reinforcement group contains the reinforcement parameters for particular surfaces.

Reinforcement Group

To create a new reinforcement group, click [New] in the *Reinforcement Group* section. The number is automatically assigned by the program. The user-defined *Description* helps you to overlook all reinforcement groups available in the design case.



Figure 3.25: Window 1.4 *Reinforcement* with three reinforcement groups

To select the desired reinforcement group, use the *No.* list or click the entries in the navigator.

By using the [Delete] button, the currently selected reinforcement group is deleted from the design case without any further warning message. Hence, surfaces that were contained in such a reinforcement group will not be designed. If you want to design them, you must reassign them to a new or an existing reinforcement group.

Applied to Surfaces

In this dialog box section, you specify the surfaces to which the parameters of the current reinforcement group apply. By default, *All* surfaces are selected. If this check box is selected, it is not possible to create any further reinforcement groups. The reason for this is that it is not possible to design a surface by different rules (this is only possible in "design cases," see chapter 8.1, page 190). Therefore, clear the *All* check box to use several reinforcement groups.

In the input field, enter the number of the surfaces to which the reinforcement parameters of the tabs below apply. You can also select them graphically in the RFEM work window by using the [↖] function. Only surface numbers that have not yet been assigned to other reinforcement groups can be entered in the input field.



3.4.1 Reinforcement Ratios

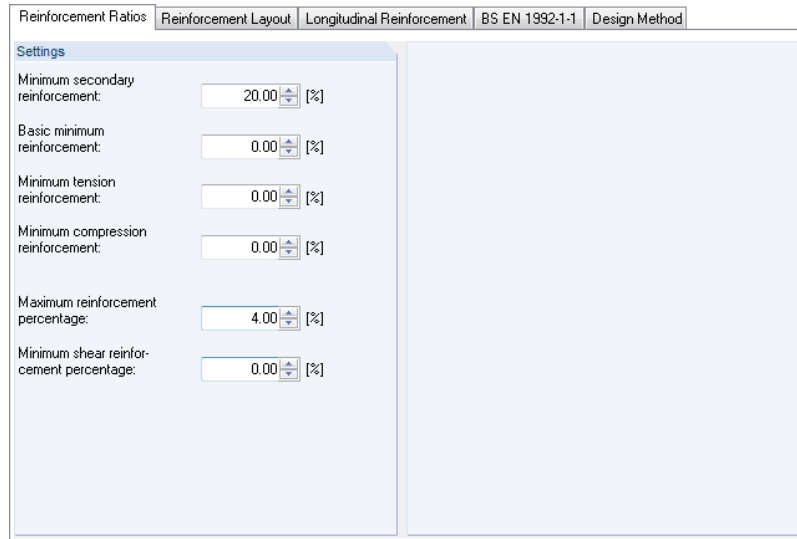


Figure 3.26: Window 1.4 Reinforcement, tab Reinforcement Ratios

This tab defines the minimum and maximum reinforcements in percentage. The *Minimum secondary reinforcement* relates to the maximum longitudinal reinforcement that is to be applied. All further reinforcement ratios are related to the cross-sectional area of a surface stripe with the width of one meter.

You can find examples for the minimum and maximum reinforcements in the chapters 2.3.7, 2.4.5, and 2.5.8 of this manual.

3.4.2 Reinforcement Layout

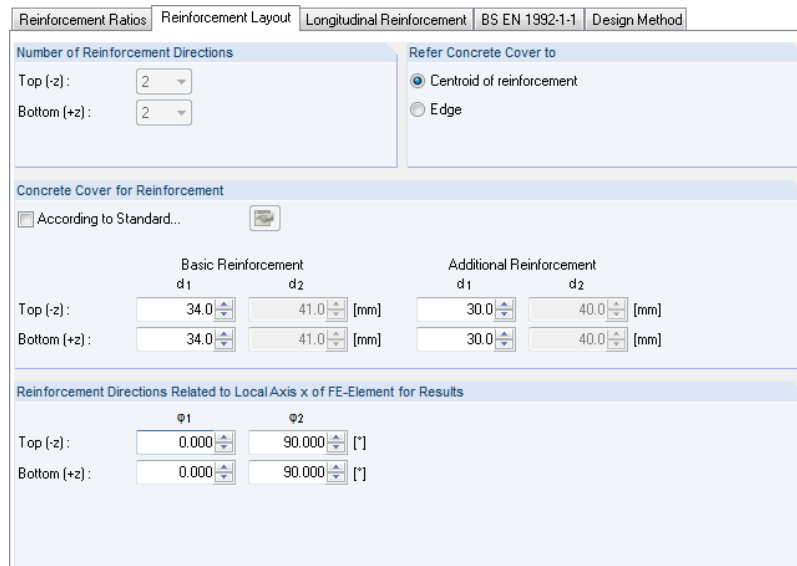


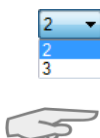
Figure 3.27: Window 1.4 Reinforcement, tab Reinforcement Layout

This tab determines the geometric specifications for the reinforcement.

Number of Reinforcement Directions

The reinforcement mesh can be defined with two or three reinforcement directions for each surface side.

For serviceability limit state designs, only a reinforcement mesh with two directions is allowed.



The definition of the "top" and "bottom" side of the surface can be found in the description of the *Concrete Cover for Reinforcement* section below.

Refer Concrete Cover to

The concrete covers that are defined in the *Concrete Cover for Reinforcement* section can be related to the reinforcement's *Centroid* or *Edge* distance.

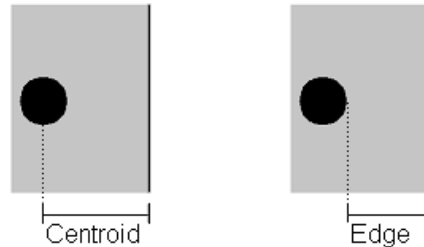


Figure 3.28: Reference of the concrete cover

If you select the *Edge* option, you have to specify the *Bar diameter D*.

Concrete Cover for Reinforcement

For both sides of the surface, you specify the concrete covers of the *Basic Reinforcement* and, if necessary, the *Additional Reinforcement*. The dimensions represent either the centroids d of the individual layers or the reinforcements' edge distances c_{nom} in direction φ_1 . The reinforcement directions are defined in the dialog box section below.

The "top" and "bottom" surface side is defined as follows: The bottom surface is defined in direction of the positive local surface axis z , the top side is defined in direction of the negative local axis z .

The RFEM graphic shows you the xyz-coordinate systems of the surfaces as soon as you move the pointer across a surface. You can also use the context menu of a surface (right-click it) to switch the axes on and off.

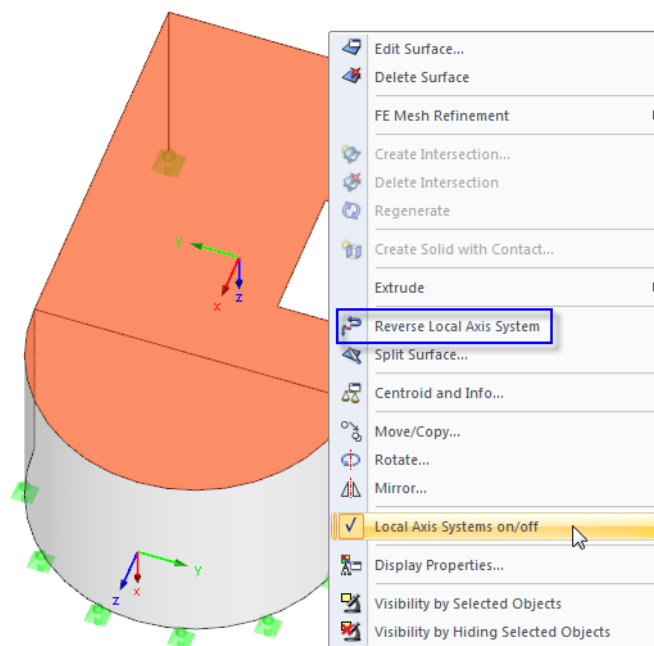
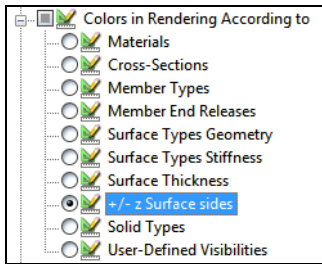


Figure 3.29: RFEM context menu of an area



To display the surface sides in different colors, select *Colors in Rendering According to* → *+/- z-Surface sides* in the *Display* navigator (see figure on the left).

You can change the orientation of the local z-axis of a surface by using the *Reverse Local Axis System* option on the context menu (see Figure 3.29). In this way, it is possible to unify, for example, the orientation of walls in order to assign the top and bottom reinforcement sides for vertical surfaces unambiguously.

The model types wall 2D - XZ ($u_x/u_z/\varphi_y$) or 2D - XY ($u_x/u_y/\varphi_z$) are models whose component's plane is exclusively subjected to compression or tension, respectively. In this case, it is not possible to create different reinforcement meshes for each surface side so that the input possibilities are limited to uniform concrete covers on both sides.

If you select the *According to Standard* check box, the [Edit] button becomes available. Click [Edit] to open the following dialog box.

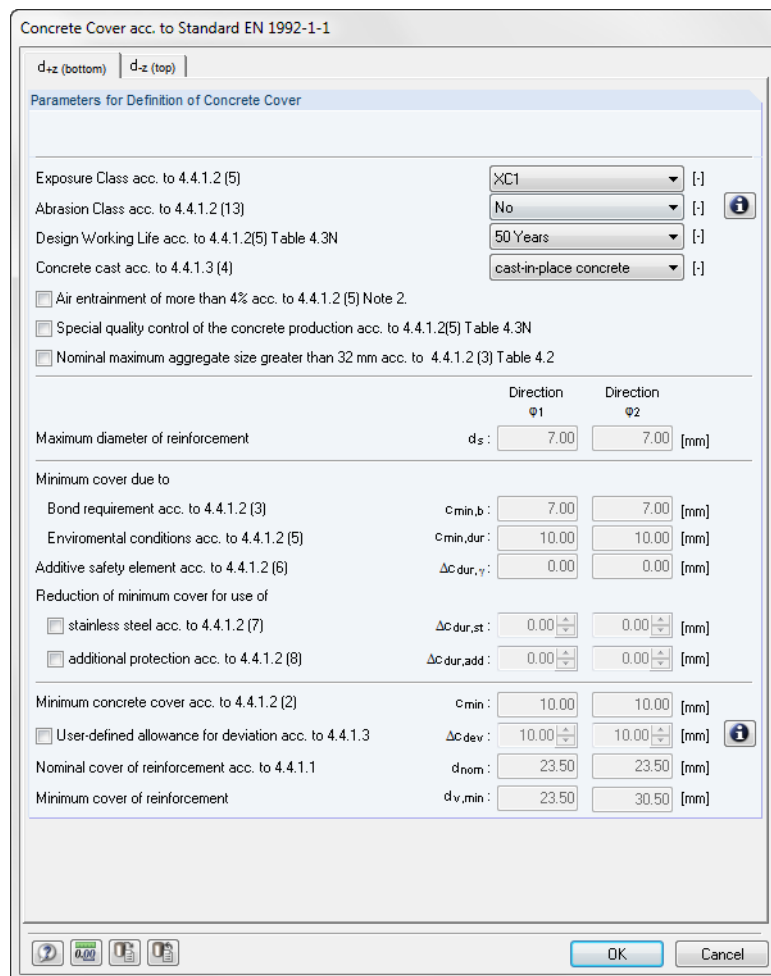
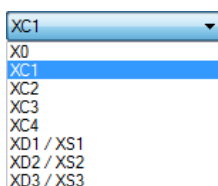


Figure 3.30: Dialog box *Cover acc. to Standard*



Selection of the exposure class

In the upper section, you can define the parameters for the design (exposure class, abrasion class, etc.) according to the standard. From these parameters, RF-CONCRETE Surfaces determines the required concrete cover.

In the two tabs, you can specify the parameters for each surface side separately.

Reinforcement Directions Related to Local Axis x of FE-Element

The reinforcement directions φ are related to the local x-axis of the finite elements. In the *Edit Surface* dialog box of RFEM, you can check and, if necessary, adjust the axes system for the results of the surfaces.

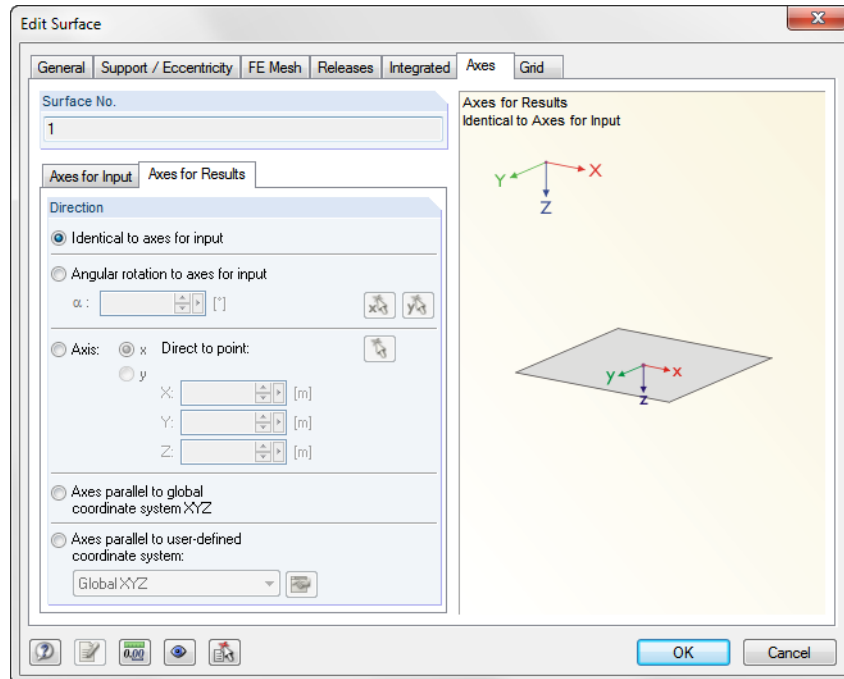


Figure 3.31: RFEM dialog box *Edit Surface*, tab *Axes* and *Axes for Results*

For curved surfaces, it is recommended to check the axes of the finite elements graphically: In the *Display* navigator of RFEM, select the option *FE Mesh* → *On Surfaces* → *FE Axis Systems x,y,z* → *Indexes* (see Figure 8.40 in chapter 8.14 of the RFEM manual).

The reinforcement directions are to be specified by means of the angle φ for each layer. Only positive angles are allowed. They represent the respective clockwise rotation of the reinforcement direction in relation to the corresponding x-axis.

For the model types wall 2D - XZ ($u_x/u_z/\varphi_x$) or 2D - XY ($u_x/u_y/\varphi_z$), it is not possible to create different reinforcement meshes for each side of the surface. Thus, the input options are limited to uniform reinforcement directions on both surface sides.

3.4.3 Longitudinal Reinforcement

Figure 3.32: Window 1.4 Reinforcement, tab Longitudinal Reinforcement for ultimate and serviceability limit state design

The sections of the tab depend on the designs selected in the 1.1 *General Data* window: If you want to carry out the ultimate limit state design exclusively, no specific reinforcement settings are required. You only need to decide which longitudinal reinforcement you want to use for the shear force check. For the serviceability limit state design, on the other hand, you have to specify reinforcement areas.

For more information on the reinforcement specifications for the serviceability limit state design, see chapter 2.6.3, page 72.

Provided Basic Reinforcement

For each surface side and each reinforcement direction, you can define a basic reinforcement that will be contained in all surfaces of the reinforcement group. In the according input fields, enter the *Reinforcement Area* and the *Diameter* relevant for the serviceability limit state design.

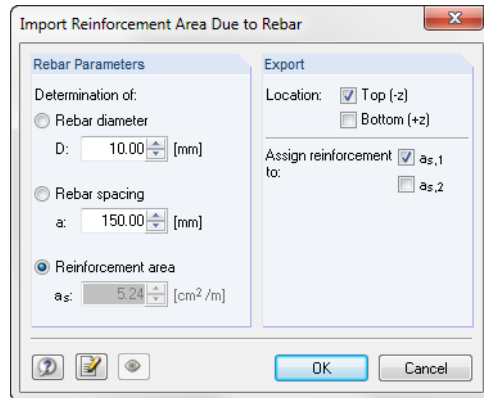
If the user-defined basic reinforcement exceeds the required reinforcement, no additional reinforcement is needed. However, large basic reinforcements are usually not applied to surfaces because this would not be efficient.

In RF-CONCRETE Surfaces, entering reinforcement areas is facilitated by libraries for rebars and mesh reinforcements. To access these libraries, use the two buttons shown on the left; they are described on the following page.





Rebars



Import Reinforcement Area Due to Rebar

Rebar Parameters

Determination of:

☐ Rebar diameter

D: 10.00 [mm]

☐ Rebar spacing

a: 150.00 [mm]

☒ Reinforcement area

a_s: 5.24 [cm²/m]

Export

Location: ☒ Top (-z)
☐ Bottom (+z)

Assign reinforcement to: ☒ a_{s,1}
☐ a_{s,2}

OK Cancel

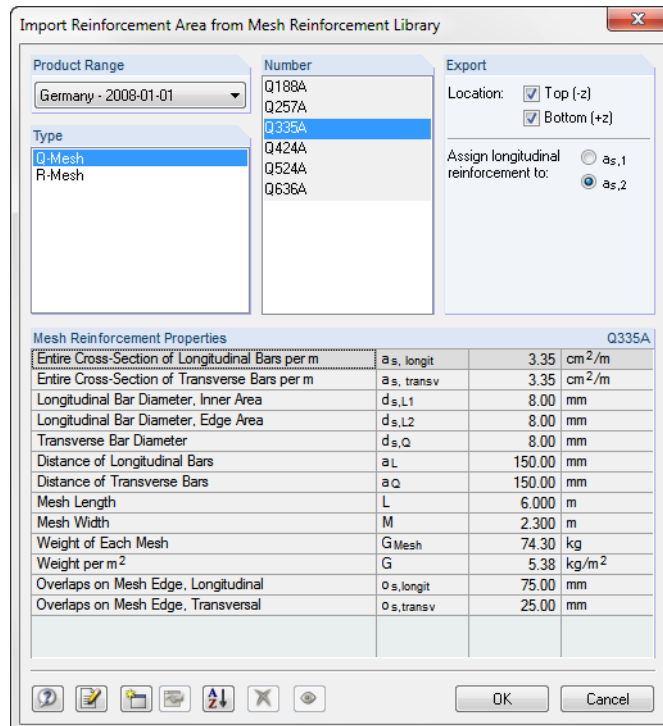
Figure 3.33: Dialog box *Import Reinforcement Area due to Rebar*

The three options in the *Rebar Parameters* section are interactive. Normally, the program determines the reinforcement area from the rebar diameter and the rebar spacing.

In the *Export* section, you decide in which input fields of the *Longitudinal Reinforcement* tab the determined reinforcement areas will be imported. The location and the reinforcement direction can be defined specifically (or generally by selecting all check boxes).



Mesh Reinforcement



Import Reinforcement Area from Mesh Reinforcement Library

Product Range

Germany - 2008-01-01

Type

☒ D-Mesh
☐ R-Mesh

Number

Q188A
Q257A
Q335A
Q424A
Q524A
Q636A

Export

Location: ☒ Top (-z)
☒ Bottom (+z)

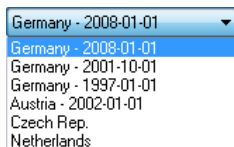
Assign longitudinal reinforcement to: ☐ a_{s,1}
☒ a_{s,2}

Mesh Reinforcement Properties

Mesh Reinforcement Properties		Q335A
Entire Cross-Section of Longitudinal Bars per m	a _{s, longit}	3.35 cm ² /m
Entire Cross-Section of Transverse Bars per m	a _{s, transv}	3.35 cm ² /m
Longitudinal Bar Diameter, Inner Area	d _{s,L1}	8.00 mm
Longitudinal Bar Diameter, Edge Area	d _{s,L2}	8.00 mm
Transverse Bar Diameter	d _{s,Q}	8.00 mm
Distance of Longitudinal Bars	a _L	150.00 mm
Distance of Transverse Bars	a _Q	150.00 mm
Mesh Length	L	6.000 m
Mesh Width	M	2.300 m
Weight of Each Mesh	G _{Mesh}	74.30 kg
Weight per m ²	G	5.38 kg/m ²
Overlaps on Mesh Edge, Longitudinal	o _{s, longit}	75.00 mm
Overlaps on Mesh Edge, Transversal	o _{s, transv}	25.00 mm

OK Cancel

Figure 3.34: Dialog box *Import Reinforcement Area from Mesh Reinforcement Library*



Germany - 2008-01-01

Germany - 2008-01-01

Germany - 2001-10-01

Germany - 1997-01-01

Austria - 2002-01-01

Czech Rep.

Netherlands

First, select the *Product Range* from the drop-down list shown on the left. Then, define the mesh *Type* and select the relevant *Number* in the section to the right. In the section below, you can check the *Mesh Reinforcement Properties*.

In the *Export* section, you decide in which input fields of the *Longitudinal Reinforcement* tab the determined reinforcement areas will be imported. The location and the reinforcement direction can be defined specifically (or generally by selecting all check boxes).

Use required reinforcement for design of serviceability

The ideal way to perform the serviceability limit state design would be the following:

1. Determine the required reinforcement by using only the load specified in the tab *Ultimate Limit State*.
2. Create a reinforcement drawing including mesh reinforcements and rebars on the bases of the colored result diagram.
3. If necessary, divide the surfaces based on the reinforcement drawing into smaller surfaces that have the same provided reinforcement area in each reinforcement direction.
4. Define the provided reinforcement area, rebar spacing, and diameter for each surface in RF-CONCRETE Surfaces.
5. Recalculate with the loads in the tab *Serviceability Limit State*.

Calculation

This procedure is cumbersome and runs contrary to the convention stating that you can determine the reinforcement and perform the serviceability limit state designs at the same time by simply using the [Calculation] button.

Therefore, you can select the check box *Use required design reinforcement for design of serviceability* to quickly obtain a provided reinforcement for the individual surfaces: The program uses the required reinforcement from the ultimate limit state design as the reinforcement to be applied. If the check box is selected, you only need to specify the rebar diameter.

Additional Reinforcement for Serviceability Limit State Design

In the areas where the statically required reinforcement exceeds the basic reinforcement, an additional reinforcement is needed. Use the drop-down list to specify which additional reinforcement is applied for the serviceability limit state design.

If you select the *Required additional reinforcement* option, the actual $A_{s,req}$ distribution is applied as additional reinforcement in the SLS design.

The *Additional reinforcement layout* is determined as difference between the greater statically required reinforcement of all surfaces of the reinforcement group and the defined basic reinforcement:

$$a_{s,add} = \max a_{s,req} - a_{s,basic}$$

Equation 3.2



Click [Info] to open the dialog box illustrating the additional reinforcement (see Figure 3.35).

To dimension the additional reinforcement, you only need to specify the rebar diameter.



You can also specify a *User-defined additional reinforcement*. For this, the program offers libraries for the rebars and mesh reinforcements, just as in the *Provided Basic Reinforcement* section.

Additional reinforcement layout
 Required additional reinforcement
 Additional reinforcement layout
 User-defined additional reinforcement

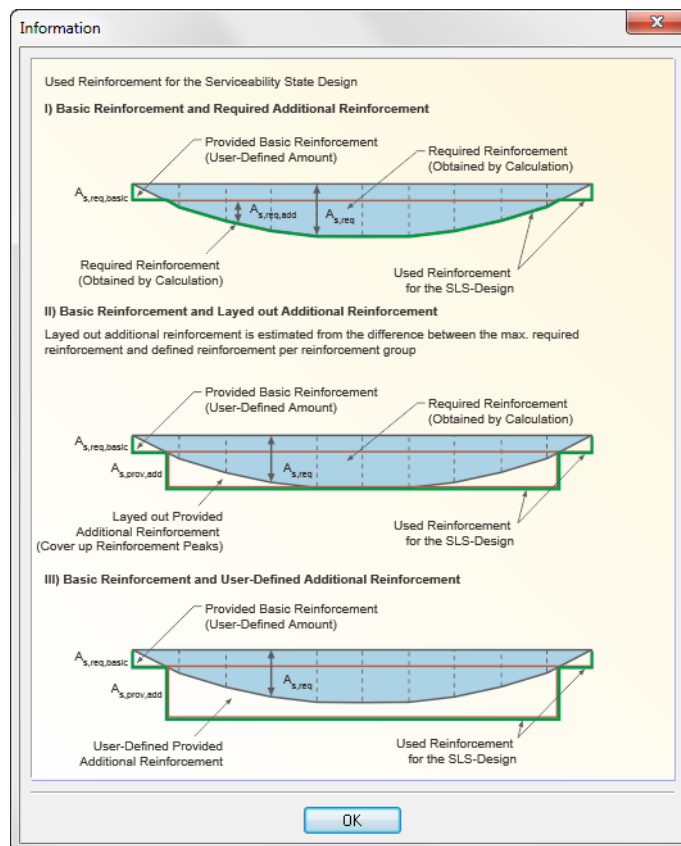


Figure 3.35: Applying additional reinforcement

Longitudinal Reinforcement for Check of Shear Resistance

The last section provides options to define the longitudinal reinforcement to be applied for the shear check without shear reinforcement.

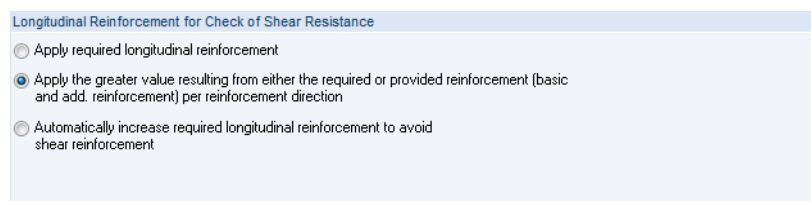


Figure 3.36: Longitudinal reinforcement for check of shear resistance

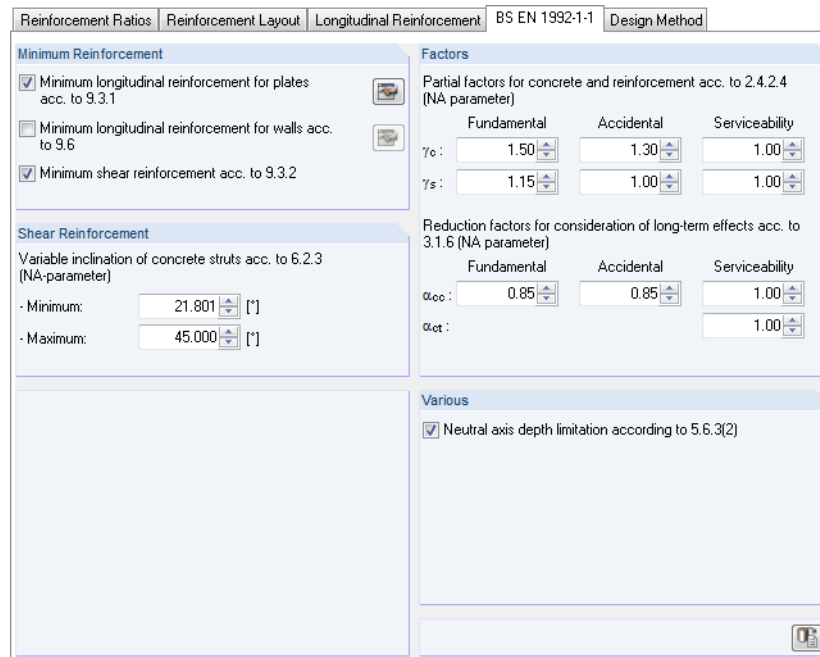
- *Apply required longitudinal reinforcement*
The check of shear resistance is carried out with the transformed provided tension reinforcement in direction of the principal shear force (see chapter 2.4.4.1, page 39).
- *Apply the greater value resulting from either the required or provided reinforcement (basic and add. reinforcement) per reinforcement direction*
For the check of shear resistance, the program uses either the statically required or the user-defined longitudinal reinforcement (see chapter 2.4.4.1, page 42).
- *Automatically increase required longitudinal reinforcement to avoid shear reinforcement*
If the required longitudinal reinforcement is not sufficient for the shear force resistance, the longitudinal reinforcement will be increased in the main shear force direction until the shear check without shear reinforcement is satisfied (see chapter 2.4.4.1, page 40).

3.4.4 Standard

The parameters of this tab depend on the standard selected in the 1.1 *General Data* window. In this tab, you specify the standard-specific reinforcement data. In the following, this is described for **EN 1992-1-1**.



At the bottom right below the table, two buttons are available. Click [Default] to reset the initial values of the current standard. Use [Set As Default] to store the defined entries as new default settings.



Reinforcement Ratios | Reinforcement Layout | Longitudinal Reinforcement | BS EN 1992-1-1 | Design Method

Minimum Reinforcement

- ☒ Minimum longitudinal reinforcement for plates acc. to 9.3.1
- ☐ Minimum longitudinal reinforcement for walls acc. to 9.6
- ☒ Minimum shear reinforcement acc. to 9.3.2

Shear Reinforcement

Variable inclination of concrete struts acc. to 6.2.3 (NA-parameter)

- Minimum: 21.801 [°]

- Maximum: 45.000 [°]

Factors

Partial factors for concrete and reinforcement acc. to 2.4.2.4 (NA parameter)

	Fundamental	Accidental	Serviceability
γ_c	1.50	1.30	1.00
γ_s	1.15	1.00	1.00

Reduction factors for consideration of long-term effects acc. to 3.1.6 (NA parameter)

	Fundamental	Accidental	Serviceability
α_{cc}	0.85	0.85	1.00
α_{ct}			1.00

Various

- ☒ Neutral axis depth limitation according to 5.6.3(2)

Figure 3.37: Window 1.4 Reinforcement, tab BS EN 1992-1-1

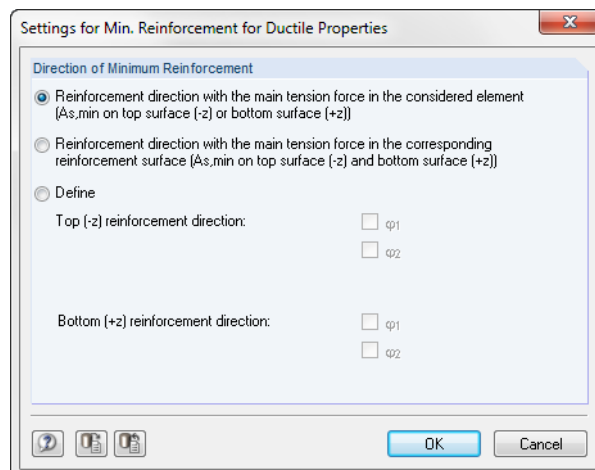
Minimum Reinforcement

In this section, you decide which provisions of the standard regarding the minimum reinforcement are to be considered in the design (see chapter 2.3.7, page 25).



For plates and walls, click [Settings] to set the direction of the minimum and main compression reinforcement.

Plates



Settings for Min. Reinforcement for Ductile Properties

Direction of Minimum Reinforcement

- ☒ Reinforcement direction with the main tension force in the considered element [As,min on top surface (-z) or bottom surface (+z)]
- ☐ Reinforcement direction with the main tension force in the corresponding reinforcement surface [As,min on top surface (-z) and bottom surface (+z)]
- ☐ Define
 - Top (-z) reinforcement direction: ☐ φ_1 ☐ φ_2
 - Bottom (+z) reinforcement direction: ☐ φ_1 ☐ φ_2

OK Cancel

Figure 3.38: Dialog box Settings for Min. Reinforcement for Ductile Properties

According to EN 1992-1-1, clause 9.3.1, the minimum reinforcement to ensure a ductile behavior of the structural component is to be placed in the main span direction of the plate. The main span direction cannot be found automatically in the determination of the reinforcement by element. However, you can control the reinforcement direction, in which you want to consider the minimum reinforcement, by using the following three options:

- *Reinforcement direction with main tension force in the considered element*
The minimum reinforcement is considered only in the reinforcement direction with the greatest tension force from all reinforcement directions of top surface (-z) and bottom surface (+z): The minimum reinforcement is placed only in one direction and on one side of the plate.
- *Reinforcement direction with main tension force in the corresponding reinforcement surface*
For each reinforcement surface, the program searches for the reinforcement direction with the greatest tension force. Then, the minimum reinforcement is determined on each surface for these directions.
- *Define*
The reinforcement direction in which you want to apply the minimum reinforcement can be specified manually.

Walls

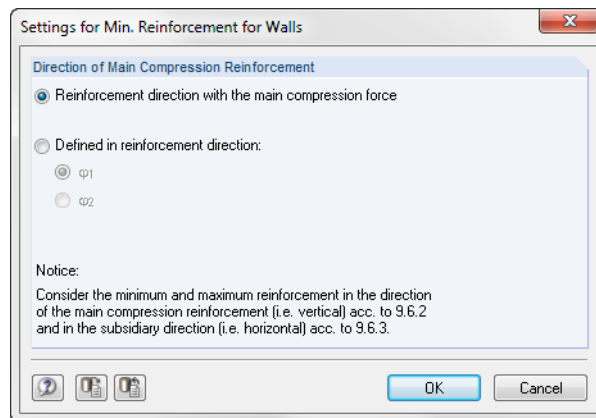


Figure 3.39: Dialog box *Settings for Min. Reinforcement for Walls*

You can specify the direction of the main compression reinforcement to determine the minimum longitudinal reinforcement for walls in the direction of the *main compression force* or *Defined*.

Shear Reinforcement

The two input fields define the allowable zone for the *inclination of concrete struts* θ . The angles are preset according to EN 1992-1-1, clause 6.2.3. If necessary, you can adjust them, but they may not lie outside the allowed limits.

Factors

The input fields above control the *Partial Safety Factors* for concrete γ_c and reinforcing steel γ_s in the design. The values for the different design situations are preset according to EN 1992-1-1, Table 2.1N.

The *reduction factors* α_{cc} or α_{ct} take account of long-term effects on the compressive or tensile strength of concrete. These coefficients are given in EN 1992-1-1, clause 3.1.6 (1) or 3.1.6 (2).

Various

Select the check box in this tab section to specify a *neutral axis depth limitation* according to EN 1992-1-1, clause 5.6.3 (2). In this case, the maximum ratio is $x_d/d = 0.45$ for concrete up to the strength class C50/60 and $x_d/d = 0.35$ for concrete from strength class C55/67.

3.4.5 Design Method



Figure 3.40: Window 1.4 Reinforcement, tab Design Method

In determining the required reinforcement, the principal internal forces are transformed into design forces (in direction of the reinforcement) and in a related concrete strut force. The sizes of these design forces depend on the presumed angle of the concrete strut that braces the reinforcement mesh.

In the load situations "tension-tension" and "tension-compression" (see Figure 2.19, page 22), the design force may become negative in one reinforcement direction for a certain inclination of the concrete strut, that is, compressive forces would occur for the tension reinforcement. Due to the optimization of design forces, the direction of the concrete strut is modified until the negative design force is zero.

In the *optimization process* of internal forces, the program determines the inclination angle of the concrete strut that produces the most favorable design result. The design moments are determined iteratively with adjusted inclination angles in order to find the smallest energy with the least required reinforcement. The optimization process is described in an example in chapter 2.4.1, page 28.



The optimization for concrete components subjected to compression (such as walls) may result in non-designable elements due to failure of the compression strut. Therefore, the optimization is not recommended for the load situation "compression-compression".

4. Calculation

Calculation

In RF-CONCRETE Surfaces, the [Calculation] is carried out with the internal forces from RFEM. If the RFEM results are not yet available, the program starts the calculation of the internal forces automatically.

4.1 Details

Details...

Click the [Details] button, which is available in the 1.1 *General Data* window, to open the corresponding dialog box where you can influence the design of result combinations, several load cases or combinations, as well as of the internal forces in the average and rip regions.

The following dialog box opens.

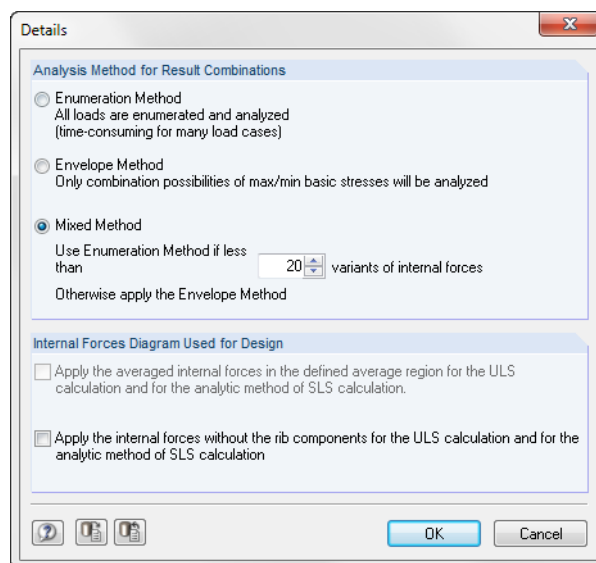


Figure 4.1: Dialog box *Details*

Analysis Method for Result Combinations

This section controls how the design internal forces of the result combinations are included in the calculation. This specification also applies when there are several load cases or load combinations in the design case to be analyzed. The *Mixed Method* is preset: Before the design, the program analyzes whether the *Enumeration Method* or the *Envelope Method* needs less computation time.

Enumeration Method

Each load case and each load combination selected in the 1.1 *General Data* window is designed individually. From the results, the reinforcement envelope is computed. For result combinations, 16 calculations are performed for the RFEM extreme values of the basic internal forces $\max/\min m_x$, $\max/\min n_x$, $\max/\min m_y$, $\max/\min n_y$, $\max/\min m_{xy}$, $\max/\min n_{xy}$, $\max/\min v_x$, and $\max/\min v_y$.

The *Enumeration Method* is very precise because every combination is calculated separately, and then the enveloping reinforcement is determined. However, the disadvantage of this method is that the number of the combinations to be analyzed increases exponentially with the number of load cases, as the program proceeds from row to row. Thus, if there are, for example, 50 selected load combinations, there will also be 50 reinforcement designs. On the other hand, all possible variants (constellations) are included in the designs.

Envelope Method

From the load cases, load combinations, and result combinations selected in the 1.1 *General Data* window, the module computes an internal forces envelope. 16 extreme value variants are analyzed. The difference to the output of extreme values of result combinations in RFEM is the following: The module analyzes not only the variants of the extreme values that are based on the maximum basic internal forces but also on their interaction (for example $m_x + m_{xy}$). With this envelope from 16 variants of extreme values, the determination of the reinforcement is started. Thus, 16 calculation runs are carried out to determine the reinforcement. Even if there are a great number of load cases, load combinations, or result combinations, the computing time is still adequate.

Since the internal forces envelope is computed with 16 extreme values, it is possible that the most unfavorable variants are not considered – unlike in the *Enumeration Method* where the load cases are computed row by row. Combinations with load cases, whose directions of action are orthogonal, are regarded as critical. In this case, a check calculation according to the *Enumeration Method* is recommended.

Mixed Method

Before the design, the module analyzes how many designs are to be performed with the load cases, load combinations, and result combinations selected in the 1.1 *General Data* window for each limit state. As mentioned in the description of the *Enumeration Method*, the module performs a separate design for each load case or each load combination. For one result combination, 16 calculations are required for the extreme value variants of the basic internal forces. If, for example, you select one result combination and 5 load combinations for the design, the program needs $16 + 5 = 21$ calculation runs. The number is greater than the preset 20 variants of internal forces. Hence, the design is carried out with the *Envelope Method*.

In the input field, you can specify the upper limit of the variants that are designed according to the precise *Enumeration Method*.

Thus, the *Mixed Method* is a compromise between precision of results and computation time.

Internal Forces Diagram Used for Design

Apply Averaged Internal Forces

Usually, the averaged internal forces from RFEM are used for the design: RF-CONCRETE Surfaces transforms the moments and axial forces in the directions of the longitudinal reinforcement and then performs the checks (see chapter 2.5.1, page 48).

If you select this check box, the design is carried out with the internal forces that are available in the average regions of RFEM. By means of the averaged results, you can reduce singularities and consider local redistribution effects in the model (see Figure 4.2).

The average regions are described in the RFEM manual, chapter 9.7.3.

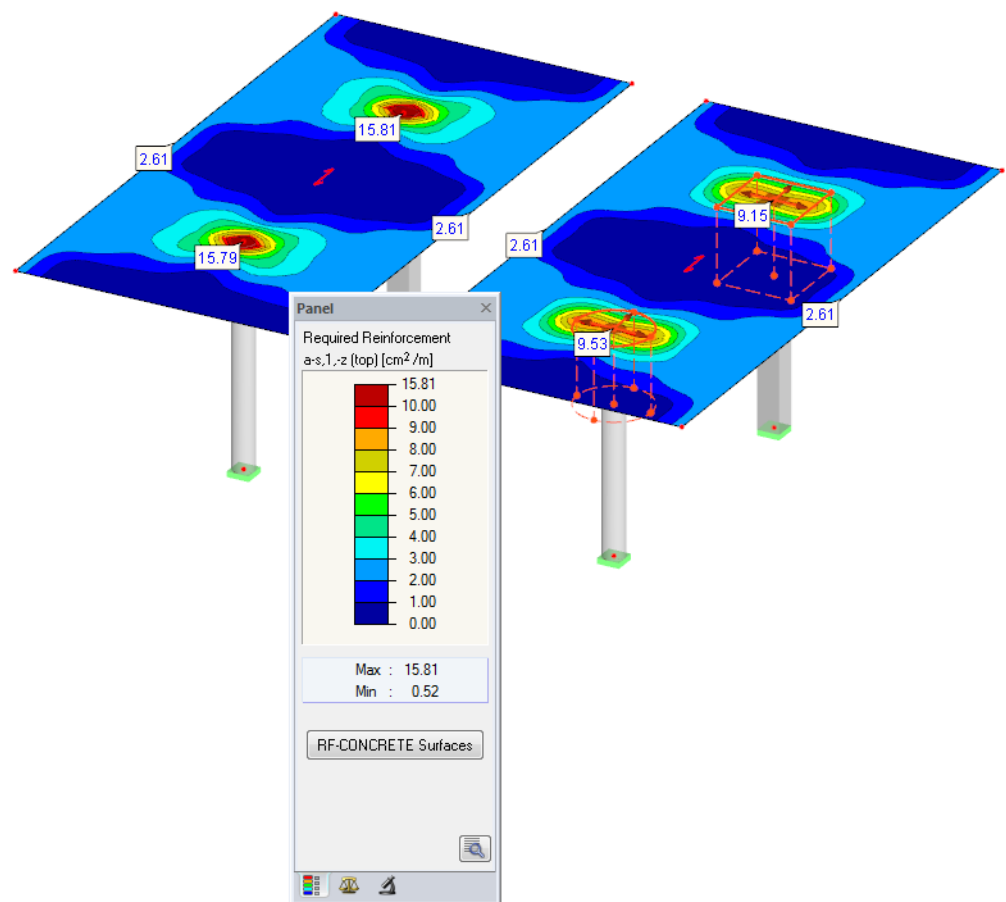


Figure 4.2: Top reinforcement for unaveraged internal forces (left) and average regions across columns (right)

Apply Internal Forces Without Rib Components

In RFEM, you can model a T-beam by using a surface and an eccentrically connected member of the "rib" type. The internal forces of the T-beam from surface component and member are determined by integration of the internal forces in surfaces as rib internal forces.

With the final check box, you decide whether the surface internal forces assigned to the rib are included in the surface design. The design with the rib component is preset.

4.2 Check

Check

Before you start the calculation, it is recommended to check if the input data is correct. The [Check] button is available in every input window of RF-CONCRETE Surfaces.

The program checks if the data required for the design is complete and if the references of the data sets are alright. If the program does not detect any input errors, it displays the following message:



Figure 4.3: Plausibility check

4.3 Start Calculation

Calculation

You can start the calculation out of each input window of the RF-CONCRETE Surfaces module by clicking [Calculation].

RF-CONCRETE Surfaces searches for the results of the load cases, load combinations, and result combinations that you want to design. If they cannot be found, the program starts the RFEM calculation to determine the design-relevant internal forces.

You can also start the calculation out of the RFEM user interface: The *To Calculate* dialog box (menu *Calculate* → *To Calculate*) lists the design cases of the add-on modules like load cases or load combinations.

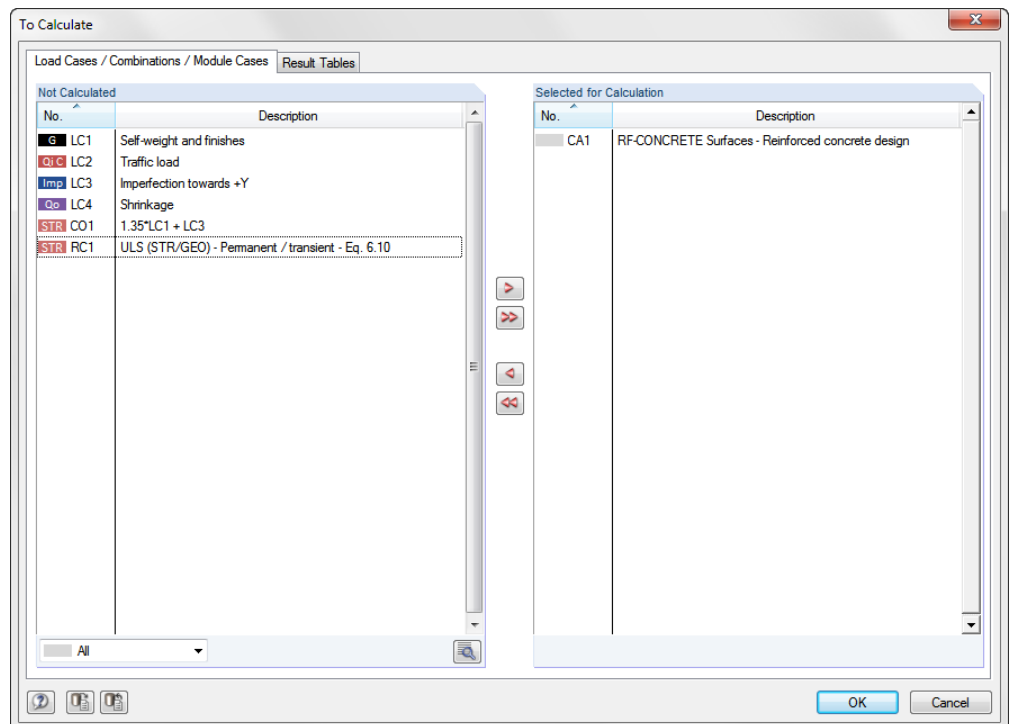


Figure 4.4: Dialog box *To Calculate*

If the design cases of RF-CONCRETE Surfaces are missing in the *Not Calculated* section, select *All* or *Add-on Modules* from the drop-down list below the section.

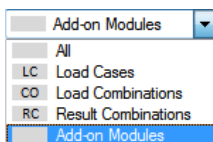
To transfer the selected RF-CONCRETE Surfaces cases to the dialog box section on the right, click [►]. To start the calculation, click [OK].

Alternatively, you can use the drop-down list in the toolbar to calculate a design case: Select the RF-CONCRETE Surfaces case, and then click [Show Results].



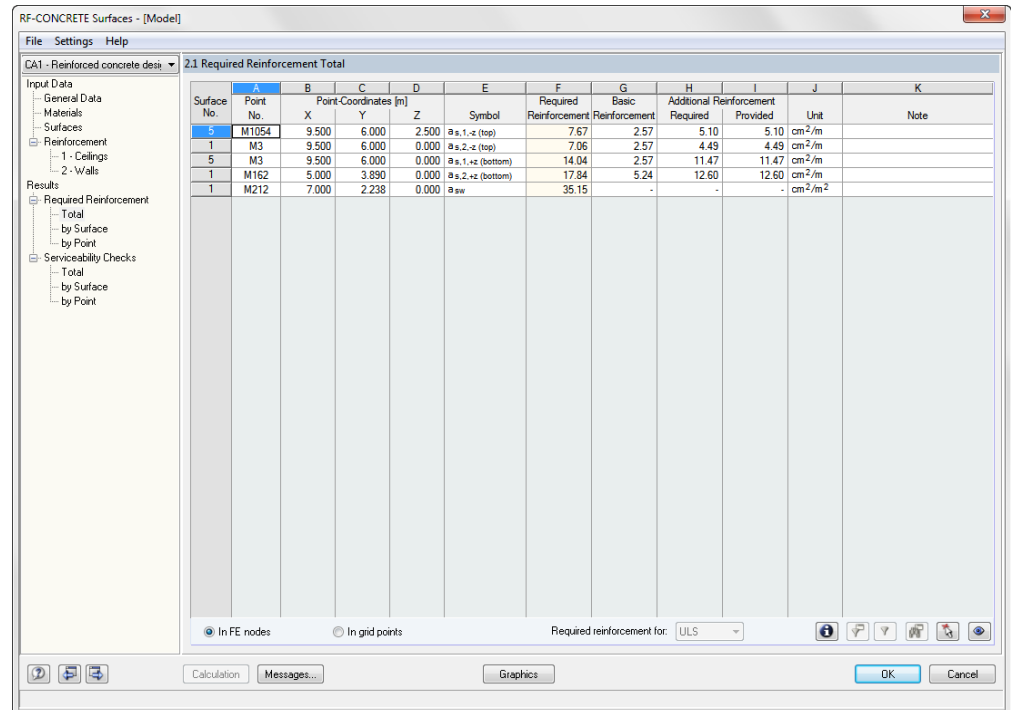
Figure 4.5: Direct calculation of an RF-CONCRETE Surfaces design case in RFEM

Subsequently, you can observe the calculation process in the solver dialog box.



5. Results

The 2.1 *Required Reinforcement Total* window is displayed immediately after the calculation.



Surface No.	Point No.	X	Y	Z	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Additional Reinforcement Provided	Unit	Note
5	M1054	9.500	6.000	2.500	as.1-z (top)	7.67	2.57	5.10	5.10	cm ² /m	
1	M3	9.500	6.000	0.000	as.2-z (top)	7.06	2.57	4.49	4.49	cm ² /m	
5	M3	9.500	6.000	0.000	as.1-z (bottom)	14.04	2.57	11.47	11.47	cm ² /m	
1	M162	5.000	3.890	0.000	as.2-z (bottom)	17.84	5.24	12.60	12.60	cm ² /m	
1	M212	7.000	2.238	0.000	asw	35.15	-	-	-	cm ² /m ²	

Figure 5.1: Results window

The ultimate limit state designs are listed in the results windows 2.1 through 2.3 by various criteria.

The windows 3.1 through 3.3 contain the results of the serviceability limit state designs.

All windows can be selected directly by clicking the according entry in the navigator. Use the buttons shown on the left to go to the previous or next window. You can also use the function keys [F2] and [F3] to select the previous or next window.

At the bottom of the windows, you find two option buttons. With these buttons, you decide whether to show the results data *In FE nodes* or *In grid points*. The results of the FE nodes are determined directly by the analysis core. The grid point results are determined by interpolation of the FE node results.

To save the results, click [OK]. Thus, you exit RF-CONCRETE Surfaces and return to the main program.

Chapter 5 *Results* presents the results windows in their proper order. Evaluating and checking results is described in chapter 6 *Results Evaluation*, page 176.



☒ In FE nodes
 ☐ In grid points

OK

5.1 Required Reinforcement Total

The table shows the maximum reinforcement areas of all analyzed surfaces. The areas are determined from the internal forces of the load cases, load combinations, and result combinations selected for the ultimate limit state design.

2.1 Required Reinforcement Total

Surface No.	A Point No.	B Point-Coordinates [m] X	C Y	D Z	E Symbol	F Required Reinforcement	G Basic Reinforcement	H Additional Reinforcement Required	I Reinforcement Provided	J Unit	K Note
5	M1054	9.500	6.000	2.500	a _{s,1,-z} (top)	7.67	2.57	5.10	5.10	cm ² /m	
1	M3	9.500	6.000	0.000	a _{s,2,-z} (top)	7.06	2.57	4.49	4.49	cm ² /m	
5	M3	9.500	6.000	0.000	a _{s,1,+z} (bottom)	14.04	2.57	11.47	11.47	cm ² /m	
1	M162	5.000	3.890	0.000	a _{s,2,+z} (bottom)	17.84	5.24	12.60	12.60	cm ² /m	
1	M212	7.000	2.238	0.000	a _{sw}	35.15	-	-	-	cm ² /m ²	

☒ In FE nodes
 ☐ In grid points
 Required reinforcement for: ULS

Figure 5.2: Window 2.1 Required Reinforcement Total

Surface No.

The column shows the numbers of surfaces containing the governing points.

Point No. / Grid Point

In these FE nodes or grid points, the greatest required reinforcement was determined for each position and direction. The reinforcement type is displayed in column E *Symbol*.

The FE mesh nodes *M* are generated automatically. In contrast to this, the grid points *G* can be controlled in RFEM, as user-defined result grids are possible for surfaces. The function is described in chapter 8.12 of the RFEM manual.

Point Coordinates X/Y/Z

The three columns show the coordinates of the governing FE nodes or grid points.

Symbol

Column E displays the reinforcement type. For the four (or six) longitudinal reinforcements, the module shows the directions (1, 2 and 3, if available) as well as the surface sides (*top* and *bottom*).

The reinforcement directions are defined in the *Reinforcement Layout* tab of the 1.4 *Reinforcement* window (see chapter 3.4.2, page 149).

The top reinforcement is defined on the surface side in direction of the negative local surface axis *z* (-*z*). Accordingly, the top reinforcement is defined in direction of the positive *z*-axis (+*z*). The Figure 3.29 on page 150 shows the axis systems of the surfaces.

The shear reinforcement is indicated as *a_{sw}*.



Required Reinforcement

This column displays the reinforcement areas that are required for the ultimate limit state design.

Basic Reinforcement

This column shows the user-defined basic reinforcement defined in the *Longitudinal Reinforcement* tab of the 1.4 *Reinforcement* window (see chapter 3.4.3, page 153).

Additional Reinforcement

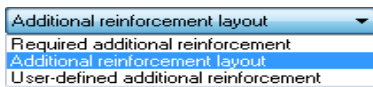
If you design the ultimate limit state exclusively, the column *Required* displays the difference between required reinforcement (column F) and provided basic reinforcement (column G).

If you design the serviceability limit state additionally, you see here the reinforcement areas that are required by the specifications in the *Longitudinal Reinforcement* tab of 1.4 *Reinforcement* module window (see chapter 3.4.3, page 153) to satisfy the serviceability limit state designs. The *Provided* column shows the reinforcement that is available as additional reinforcement for the serviceability limit state design according to the specification in the *Longitudinal Reinforcement* tab of 1.4 *Reinforcement* window.

Note

The final column indicates non-designable situations or notes referring to design issues. The numbers are explained in the status bar.

The button shown on the left allows you to view all [Messages] of the current design case. A dialog box appears showing the relevant messages.



Messages...

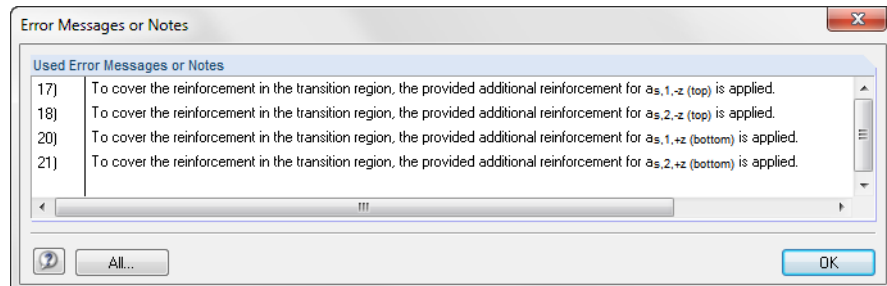


Figure 5.3: Dialog box *Error Messages or Notes*



The buttons are described in chapter 6 *Results Evaluation* on page 176.

5.2 Required Reinforcement by Surface

2.2 Required Reinforcement by Surface

Surface No.	A Point No.	B Point-Coordinates [m] X	C Y	D Z	E Symbol	F Required Reinforcement	G Basic Reinforcement	H Additional Reinforcement Required	I Reinforcement Provided	J Unit	K Note
1	M20	6.000	6.000	0.000	a _{s,1,-z} (top)	5.72	2.57	3.15	3.15	cm ² /m	
	M3	9.500	6.000	0.000	a _{s,2,-z} (top)	7.06	2.57	4.49	4.49	cm ² /m	
	M169	5.110	4.000	0.000	a _{s,1,+z} (bottom)	10.69	5.24	5.45	5.45	cm ² /m	
	M162	5.000	3.890	0.000	a _{s,2,+z} (bottom)	17.84	5.24	12.60	12.60	cm ² /m ²	
	M212	7.000	2.238	0.000	a _{sw}	35.15	-	-	-	cm ² /m ²	
2	M678	9.926	5.785	0.000	a _{s,1,-z} (top)	5.72	2.57	3.15	3.15	cm ² /m	
	M678	9.926	5.785	0.000	a _{s,2,-z} (top)	6.54	2.57	3.97	4.49	cm ² /m	
	M3	9.500	6.000	0.000	a _{s,1,+z} (bottom)	2.00	5.24	0.00	0.00	cm ² /m	
	M74	9.500	4.500	0.000	a _{s,2,+z} (bottom)	2.45	5.24	0.00	0.00	cm ² /m	
	M3	9.500	6.000	0.000	a _{sw}	13.66	-	-	-	cm ² /m ²	
3	M10	0.000	2.500	2.000	a _{s,1,-z} (top)	2.51	2.57	0.00	0.00	cm ² /m	
	M748	0.000	3.000	0.000	a _{s,2,-z} (top)	3.59	2.57	1.02	2.91	cm ² /m	
	M717	0.000	0.000	0.500	a _{s,1,+z} (bottom)	2.53	2.57	0.00	0.00	cm ² /m	
	M717	0.000	0.000	0.500	a _{s,2,+z} (bottom)	0.88	2.57	0.00	0.00	cm ² /m	
	M1	0.000	0.000	0.000	a _{sw}	0.00	-	-	-	cm ² /m ²	
4	M17	9.500	0.000	4.000	a _{s,1,-z} (top)	2.51	2.57	0.00	0.00	cm ² /m	
	M4	9.500	0.000	0.000	a _{s,2,-z} (top)	5.48	2.57	2.91	2.91	cm ² /m	
	M821	9.500	0.000	0.500	a _{s,1,+z} (bottom)	3.84	2.57	1.27	11.47	cm ² /m	
	M878	8.500	0.000	3.500	a _{s,2,+z} (bottom)	2.51	2.57	0.00	0.00	cm ² /m	
	M1	0.000	0.000	0.000	a _{sw}	0.00	-	-	-	cm ² /m ²	
5	M1054	9.500	6.000	2.500	a _{s,1,-z} (top)	7.67	2.57	5.10	5.10	cm ² /m	
	M821	9.500	0.000	0.500	a _{s,2,-z} (top)	3.38	2.57	0.81	2.91	cm ² /m	
	M3	9.500	6.000	0.000	a _{s,1,+z} (bottom)	14.04	2.57	11.47	11.47	cm ² /m	
	M3	9.500	6.000	0.000	a _{s,2,+z} (bottom)	3.96	2.57	1.39	1.39	cm ² /m	
	M3	9.500	6.000	0.000	a _{sw}	8.78	-	-	-	cm ² /m ²	

☒ In FE nodes
 ☐ In grid points
 Required reinforcement for: ULS

Figure 5.4: Window 2.2 Required Reinforcement by Surface

This window shows the maximum reinforcement areas that are required for each of the designed surfaces. The columns are described in chapter 5.1.

5.3 Required Reinforcement by Point

2.3 Required Reinforcement by Point

Surface No.	Point No.	Point-Coordinates [m]	Symbol	Required Reinforcement	Basic Reinforcement	Additional Reinforcement Required	Reinforcement Provided	Unit	Note
		X Y Z							
1	M1	0.000 0.000 0.000	$a_{s,1,-z}$ (top)	1.00	2.57	0.00	3.15	cm ² /m	17)
			$a_{s,2,-z}$ (top)	2.00	2.57	0.00	4.49	cm ² /m	18)
			$a_{s,1,+z}$ (bottom)	2.00	5.24	0.00	0.00	cm ² /m	
			$a_{s,2,+z}$ (bottom)	2.93	5.24	0.00	0.00	cm ² /m	
			a_{sw}	0.00	-	-	-	cm ² /m ²	
			$n_{1,-z}$ (top)	-27.337	-	-	-	kN/m	
			$n_{2,-z}$ (top)	-28.473	-	-	-	kN/m	
			$n_{1,+z}$ (bottom)	13.115	-	-	-	kN/m	
			$n_{2,+z}$ (bottom)	13.792	-	-	-	kN/m	
			V_{Ed}	46.858	-	-	-	kN/m	
			$V_{Rd,c}$	102.692	-	-	-	kN/m	
			$V_{Rd,max}$	627.427	-	-	-	kN/m	
			$V_{Rd,s}$	0.000	-	-	-	kN/m	
			Theta	21.801	-	-	-	°	
1	M2	0.000 6.000 0.000	$a_{s,1,-z}$ (top)	2.93	2.57	0.36	3.15	cm ² /m	
			$a_{s,2,-z}$ (top)	2.00	2.57	0.00	0.00	cm ² /m	
			$a_{s,1,+z}$ (bottom)	2.96	5.24	0.00	0.00	cm ² /m	
			$a_{s,2,+z}$ (bottom)	2.00	5.24	0.00	0.00	cm ² /m	
			a_{sw}	0.00	-	-	-	cm ² /m ²	
			$n_{1,-z}$ (top)	40.556	-	-	-	kN/m	
			$n_{2,-z}$ (top)	35.016	-	-	-	kN/m	
			$n_{1,+z}$ (bottom)	70.696	-	-	-	kN/m	
			$n_{2,+z}$ (bottom)	56.699	-	-	-	kN/m	
			V_{Ed}	62.876	-	-	-	kN/m	
			$V_{Rd,c}$	93.442	-	-	-	kN/m	
			$V_{Rd,max}$	919.695	-	-	-	kN/m	
			$V_{Rd,s}$	0.000	-	-	-	kN/m	
			Theta	45.000	-	-	-	°	
1	M3	9.500 6.000 0.000	$a_{s,1,-z}$ (top)	3.75	2.57	1.18	3.15	cm ² /m	
			$a_{s,2,-z}$ (top)	7.06	2.57	4.49	4.49	cm ² /m	
			$a_{s,1,+z}$ (bottom)	4.97	5.24	0.00	0.00	cm ² /m	
			$a_{s,2,+z}$ (bottom)	2.00	5.24	0.00	0.00	cm ² /m	
			a_{sw}	0.00	-	-	-	cm ² /m ²	
			$n_{1,-z}$ (top)	170.290	-	-	-	kN/m	

☒ In FE nodes
 ☐ In grid points
 Required reinforcement for: ULS

Figure 5.5: Window 2.3 Required Reinforcement by Point

☒ In FE nodes
☐ In grid points

This results window lists the maximum reinforcement areas for all FE nodes or grid points of each surface. The columns are described in detail in chapter 5.1.

In addition to the longitudinal and shear reinforcements, the table display design-relevant values of the actions and resistances. For EN 1992-1-1, these are the following:

Symbol	Meaning
$n_{1,-z}$ (top)	Axial force or membrane force for design of reinforcement in first reinforcement direction on top side of the surface
$n_{2,-z}$ (top)	Axial force or membrane force for design of reinforcement in second reinforcement direction on top side of the surface
$n_{1,+z}$ (bottom)	Same as $n_{1,-z}$ (top), but on bottom side of the surface
$n_{2,+z}$ (bottom)	Same as $n_{2,-z}$ (top), but on bottom side of the surface
$m_{1,-z}$ (top) $m_{2,-z}$ (top)	Only for type of model 2D - XY ($u_z/\varphi_x/\varphi_y$): moment for design of reinforcement in first or second reinforcement direction on top side of the surface
$m_{1,-z}$ (bottom) $m_{2,-z}$ (bottom)	Same as $m_{1,-z}$ (top) / $m_{2,-z}$ (top), but on bottom side of the surface
V_{Ed}	Design value of applied shear force
$V_{Rd,c}$	Design shear resistance without shear reinforcement
$V_{Rd,max}$	Design shear resistance of concrete strut
$V_{Rd,s}$	Design shear resistance of shear reinforcement
Theta	Inclination angle of concrete strut θ

Table 5.1: Output values in window 2.3 for EN 1992-1-1



The search function, which you can start by clicking the button shown on the left, helps you find specific FE nodes or grid points quickly (see Figure 6.7, page 182).

5.4 Serviceability Checks Total

The upper part of the window shows a summary of the governing serviceability limit state designs. The lower part displays the intermediate results of the current FE node or grid point (that is, of the point selected in the upper table) including all design-relevant parameters. You can expand or reduce the entries by clicking [+] or [-], respectively.

3.1 Serviceability Checks Total

Surface No.	Point No.	Point-Coordinates [m]			Loading	Symbol	Exist. Value	Design Limit Value	Unit	Ratio	Note
		X	Y	Z							
5	M3	9.500	6.000	0.000	RC2	σ_c	-4.98	-13.50	N/mm ²	0.37	
5	M3	9.500	6.000	0.000	RC2	σ_s	322.85	400.00	N/mm ²	0.81	
1	M1	0.000	0.000	0.000	RC2	$\sigma_{s,min}$	3.35	3.85	cm ² /m	1.16	207)
5	M3	9.500	6.000	0.000	RC2	$\lim d_s$	10.0	9.9	mm	1.02	211)
5	M3	9.500	6.000	0.000	RC2	$\lim s_1$	0.095	0.096	m	0.99	
5	M3	9.500	6.000	0.000	RC2	w_k	0.282	0.300	mm	0.95	

☒ In FE nodes ☐ In grid points Max: 1.16 > 1

Intermediate Results - Surface No. 5 - FE Mesh Point No. 3

☐ Check Concrete Compressive Stress

☐ Bottom surface (+z)

Concrete cracks. Longitudinal reinforcement is activated.

☐ Concrete compressive stress in strut direction on opposite

$\sigma_{c,strut}$	-1.53	N/mm ²
--------------------	-------	-------------------

Governing Strut Force

$n_{pressure,+z}$	-160.68	kNm/m
-------------------	---------	-------

Area of Concrete

A_c	1050.00	cm ²
-------	---------	-----------------

☐ Top surface (z)

Concrete does not crack on this side.

☐ Determination of concrete compressive stress in particular reinforcement directions

☐ Bottom surface (+z)

Concrete Compressive Stress in Direction 1

$\sigma_{c,+z,\phi 1}$	0.00	N/mm ²
------------------------	------	-------------------

Concrete Compressive Stress in Direction 2

$\sigma_{c,+z,\phi 2}$	0.00	N/mm ²
------------------------	------	-------------------

☐ Top surface (z)

Concrete Compressive Stress in Direction 1

$\sigma_{c,z,\phi 1}$	-4.98	N/mm ²
-----------------------	-------	-------------------

Concrete Compressive Stress in Direction 2

$\sigma_{c,z,\phi 2}$	0.00	N/mm ²
-----------------------	------	-------------------

Maximum Concrete Compressive Stress

$\max \sigma_c$	-4.98	N/mm ²
-----------------	-------	-------------------

☐ Allowable Concrete Compressive Stress

Parameter in National Annex

k_2	0.450	
-------	-------	--

Characteristic Concrete Compressive Strength

f_{ck}	30.00	N/mm ²
----------	-------	-------------------

Allowable Concrete Compressive Stress

$\text{perm } \sigma_c$	-13.50	N/mm ²
-------------------------	--------	-------------------

☐ Check

Maximum Concrete Compressive Stress

$\max \sigma_c$	-4.98	N/mm ²
-----------------	-------	-------------------

Figure 5.6: Window 3.1 *Serviceability Checks Total*

Figure 5.6 shows the results window of an analytical serviceability limit state check. Chapter 5.7 on page 173 describes the results windows that appear when a nonlinear serviceability limit state calculation was carried out.

The method of check is defined in the *Serviceability Limit State* tab of the 1.1 *General Data* window (see Figure 3.8, page 131).

Surface No.

The column shows the numbers of surfaces containing the governing points.

Point No. / Grid Point

These FE nodes or grid points provide the maximum ratios for the required checks. The type of check is displayed in column F *Symbol*.

The FE mesh nodes *M* are generated automatically. The grid points *G* can be controlled in RFEM (see chapter 8.12 of the RFEM manual).

Point Coordinates X/Y/Z

The three columns show the coordinates of the governing FE nodes or grid points.

☒ In FE nodes ☐ In grid points

Loading

Column E displays the load cases, load or result combinations whose internal forces produce the greatest ratios for the respective serviceability limit state design.

Symbol

Column F shows the type of the serviceability limit state design. If you have selected the analytical method, this column will show up to six types of check. These types are described by an example in chapter 2.6.4, page 77.

The symbols stand for the following types of checks:

Symbol	Design SLS
σ_c	Limitation of concrete compressive stress (→ chapter 2.6.4.7, page 77) according to specifications in the 1.3 <i>Surfaces</i> window (see Figure 3.20, page 144)
σ_s	Limitation of reinforcing steel stress (→ chapter 2.6.4.8, page 80) according to specifications in the 1.3 <i>Surfaces</i> window (see Figure 3.19, page 141)
$a_{s,min}$	Minimum reinforcement for crack width limitation (→ chapter 2.6.4.9, page 81) according to specifications in the 1.3 <i>Surfaces</i> window (see Figure 2.97, page 82)
lim d_s	Limitation of rebar diameter (→ chapter 2.6.4.10, page 84) according to specifications in the 1.4 <i>Surfaces</i> window (see Figure 3.32, page 153)
lim s_l	Limitation of rebar spacing (→ chapter 2.6.4.11, page 86) according to specifications in the 1.4 <i>Surfaces</i> window (see Figure 3.32, page 153)
w_k	Limitation of crack width (→ chapter 2.6.4.12, page 87) according to specifications in the 1.3 <i>Surfaces</i> window (see Figure 3.19, page 141)

Table 5.2: Serviceability limit state designs according to analytical method

Existing Value

This column displays the values of all surfaces that are governing for the serviceability limit state designs.

Limit Value

The design limit values are determined from the standard specifications and the load situation. The determination of the limit values is described in chapter 2.6.4, page 77f.

Ratio

Column J shows the ratio of the existing value (column G) and the limit value (column H). Ratios greater than 1 mean that the design criterion is not satisfied. The length of the colored scale illustrates the respective ratio graphically.

For the serviceability limit state designs, not all types of checks must be satisfied (see explanation in Figure 3.10, page 133).

Note

The final column indicates non-designable situations or notes referring to design issues. The numbers are explained in the status bar.

To display all messages of the currently selected design case, use the [Messages] button shown on the left. A dialog box showing the relevant messages appears (see Figure 5.3, page 166).

The buttons are described in chapter 6 *Results Evaluation*, page 176.

Max: 0.85 ≤ 1 



Messages...



5.5 Serviceability Checks by Surface

3.2 Serviceability Checks by Surface

Surface No.	Point No.	Point-Coordinates [m]	X	Y	Z	Loading	Symbol	Exist. Value	Limit Value	Unit	Ratio	Note
1	M195	7.000	3.890	0.000	RC2	σ_c	-3.57	-13.50	N/mm ²		0.27	
	M162	5.000	3.890	0.000	RC2	σ_s	322.60	400.00	N/mm ²		0.81	
	M1	0.000	0.000	0.000	RC2	$a_{s,min}$	3.35	3.85	cm ² /m		1.16	207)
	M162	5.000	3.890	0.000	RC2	$lim\ d_s$	9.3	9.9	mm		0.95	
	M6	5.000	4.000	0.000	RC2	$lim\ s_i$	0.110	0.117	m		0.94	
2	M162	5.000	3.890	0.000	RC2	w_k	0.241	0.300	mm		0.81	
	M3	9.500	6.000	0.000	RC2	σ_c	-2.59	-13.50	N/mm ²		0.20	
	M3	9.500	6.000	0.000	RC2	σ_s	0.00	400.00	N/mm ²		0.00	226)
	M3	9.500	6.000	0.000	RC2	$a_{s,min}$	3.35	3.85	cm ² /m		1.16	207) 208)
	M3	9.500	6.000	0.000	RC2	$lim\ d_s$	8.8	0.0	mm		0.00	226)

☒ In FE nodes
☐ In grid points
Max: 1.16 > 1

Intermediate Results - Surface No. 1 - FE Mesh Point No. 195

☒ Determination of minimum reinforcement

☒ Bottom surface (+z)

Minimum Reinforcement into Direction 1	$a_{s,min,+z,1}$	3.85	cm ² /m
Minimum Reinforcement into Direction 2	$a_{s,min,+z,2}$	4.16	cm ² /m

☒ Top surface (-z)

Minimum Reinforcement into Direction 1	$a_{s,min,-z,1}$	4.04	cm ² /m
Minimum Reinforcement into Direction 2	$a_{s,min,-z,2}$	3.97	cm ² /m

☒ Check

Minimum Reinforcement at the Bottom (+z) Surface in Direct	$a_{s,min,+z,1}$	3.85	cm ² /m
Existing Reinforcement at the Bottom (+z) Surface in Direct	$a_{s,exist,+z,1}$	3.35	cm ² /m
Criterion of Check	Criterion	1.150	

Figure 5.7: Window 3.2 Serviceability Checks by Surface

This window lists the maximum ratios of each designed surface resulting for the serviceability limit state designs. The columns are described in detail in chapter 5.4.

5.6 Serviceability Checks by Point

3.3 Serviceability Checks by Point

Surface No.	A Point No.	B Point-Coordinates [m] X	C Y	D Z	E Loading	F Symbol	G Exist. Value	H Limit Value	I Unit	J Ratio	K Note
1	M6	5.000	4.000	0.000	RC2	σ_c	-2.14	-13.50	N/mm ²	0.16	
					RC2	σ_s	318.26	400.00	N/mm ²	0.80	
					RC2	$\sigma_{s,min}$	3.35	3.85	cm ² /m	1.16	209) 210)
					RC2	lim d_s	9.3	10.1	mm	0.92	
					RC2	lim s_l	0.110	0.117	m	0.94	
					RC2	w_k	0.237	0.300	mm	0.79	
1	M7	7.000	4.000	0.000	RC2	σ_c	-2.00	-13.50	N/mm ²	0.15	
					RC2	σ_s	318.73	400.00	N/mm ²	0.80	
					RC2	$\sigma_{s,min}$	3.35	3.85	cm ² /m	1.16	209) 210)
					RC2	lim d_s	9.3	10.1	mm	0.93	

☒ In FE nodes ☐ In grid points Max: 1.16 > 1

Intermediate Results - Surface No. 1 - FE Mesh Point No. 6

☒ Determination of calculated value of crack width

	Ratio	
☒ Bottom surface (+z)	0.789	
Calculated Value of Crack Width into Reinforcement Direction 1	$w_{k,+z,\phi 1}$	0.237 mm
Calculated Value of Crack Width into Reinforcement Direction 2	$w_{k,+z,\phi 2}$	0.236 mm
Crack width into the resulting difference in the main strain	$w_{k,+z,res}$	0.233 mm
☒ Top surface (-z)	0.000	
Concrete does not crack on this side.		

☒ Check

Crack Width at the Bottom (+z) Surface in Direction 1	$w_{k,+z,\phi 1}$	0.237 mm
Maximum allowable crack width at the bottom (+z) surface acc. to user input	$w_{k,+z,limit}$	0.300 mm
Criterion of Check	Criterion	0.789

Figure 5.8: Window 3.3 Serviceability Checks by Point

☒ In FE nodes ☐ In grid points


This results window lists the maximum ratios for all FE nodes or grid points of each surface. The columns are described in detail in chapter 5.4.

The search function, which you can start by clicking the button shown on the left, helps you find specific FE nodes or grid points quickly (see Figure 6.7, page 182).

5.7 Nonlinear Calculation Total

The upper part of the window presents a summary of the governing serviceability limit state designs. The lower part displays the intermediate results of the current FE nodes or grid point (that is, the entry selected above) including all design-relevant parameters. You can expand or reduce the entries by clicking [+] or [-], respectively.

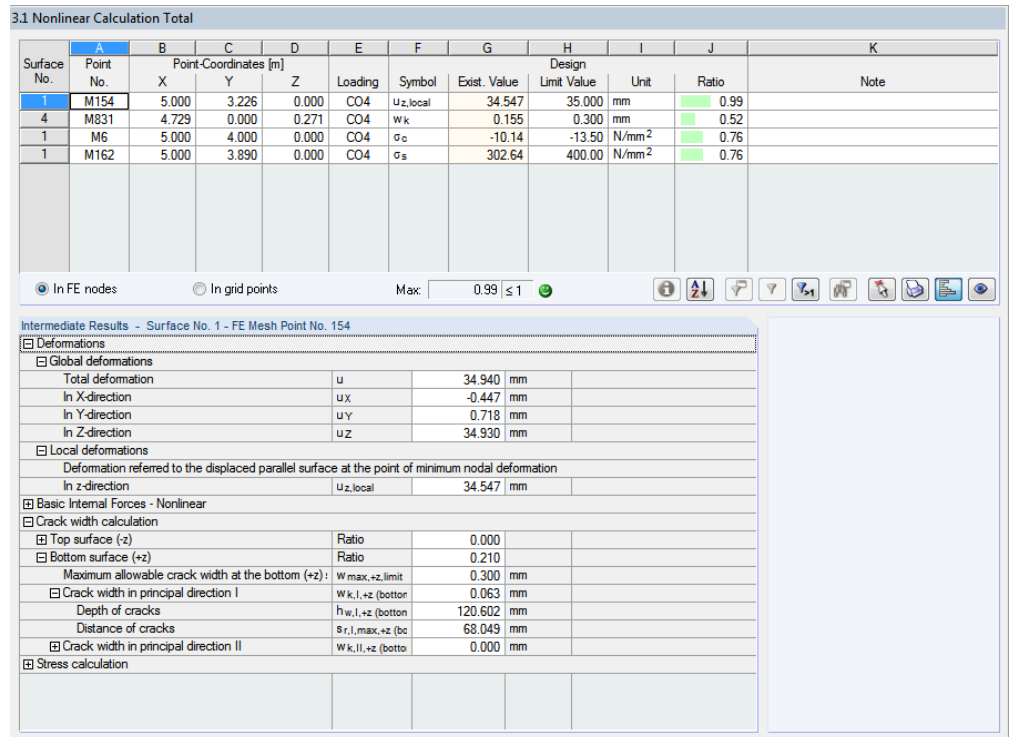


Figure 5.9: Window 3.1 *Nonlinear Calculation Total*

Figure 5.9 shows the results window of a nonlinear serviceability limit state design. The method of check is specified in the *Serviceability Limit State* tab of the 1.1 *General Data* window (see Figure 3.8, page 131).

The columns are described in chapter 5.4, page 169.

The symbols stand for the following types of checks:

Symbol	Design SLS
$u_{z,local}$	Deformation in cracked state (→ chapter 2.8.2.4, page 111) according to specifications in the window 1.3 <i>Surfaces</i>
w_k	Limitation of crack width (→ chapter 2.6.4.12, page 87) according to specifications in the 1.3 <i>Surfaces</i> window (see Figure 3.19, page 141)
σ_c	Limitation of concrete compression stress (→ chapter 2.6.4.7, page 77) according to specifications in the 1.3 <i>Surfaces</i> window (see Figure 3.20, page 144)
σ_s	Limitation of reinforcing steel stress (→ chapter 2.6.4.8, page 80) according to specifications in the 1.3 <i>Surfaces</i> window (see Figure 3.20, page 144)

Table 5.3: Serviceability limit state designs according to nonlinear method



The values of the deformations, crack widths, and stresses represent the results in cracked sections (state II).

The crack widths w_k in the intermediate results refer to the reinforcement directions. For example, the value for $w_{k,I,-z}$ (top) represents the crack width for the first direction of reinforcement on the top side of the surface; the crack runs perpendicular to the reinforcement direction 1.

5.8 Nonlinear Calculation by Surface

3.2 Nonlinear Calculation by Surface

Surface No.	Point No.	Point-Coordinates [m]			E	F	G	H	I	J	K
		X	Y	Z	Loading	Symbol	Exist. Value	Limit Value	Unit	Ratio	Note
1	M154	5.000	3.226	0.000	CO4	$u_{z,local}$	34.547	35.000	mm	0.99	
	M28	1.000	6.000	0.000	CO4	w_k	0.131	0.300	mm	0.44	
	M6	5.000	4.000	0.000	CO4	σ_c	-10.14	-13.50	N/mm ²	0.76	
	M162	5.000	3.890	0.000	CO4	σ_s	302.64	400.00	N/mm ²	0.76	
2	M81	9.500	3.000	0.000	CO4	$u_{z,local}$	11.044	60.000	mm	0.19	
	M677	9.876	5.378	0.000	CO4	w_k	0.106	0.300	mm	0.36	
	M677	9.876	5.378	0.000	CO4	σ_c	-5.49	-13.50	N/mm ²	0.41	
	M705	10.884	4.020	0.000	CO4	σ_s	96.73	400.00	N/mm ²	0.25	
3	M808	0.000	3.987	1.005	CO4	$u_{z,local}$	-1.492	40.000	mm	0.04	
	M778	0.000	3.483	0.513	CO4	w_k	0.042	0.300	mm	0.14	

☒ In FE nodes
 ☐ In grid points
 Max: 0.99 ≤ 1

Intermediate Results - Surface No. 1 - FE Mesh Point No. 28

☒ Deformations
☒ Basic Internal Forces - Nonlinear
☒ Crack width calculation

Top surface (+z)		Ratio	0.002
Maximum allowable crack width at the top (+z) surfa		$w_{max,-z,limit}$	0.300 mm
☐ Crack width in principal direction I		$w_{k,I,-z}$ (top)	0.000 mm
Depth of cracks		$h_{w,I,-z}$ (top)	0.000 mm
Distance of cracks		$s_{r,I,max,-z}$ (top)	0.000 mm
☐ Crack width in principal direction II		$w_{k,II,-z}$ (top)	0.001 mm
Bottom surface (+z)		Ratio	0.435
Maximum allowable crack width at the bottom (+z):		$w_{max,+z,limit}$	0.300 mm
☐ Crack width in principal direction I		$w_{k,I,+z}$ (bottom)	0.131 mm
Depth of cracks		$h_{w,I,+z}$ (bottom)	141.886 mm
Distance of cracks		$s_{r,I,max,+z}$ (bc)	178.740 mm
☐ Crack width in principal direction II		$w_{k,II,+z}$ (bottom)	0.000 mm

☒ Stress calculation

Figure 5.10: Window 3.2 Nonlinear Calculation by Surface

This window lists the maximum ratios of each designed surface that result in the serviceability limit state design. The columns are described in the chapters 5.4 and 5.7.

5.9 Nonlinear Calculation by Point

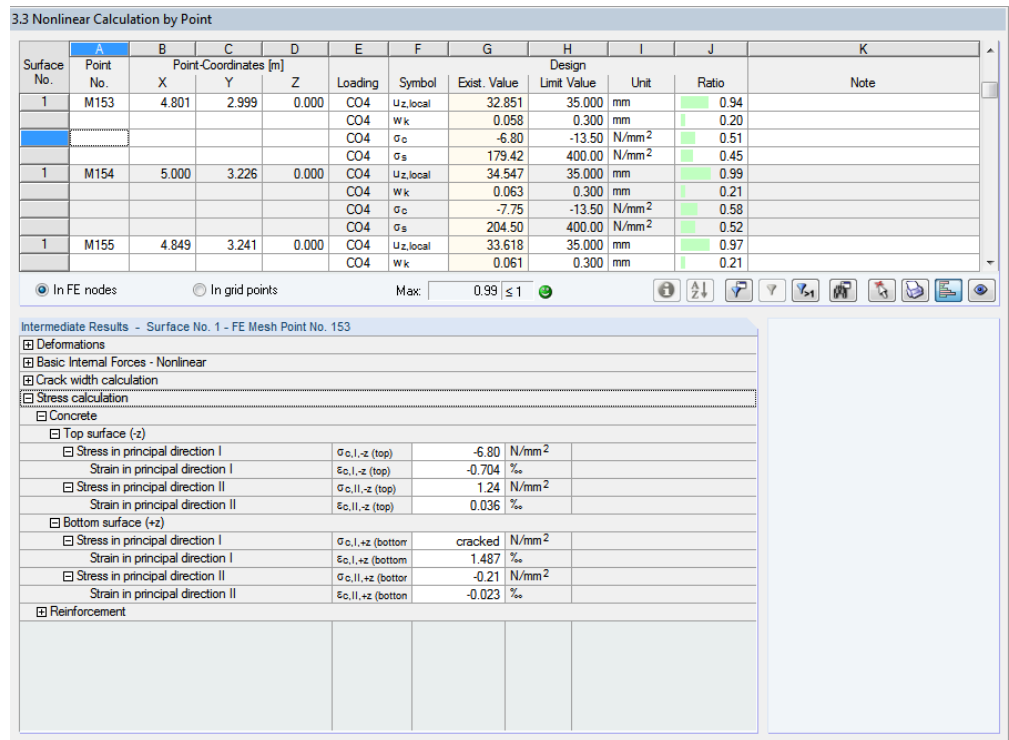


Figure 5.11: Window 3.3 Nonlinear Calculation by Point

☒ In FE nodes ☐ In grid points


This results window lists the maximum ratios for all FE nodes or grid points of each surface. The columns are described in the chapters 5.4 and 5.7.

The search function that you can start by clicking the button shown on the left helps you find FE nodes or grid points quickly (see Figure 6.7, page 182).

6. Results Evaluation

The design results can be evaluated in various ways. For this purpose, you can use the buttons below the tables.

3.2 Serviceability Checks by Surface

Surface No.	A Point No.	B Point-Coordinates [m] X	C Y	D Z	E Loading	F Symbol	G Exist. Value	H Design Limit Value	I Unit	J Ratio	K Note
1	M195	7.000	3.890	0.000	RC2	σ_c	-3.57	-13.50	N/mm ²	0.27	
	M162	5.000	3.890	0.000	RC2	σ_s	322.60	400.00	N/mm ²	0.81	
	M1	0.000	0.000	0.000	RC2	$\sigma_{s,min}$	3.35	3.85	cm ² /m	1.16	207)
	M162	5.000	3.890	0.000	RC2	lim d _s	9.3	9.9	mm	0.95	
	M6	5.000	4.000	0.000	RC2	lim s _i	0.110	0.117	m	0.94	
	M162	5.000	3.890	0.000	RC2	w _k	0.241	0.300	mm	0.81	
2	M3	9.500	6.000	0.000	RC2	σ_c	-2.59	-13.50	N/mm ²	0.20	
	M3	9.500	6.000	0.000	RC2	σ_s	0.00	400.00	N/mm ²	0.00	226)
	M3	9.500	6.000	0.000	RC2	$\sigma_{s,min}$	3.35	3.85	cm ² /m	1.16	207), 208)
	M3	9.500	6.000	0.000	RC2	lim d _s	8.8	0.0	mm	0.00	226)

☒ In FE nodes
☐ In grid points
Max: 1.16 > 1

Figure 6.1: Buttons for results evaluation

The buttons have the following functions:

Button	Description	Function
	Design Details	Opens the dialog box <i>Design Details</i> → chapter 6.1, page 177
	Sorts Results	Sorts the results by the maximum ratios (column J) or maximum values (column G) → chapter 6.3, page 182
	Filter	Opens the <i>Filter Points</i> dialog box to select FE nodes or grid points by certain criteria → chapter 6.3, page 182
	Only Designable Results	Hides the rows with the non-designable situations
	Exceeding	Shows only rows with a ratio > 1 (design not fulfilled)
	Find	Opens the <i>Find FE Node / Grid Point</i> dialog box to find a certain results row → chapter 6.3, page 182
	Surface Selection	Allows you to graphically select a surface to show its results in the table
	Print	Includes the intermediate results of the current FE node or grid point in printout report
	Show Color Bars	Shows or hides the colored reference scales in the results windows
	View Mode	Jumps to the RFEM work window to adjust the view

Table 6.1: Buttons in the results windows

6.1 Design Details



Click [Info] (available in all results windows) to check on the design details of the currently selected grid point, that is, the table row where the cursor is placed.

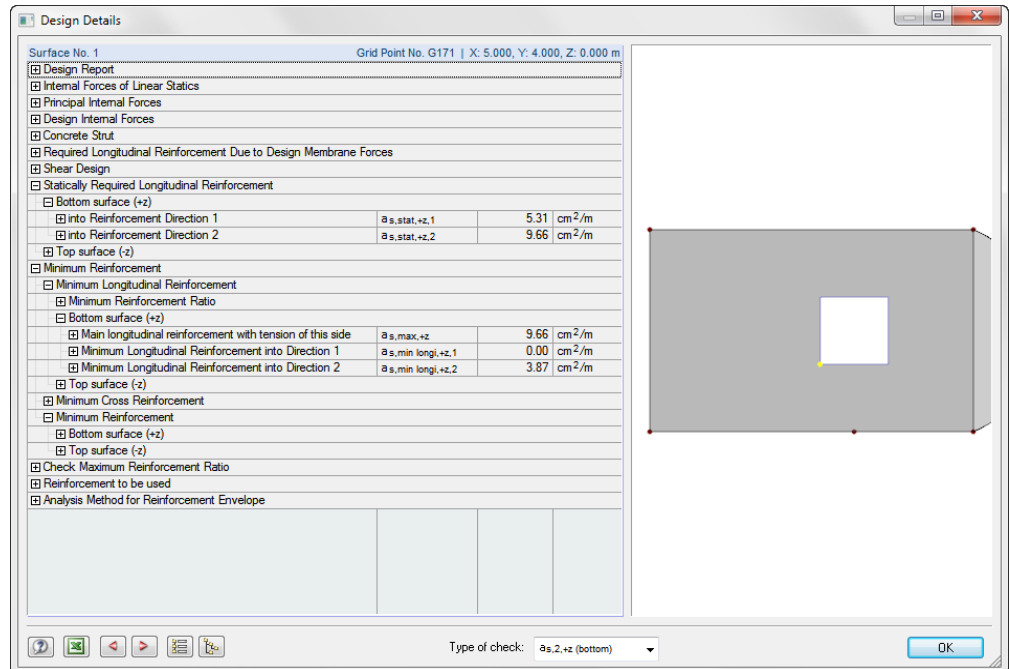


Figure 6.2: Dialog box *Design Details* for ultimate limit state design



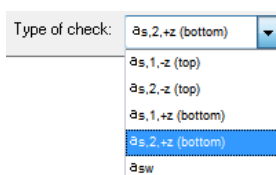
The design details are listed in a tree structure. You can expand or reduce the entries by clicking [+] or [-], respectively. The buttons shown on the left [Close] or [Open] the subentries in the directory tree, respectively.

The graphic on the right of the dialog box shows the location of the point in the model.

The following details are displayed for the ultimate limit state design (see chapter 2.5):

- Design report
- Internal forces of linear statics
- Principal internal forces
- Design internal forces
- Concrete strut
- Required longitudinal reinforcement
- Shear design
- Statically required longitudinal reinforcement
- Minimum reinforcement
- Check maximum reinforcement ratios
- Reinforcement to be used
- Analysis method for reinforcement envelope

The design details depend on the selected *Type of check*. Use this drop-down list at the bottom of the dialog box to select the results that you want to display.





In the serviceability limit state design, many detailed intermediate results are shown in the bottom part of the window (see Figure 5.6, page 169). Click [Info] to view the list of the design details that are available for the current point. This possibility is only available for results from the analytical method.

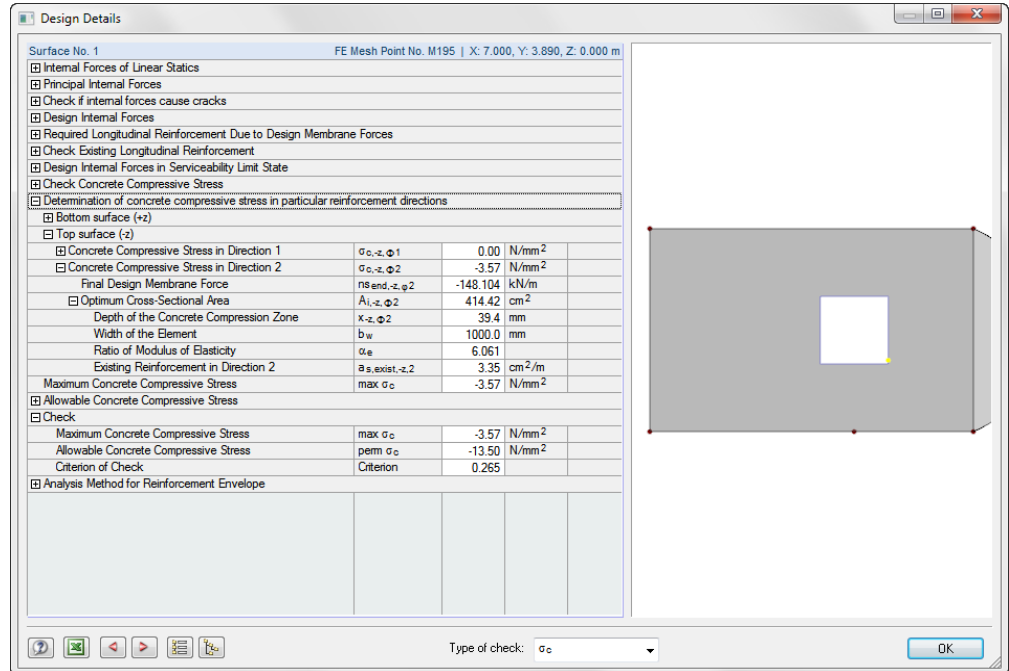
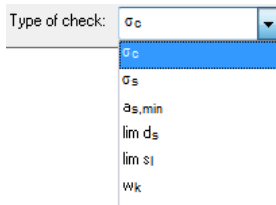


Figure 6.3: Dialog box *Design Details* for serviceability limit state design



A tree structure shows all design details relevant for each *Type of check*. Use this drop-down list at the bottom of the dialog box to select the results that you want to display.

Method of check	Type of check
Analytical	σ_c σ_s $a_{s,min}$ $\lim d_s$ $\lim s_l$ w_k
	} see Table 5.2, page 170

Table 6.2: *Type of check* for serviceability limit state designs



Click [◀] to go to the previous FE node or grid point. Click [▶] to select the next point.

6.2 Results on the RFEM Model

To evaluate results, you can also use the RFEM work window.

RFEM background graphic and view mode

The RFEM work window in the background helps you find the location of an FE node or grid point in the model. An arrow in the RFEM background graphic indicates the point selected in the results window of RF-CONCRETE Surfaces; the surface is highlighted in the selection color.

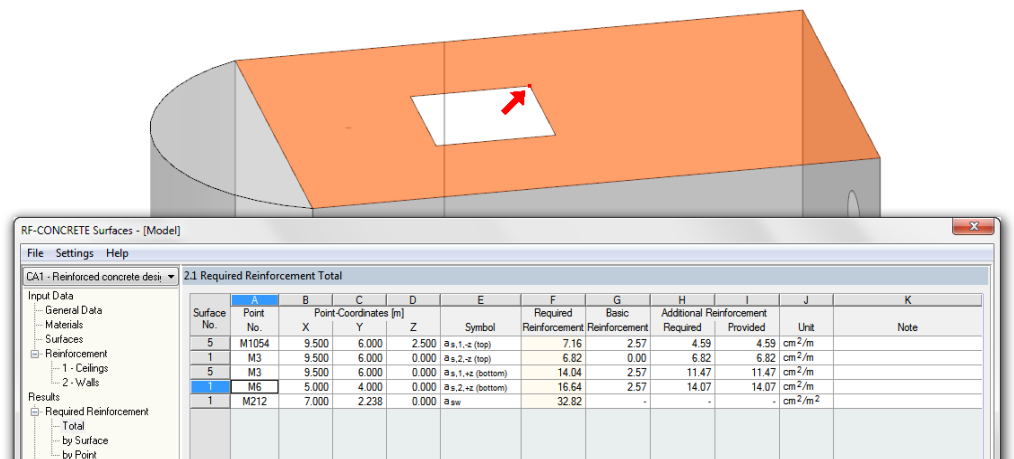


Figure 6.4: Highlighted surface and current FE node in the RFEM model

If you cannot improve the view by moving the RF-CONCRETE Surfaces module window, click [Jump to graphic to change view] to activate the *View Mode*: Now, the module window is hidden so that you can adjust the view in the RFEM work window. The view mode provides only the functions of the *View* menu, for example zooming, moving, or rotating the display. The arrow remains visible.

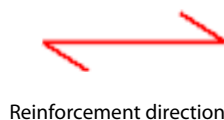
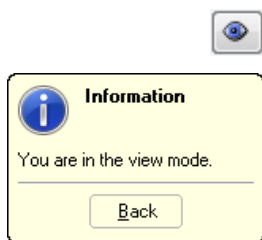
To return to RF-CONCRETE Surfaces, click [Back].

RFEM work window

You can also graphically check the reinforcements and design ratios in the RFEM model. Click [Graphic] to exit the design module. The work window of RFEM now shows all design results like the internal forces of a load case.

Results navigator

The *Results* navigator is adjusted to the RF-CONCRETE Surfaces add-on module: It allows you to graphically show the results of the longitudinal reinforcements for each reinforcement direction and layer, shear reinforcement, the design internal forces or the ratios, and detail results of the serviceability limit state designs.



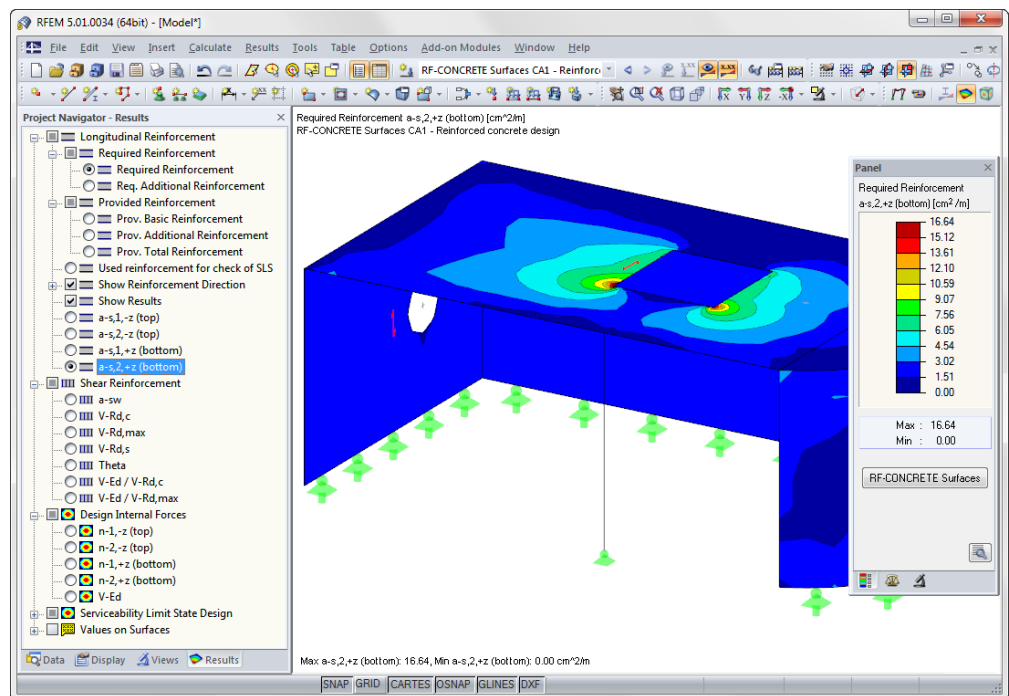


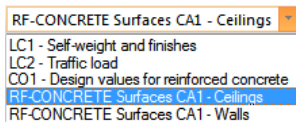
Figure 6.5: RFEM work window with *Results* navigator for RF-CONCRETE Surfaces



Similar to the display of internal forces, the [Show Results] button shows or hides the display of the design results.



Since the RFEM tables are of no relevance for the evaluation of the design results, you may hide them.



You can select the design cases in the drop-down list in the RFEM toolbar.

Panel

The panel providing the usual control options is also available for the results evaluation. The functions are described in the RFEM manual, chapter 3.4.6. In the second tab, you can set the *Display Factors* for the reinforcements, internal forces, or ratios. The third tab of the panel allows you to display the results of selected surfaces (see the RFEM manual, chapter 9.9.3):

Values on surfaces

You can use all possibilities provided by RFEM in order to display the result values of the reinforcements and ratios on the surfaces. This function is described in the RFEM manual, chapter 9.4. The following figure shows the *bottom (+z) reinforcement* that must be placed additionally to the specified basic reinforcement. The values are applied in the reinforcement directions 1 and 2, respectively.

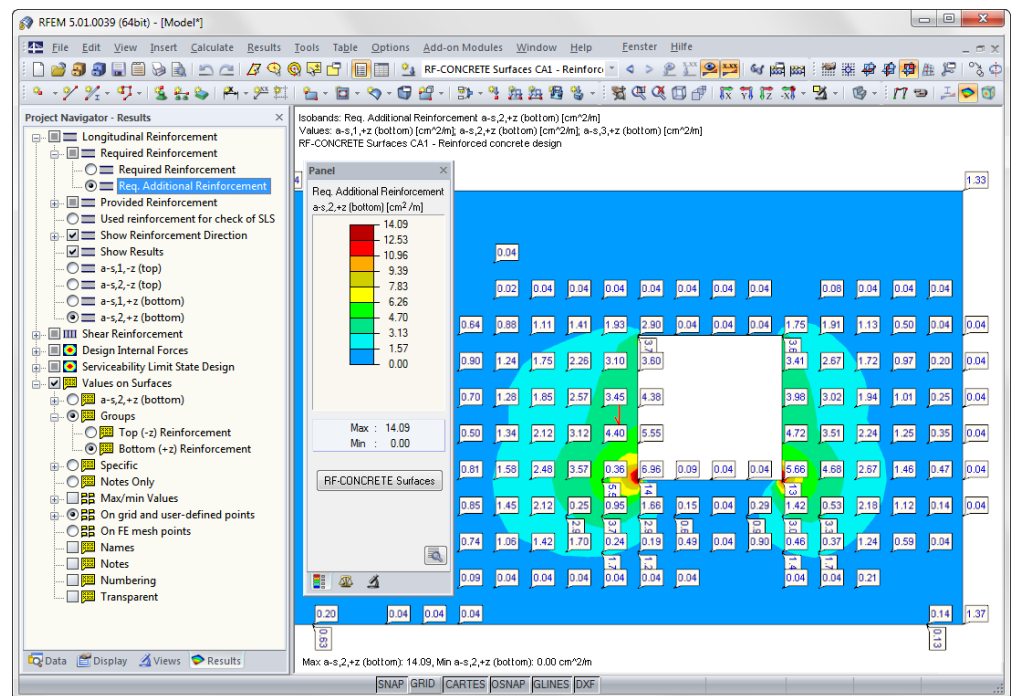


Figure 6.6: Group bottom (+z) reinforcement for required additional reinforcement

The graphics of the design results can be transferred to the printout report (see chapter 7.2, page 188).

To return to the add-on module click [RF-CONCRETE Surfaces].

RF-CONCRETE Surfaces

6.3 Filter for Results

The results windows of RF-CONCRETE Surfaces allow you to filter the results by various criteria. In addition, the module provides filter options to evaluate the design results graphically. These options are described in the RFEM manual, chapter 9.9.

For RF-CONCRETE Surfaces, you can also use the *Visibilities* (see RFEM manual, chapter 9.9.1) to filter the surfaces for the evaluation.

Likewise, you can use the *Sections* in the RFEM model or create new ones (see RFEM manual, chapter 9.6.1). This option allows you to evaluate the results more specifically. You can redistribute the reinforcement peaks that stem from singularities by using the smoothing function.

Find point

The results windows 2.2 and 2.3 (reinforcement) as well as 3.2 and 3.3 (serviceability) provide a search function for FE nodes and grid points. Click the button shown on the left (see Figure 6.1, page 176) to open the following dialog box.

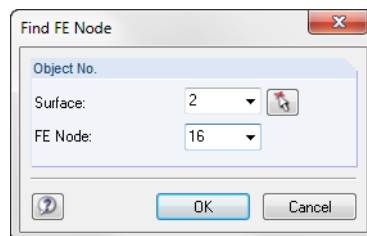


Figure 6.7: Dialog box *Find Grid Point*

First, enter the number of the surface manually or click [>] to select it graphically. Then, you can enter the number of the grid point or FE node, or select it in the list.

After clicking [OK], the current results window shows you the results row of this point.

Sort results

By default, window 3.1 and 3.2 show the results arranged by to the maximum design ratios: The sorting conforms to table column J.

You can also sort the results by existing values from column G. The greatest ratio of the deformation, for example, does not necessarily represent the maximum deformation because the limit values can be defined differently for each surface. Click the [Sort Results] button to switch between these two types of result arrangement.

Filter points

The button shown on the left is available in the results windows 2.2 and 2.3 (reinforcement) as well as 3.2 and 3.3 (serviceability). By clicking it, you open the following dialog box.

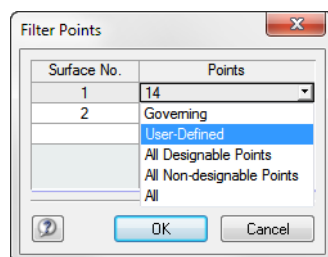
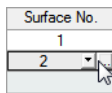


Figure 6.8: Dialog box *Filter Points*



In the *Surface No.* column, enter the number of the corresponding surface. Alternatively, you can select the surface graphically in the RFEM work window. To use this function, click into the input field, and then click [...].

The *Points* column offers several criteria for filtering. In addition to *All Designable* and *All Non-designable Points*, you can select the *Governing* points. These points provide the maximum reinforcement areas or ratios for the respective type of check. You can also enter the numbers of the points as *User-defined*.

Display only designable or non-designable results

The two buttons shown on the left allow you to display only designable results or only failed designs in the windows. Thus, you can, for example, hide failed designs due to singularities or analyze the causes for design problems.



Filtering comments in work window

The reinforcements and ratios can be used as filter criteria in the RFEM work window. To return to the RFEM work window, click [Graphics]. To apply this filter function, the panel must be displayed. If the panel is not active, select in the RFEM menu

View → Control Panel (Color Scale, Factors, Filter)

or use the toolbar button shown on the left.



The panel is described in the RFEM manual, chapter 3.4.6. You can change the filter settings for the results in the first panel tab (color scale).

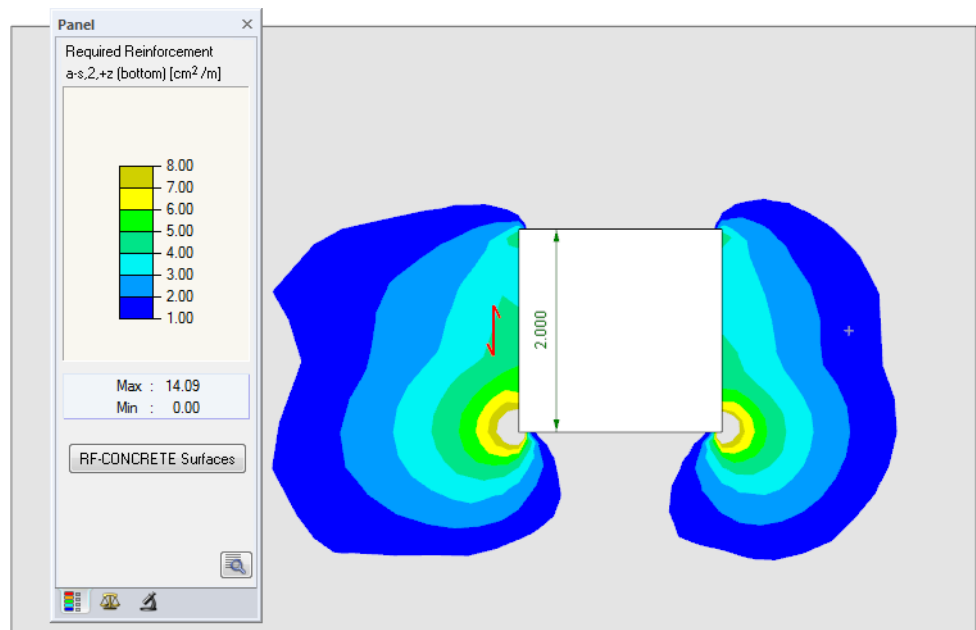


Figure 6.9: Filtering additional reinforcement with adjusted color spectrum

As the figure above shows, you can set the value scale of the panel in such a way that only reinforcements greater than 1.00 cm²/m are shown. The scale's color range is set in levels of 1.00 cm²/m; the maximum value of 8.00 cm²/m suppresses the effects of singularities.

The following chapter 6.4 describes how to adjust the value and color spectra to the diameters and spacings of the rebars.

The general control functions of RFEM are available for the graphical display of the grid point or FE node values. The options are described in detail in the RFEM manual, chapter 9.4.

Filtering surfaces in the work window



In the *Filter* tab of the control panel, you can enter the numbers of selected surfaces to display their result diagrams in a filtered display. The functions are described in detail in the RFEM manual, chapter 9.9.3.

Required Reinforcement a-s,2,+z (bottom) [cm²/m]
 RF-CONCRETE Surfaces CA1 - Reinforced concrete design

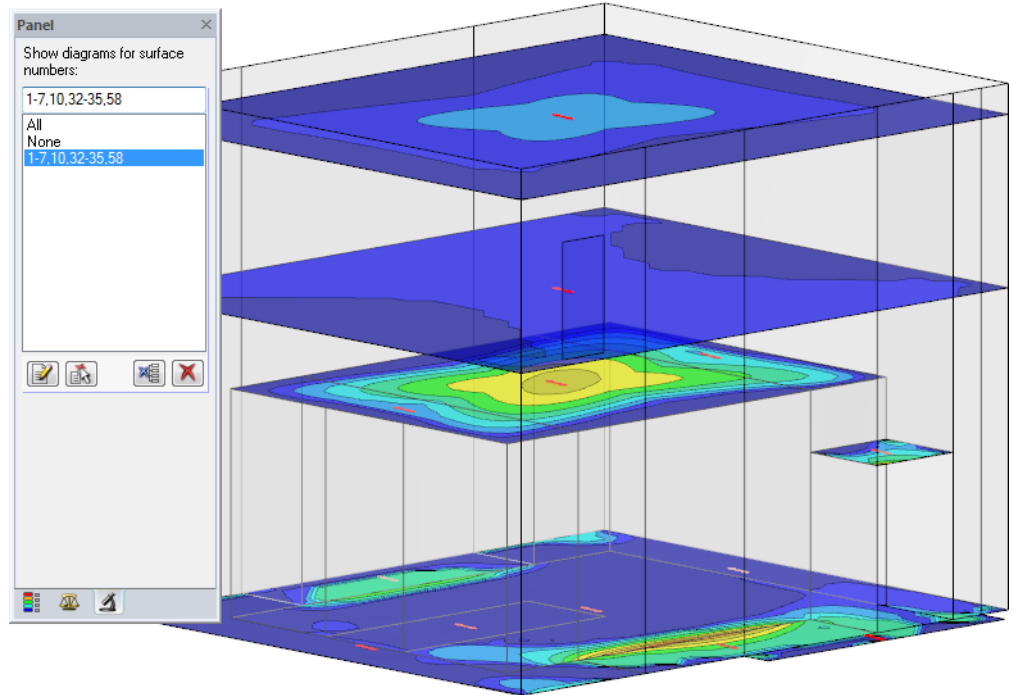


Figure 6.10: Surface filter for reinforcement of floor slabs and ceilings

In contrast to the visibility function, the model is displayed completely in the graphic. The figure above shows the reinforcement of the horizontal surfaces of a building. The remaining surfaces are shown in the model but are displayed without the reinforcements.

6.4 Configuring the Panel

The reinforcement results can be shown graphically as isobands or isolines. Twelve color zones are set by default for the value spectrum, covering a range between minimum and maximum value. You can also adjust the value spectrum regarding the definition of the reinforcement in order to prepare the graphical results for a reinforcement drawing.

To adjust the panel, double click one of the colors. You can also use the [Options] button available in the panel: The *Options* dialog box opens, where you can click the [Edit] button to access another dialog box for changing the ranges of colors and values.

In the *Edit Isoband Value and Color Spectra* dialog box, you can click [Edit] to open the *Edit Value Spectrum with Reinforcement Definition* dialog box.

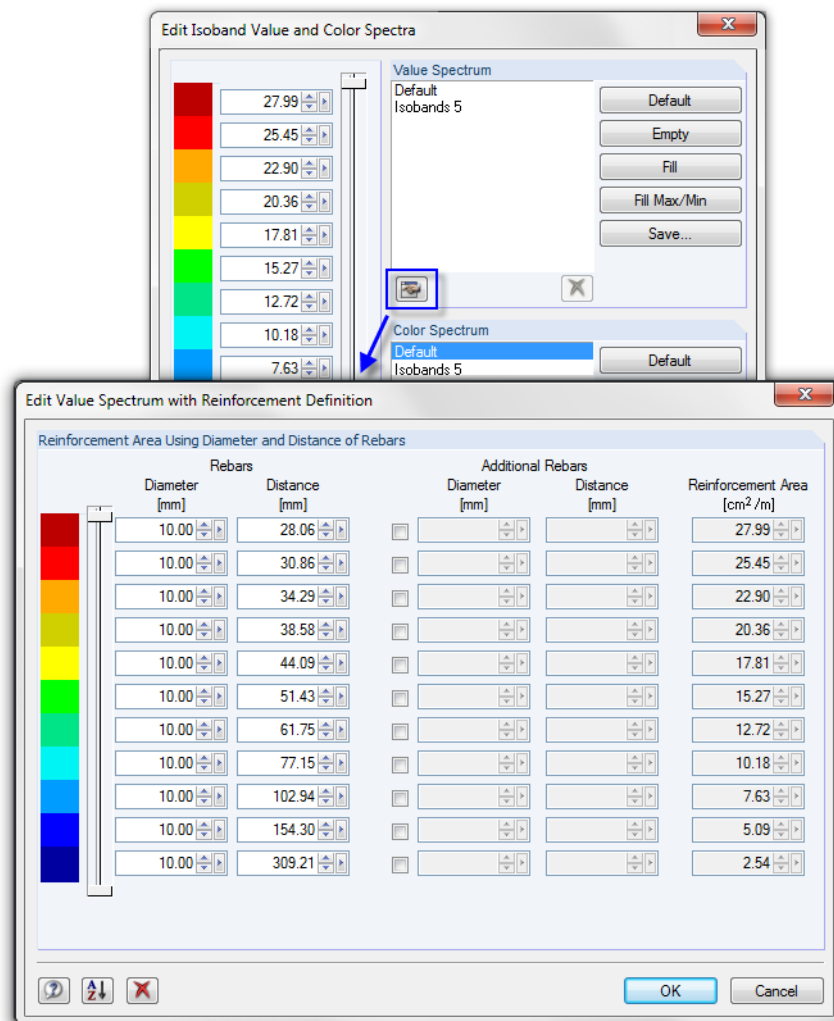
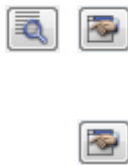


Figure 6.11: Dialog boxes *Edit Isoband Value and Color Spectra* and *Edit Value Spectrum with Reinforcement Definition*

This dialog box determines the reinforcement area per meter from the *Diameter* and the *Distance* of the rebars. In the *Additional Rebars* columns, you can assign further rebar diameters and distances (see Figure 6.12). Thus, you can set user-defined value spectra for the reinforcement, which can only be used for the reinforcement drawing.

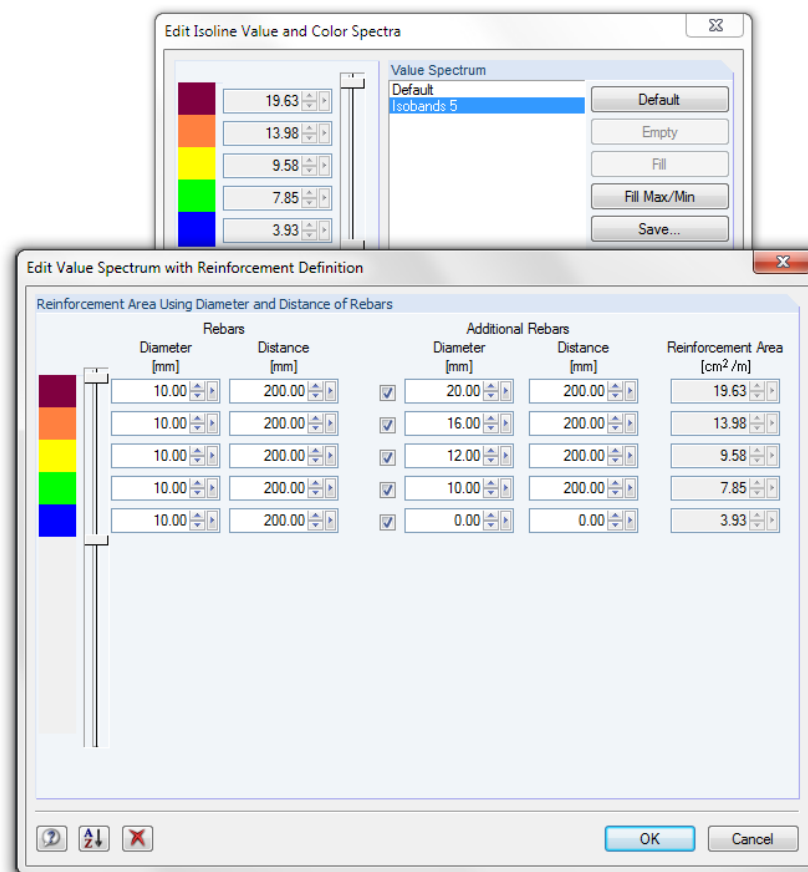


Figure 6.12: Dialog box *Edit Value Spectrum with Reinforcement Definitions* with rebar diameters and distances

Click [OK] to import the reinforcement areas resulting from the defined rebar diameters and rebar distances in the *Edit Isoband Value and Color Spectra* dialog box.

In the panel, the diameters of the rebars with the according distances that are to be specified for the individual value ranges.

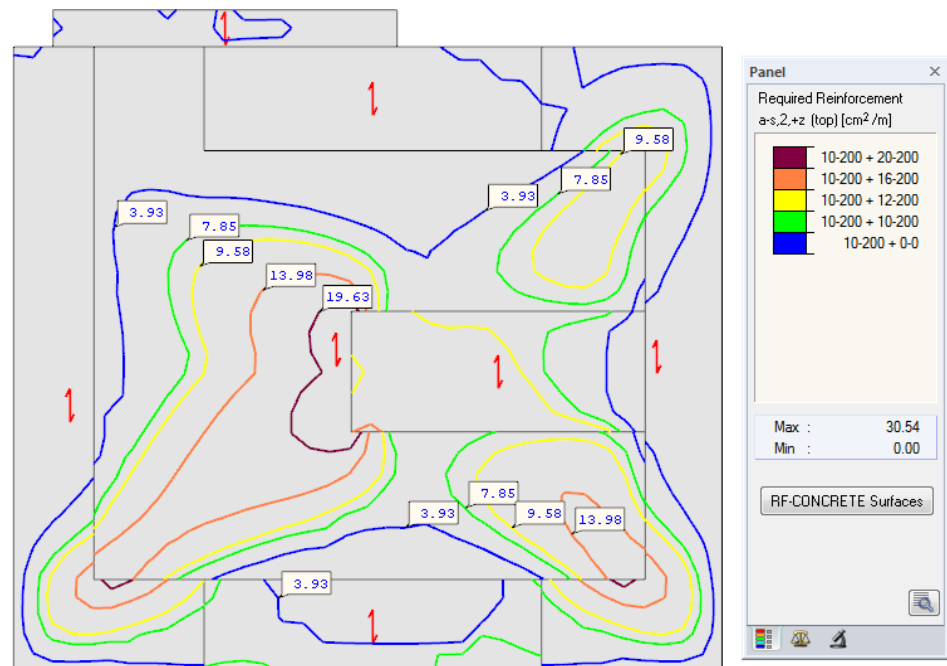


Figure 6.13: Graphic and panel with user-defined reinforcement zones

7. Printout

7.1 Printout Report

The creation of printouts for the data of RF-CONCRETE Surfaces is similar to the procedure in RFEM: The program generates a printout report for the RF-CONCRETE Surfaces results; graphics and descriptions can be added subsequently. The selection in the printout report controls the design module data that will finally appear in the printout.



The printout report is described in the RFEM manual. Chapter 10.1.3.4 *Selecting Data of Add-on Modules* describes how the input and output data can be prepared for the printout.

A special selection option is available for the intermediate results of serviceability limit state designs according to the analytical method. In the *Points* column, you can select all designable, all non-designable, or only the *Governing* points: These points provide the greatest reinforcement areas or ratios. You can also enter *User-defined* point numbers.

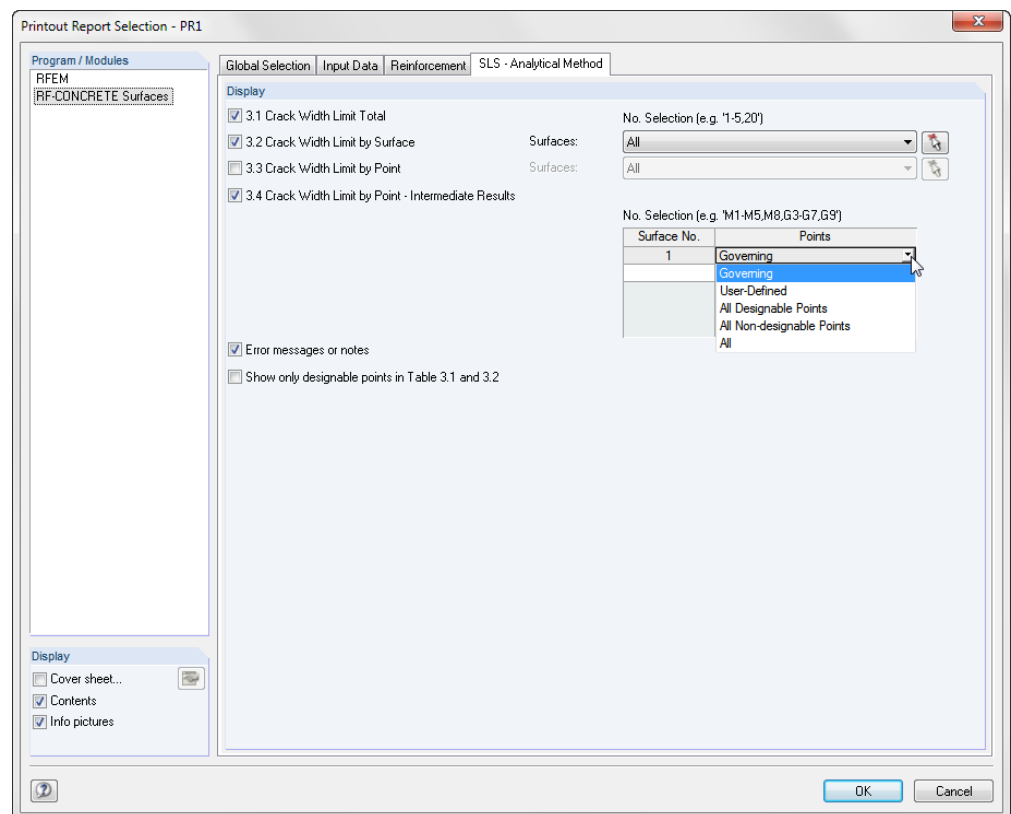


Figure 7.1: Dialog box *Printout Report Selection*, tab *SLS - Analytical Method*

For complex structural systems with a high number of design cases, it is recommended to split the data into several printout reports to make data arrangement clearer.

7.2 Graphic Printout

In RFEM, you can transfer every figure shown in the work window to the printout report or directly to the printer. Thus, you can also prepare the reinforcements and ratios shown in the RFEM model for the printout report.

Printing graphics is described in chapter 10.2 of the RFEM manual.

Analyses on RFEM model

To print the current graphic of the design ratios, click

File → Print Graphic

or use the toolbar button shown on the left.

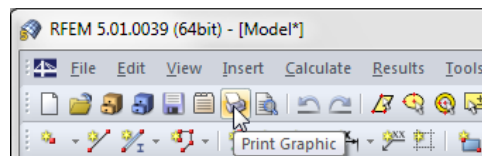


Figure 7.2: Button *Print Graphic* in RFEM toolbar

The following dialog box appears.

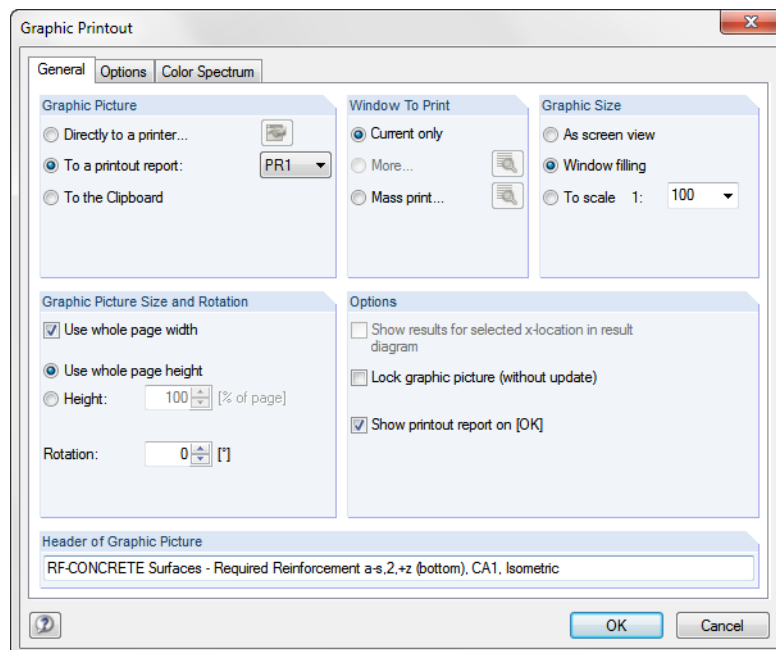
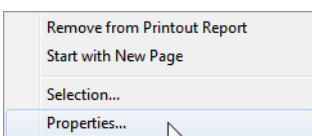
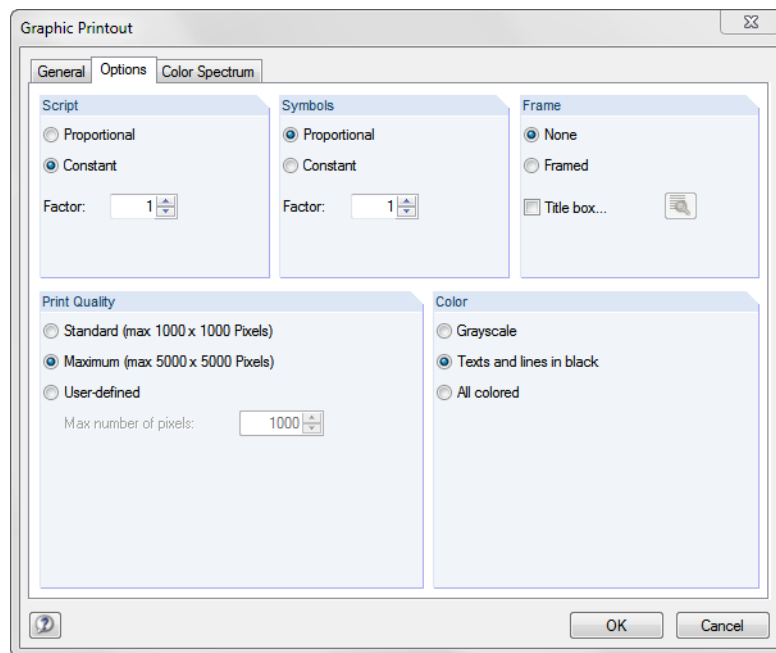


Figure 7.3: Dialog box *Graphic Printout*, tab *General*

The dialog box is described in detail in the RFEM manual, chapter 10.2. You can move a graphic that has been integrated in the printout report within the report by using the drag-and-drop function.

To adjust a graphic subsequently in the printout report, right-click the corresponding entry in the report navigator. Click the *Properties* option on the context menu to open the *Graphic Printout* dialog box where you can adjust the relevant options.



Figure 7.4: Dialog box *Graphic Printout*, tab *Options*

8. General Functions

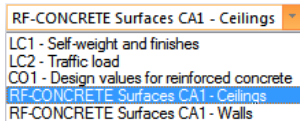
This chapter describes some useful menu functions as well as export options for the designs.

8.1 Design Cases

Design cases allow you to group surfaces for the designs or to design different variants (for example, modified materials of reinforcement specifications, nonlinear analysis).

It is no problem to analyze a surface in different design cases.

The design cases of RF-CONCRETE Surfaces can also be selected in RFEM by using the load case list in the toolbar.



Creating new design case

To create a new design case in RF-CONCRETE Surfaces, click

File → New Case.

The following dialog box appears:

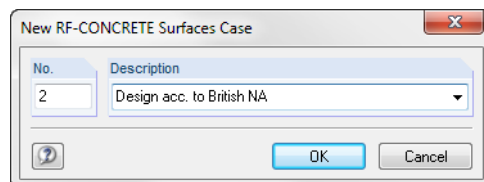


Figure 8.1: Dialog box *New RF-CONCRETE Surfaces Case*

In this dialog box, you enter a *No.* (one that is not yet assigned) for the new design case. A *Description* will make the selection in the load case list easier.

After you click [OK], the 1.1 *General Data* window opens where you can enter the new design data.

Renaming design case

To change the description of a design case, click

File → Rename Case.

The following dialog box appears:

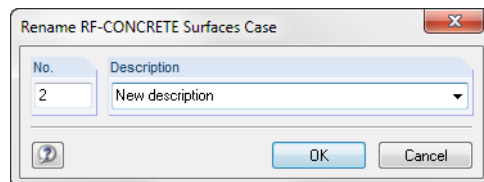


Figure 8.2: Dialog box *Rename RF-CONCRETE Surfaces Case*

Here, you can specify a different *Description* but also a different *No.* for the design case.

Copying design case

To copy the input data of the current design case, , click

File → Copy Case.

The following dialog box appears:

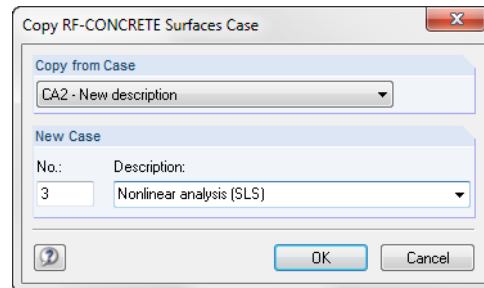


Figure 8.3: Dialog box *Copy RF-CONCRETE Surfaces Case*

A new *No.* and, if necessary, a new *Description* must be specified for the new case.

Deleting design cases

To delete a design case in RF-CONCRETE Surfaces, click

File → Delete Case.

The following dialog box appears:

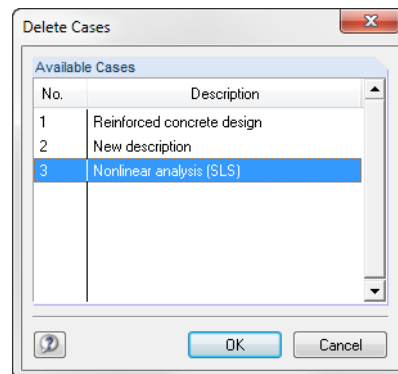


Figure 8.4: Dialog box *Delete Case*

You can select the design case in the *Available Cases* list. To delete the selected case, click [OK].

8.2 Units and Decimal Places

The units and decimal places for RFEM and the add-on modules are managed in one dialog box. In the RF-CONCRETE Surfaces add-on module, you can open the dialog box to define the units by clicking

Settings → Units and Decimal Places.

The dialog box known from RFEM appears. RF-CONCRETE Surfaces is preset in the *Program / Module* list.

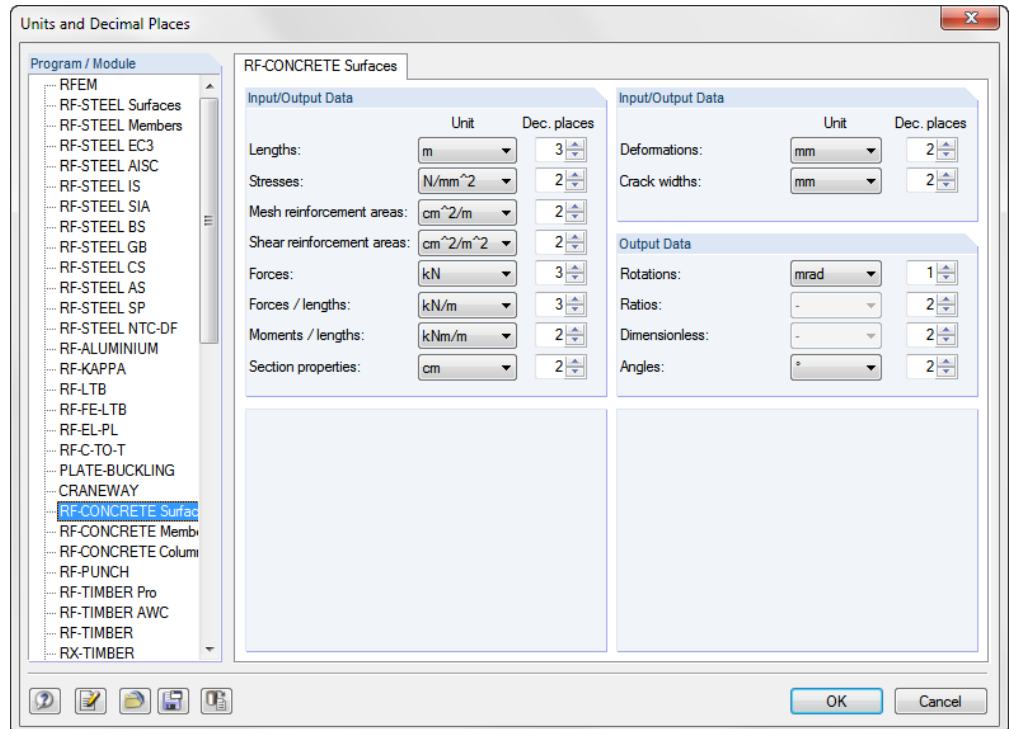


Figure 8.5: Dialog box *Units and Decimal Places*



The settings can be saved as user profile to reuse them in other models. These functions are described in the RFEM manual, chapter 11.1.3.

8.3 Export of Results

The results of RF-CONCRETE Surfaces can also be used in other programs.

Clipboard

To copy cells selected in the results windows of RF-CONCRETE Surfaces to the Clipboard, press [Ctrl]+[C]. To insert the cells, for example in a word processing program, press [Ctrl]+[V]. The headers of the table columns are not transferred.

Printout Report

The data of RF-CONCRETE Surfaces can be printed into the global printout report (see chapter 7.1, page 187), and then exported. In the printout report, click

File → Export in RTF.

The function is described in the RFEM manual, chapter 10.1.11.

Excel / OpenOffice

The module RF-CONCRETE Surfaces provides a function for the direct data export to MS Excel, OpenOffice.org Calc, or in the CSV format. To open the corresponding dialog box, click

File → Export Tables.

The following export dialog box appears.

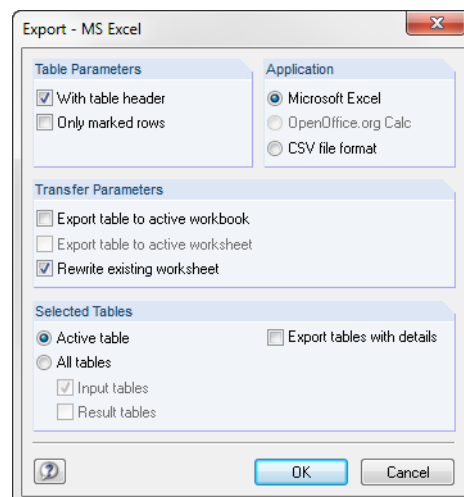


Figure 8.6: Dialog box *Export - MS Excel*

If your selection is complete, click [OK] to start the export. Excel or OpenOffice are started automatically, that is, you do not need to open the programs first.

Excel table for RF-CONCRETE Surfaces.xlsx - Microsoft Excel

File

Home

Insert

Page Layout

Formulas

Data

Review

View

Add-Ins

Figure 8.7: Results in Excel

CAD programs

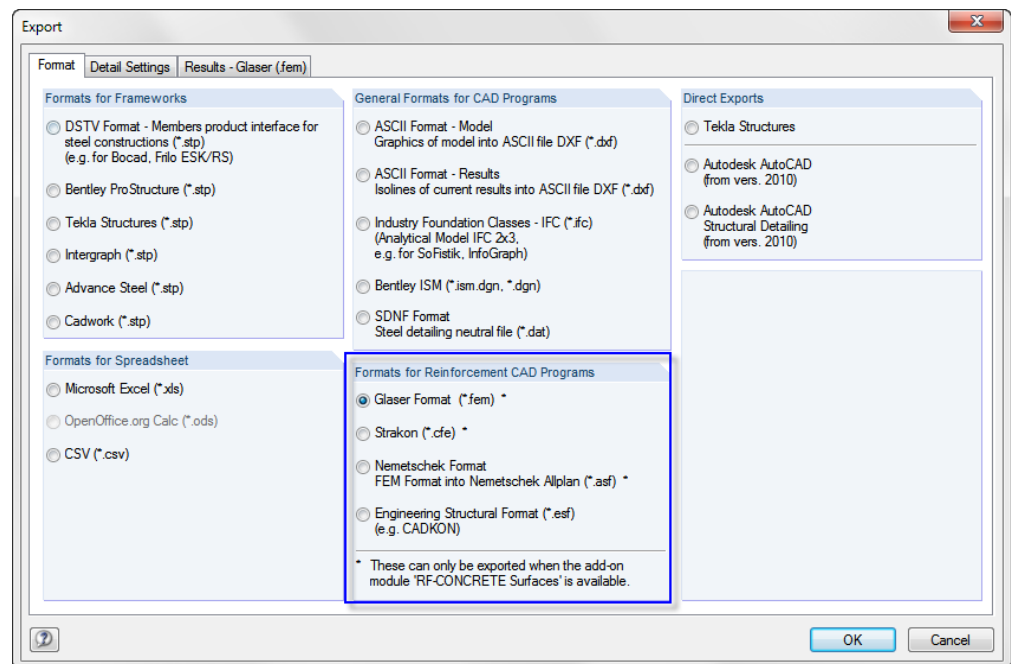
The reinforcement areas determined in RF-CONCRETE Surfaces can also be used in CAD programs. RFEM provides interfaces to the following programs:

- Nemetschek (FEM format for Allplan *.asf)
- Glaser (Format *.fem)
- Strakon (Format *.cfe)

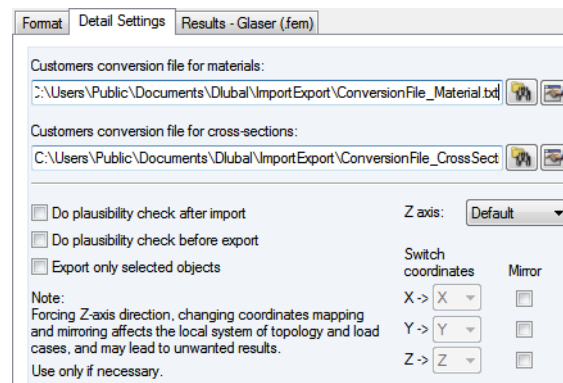
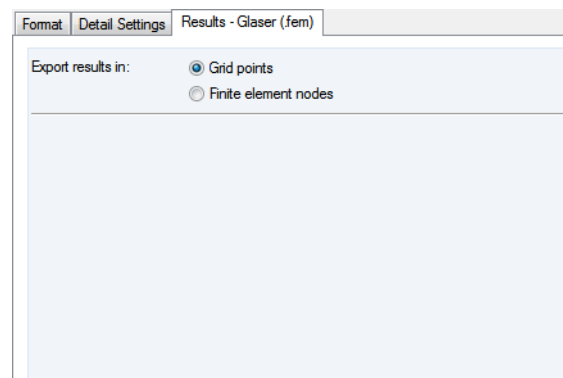
To use this export function in RFEM, click

File → Export.

The *Export* dialog box appears where you can select the relevant interface. The dialog box is described in detail in the RFEM manual, chapter 12.5.2

Figure 8.8: RFEM dialog box *Export*, tab *Format*

Depending on the interface, further tabs with specific settings are available for the export of reinforcements.

Figure 8.9: RFEM dialog box *Export*, tab *Details*Figure 8.10: RFEM dialog box *Export*, tab *Results - Glaser (*.fem)*

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